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Effects of Viscous Dissipation and Concentration Based Internal Heat Source on Convective Instability in a Porous Medium with Throughflow

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Abstract—Linear stability analysis of double diffusive convection in a horizontal porous layer saturated with fluid is examined by considering the effects of viscous dissipation, concentration based internal heat source and vertical throughflow. The basic steady state solution for Governing equations is derived. Linear stability analysis has been implemented numerically by using shooting and Runge-kutta methods. Critical thermal Rayleigh number Ra_c is obtained for various values of solutal Rayleigh number Sa, vertical Peclet number Pe, Gebhart number Ge, Lewis number Le and measure of concentration based internal heat source γ. It is observed that Ge has destabilizing effect for upward throughflow and stabilizing effect for downward throughflow. And γ has considerable destabilizing effect for upward throughflow and insignificant destabilizing effect for downward throughflow.

Keywords—Porous medium, concentration based internal heat source, vertical throughflow, viscous dissipation.

I. Introduction

DOUBLE diffusive convection in a fluid saturated porous media is a subject extensively investigated over last the few decades since it has many geophysical, engineering, biological applications which includes energy storage and recovery, nuclear waste disposal, insulation of buildings etc.

Study of viscous dissipation effect is noteworthy in natural convection process in a variety of devices which are subject to stronger gravitational field, high speed of rotation, high mass flow rate or larger length scale problems. Effect of viscous dissipation on convection was analysed in [1], [2] and [3]. Viscous dissipation effect on thermal convection in an inclined porous layer is studied in [4]. Thermosolutal convection with the inclusion of viscous heating contribution has been examined in [5]. From the above works they have concluded that instabilities caused by viscous dissipation might arise even there is no temperature gradient in the vertical direction of the porous layer. All the progresses in this area of research have been included in the book [6].

The study of vertical throughflow is important since it alters dimensionless temperature gradient across the porous layer. By altering throughflow, there is a chance to regulate the convective instability. In [7] throughflow effect on convection in superposed fluid and porous layer is studied. Throughflow effect on thermal convective instability has been investigated in [8], whereas this effect on double diffusive convection is provided in [9]. Vertical throughflow effect on thermal convection under the consideration of viscous dissipation is analyzed in [10], where as this throughlow effect has been studied in [11] with the assumption of a composite porous medium containing two horizontal porous layers.

Convection with internal heat source is studied by several researchers [12], [13]. This is similar to the model of earth's mantle which is heated internally by radioactive material. This type of convection is little complicated because internal heat being generated, strongly depends on the vertical motion. Convective instability in a porous layer including internal heat source with non uniform boundary conditions is discussed in [14]. Thermosolutal convection with concentration based internal heat source by using linear and nonlinear stability theories have been analyzed in [15] and it extended to [16] with the use of operative method to get sharp thresholds.

In the present article, double diffusive convection in a homogeneous porous layer is examined by taking into account of concentration based internal heat source, viscous heating contribution and vertical throughflow.

II. Mathematical Formulation

The basic model consists of a fluid saturated homogeneous porous layer with height H and extended up to infinity in the horizontal directions. $Ox^*y^*z^*$ be the Cartesian frame of reference such that y^*-axis to be in vertical direction. Porous layer is supposed to be confined between two permeable isothermal planes $y^* = 0$ and $y^* = H$. Temperature and concentration at the lower plane be ${T_0}^*$, ${C_0}^*$ and at the upper plane be ${T_1}^*$, ${C_1}^*$. Oberbeck-boussinesq approximation and Darcy law are valid. Viscous heating contribution in the energy balance is considered. The medium is heated due to internal heat source which varies linearly with concentration. The governing equations in dimensional form are

$$\nabla^* \cdot \mathbf{v}^* = 0, \tag{1}$$

$$\frac{\mu}{K}\mathbf{v}^* = -\nabla^* P^* + {\rho_f}^* g, \tag{2}$$

$$\sigma\frac{\partial T^*}{\partial t^*} + \mathbf{v}^* \cdot \nabla^* T^* = \alpha {\nabla^*}^2 T^* + \frac{\nu}{Kc}\mathbf{v}^* \cdot \mathbf{v}^* + \beta(C^* - {C^*}_0), \tag{3}$$

$$\phi\frac{\partial C^*}{\partial t^*} + \mathbf{v}^* \cdot \nabla^* C^* = D{\nabla^*}^2 C^*, \tag{4}$$

where $\mathbf{v}^*$ be the Darcy velocity, P^* be the pressure, ${\rho_f}^*$ be the fluid density. β is a proportionality constant of internal

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heat source. T^* and C^* the temperature and concentration, respectively. μ, K, ϕ, c, α, and D stands for viscosity, permeability of the medium, porosity, specific heat, thermal diffusivity and solutal diffusivity, respectively. Assuming throughflow in the vertical direction, the boundary conditions may be taken as

$$y^* = 0: \quad v^* = {v_0}^*, \quad T^* = {T_1}^*, \quad C^* = {C_1}^*,$$
$$y^* = H: \quad v^* = {v_0}^*, \quad T^* = {T_0}^*, \quad C^* = {C_0}^*. \tag{5}$$

Introducing dimensionless quantities

$$(x, y, z) = \frac{1}{H}(x^*, y^*, z^*), \; t = \frac{\alpha}{\sigma H^2} t^*,$$
$$(u, v, w) = \mathbf{v} = \frac{H}{\alpha}\mathbf{v}^*, \; P = \frac{K(P^* + \rho_0 g y^*)}{\mu\alpha},$$
$$T = \frac{T^* - {T_0}^*}{{T_1}^* - {T_0}^*}, \; C = \frac{C^* - {C_0}^*}{{C_1}^* - {C_0}^*}, \; \sigma = \frac{(\rho c)_m}{(\rho c_p)_f},$$

Equations (1)-(5) take the dimensionless form

$$\nabla \cdot \mathbf{v} = 0, \tag{6}$$
$$\mathbf{v} = -\nabla P + [RaT + \frac{1}{Le} SaC]\hat{\mathbf{j}}, \tag{7}$$
$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + \frac{Ge}{Ra}\mathbf{v} \cdot \mathbf{v} + \gamma C, \tag{8}$$
$$\frac{\phi}{\sigma}\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = \frac{1}{Le}\nabla^2 C, \tag{9}$$

and boundary conditions

$$y = 0: \quad v = Pe, \quad T = 1, \quad C = 1,$$
$$y = 1: \quad v = Pe, \quad T = 0, \quad C = 0. \tag{10}$$

where

$$Ra = \frac{g_0 \beta_T H K({T_1}^* - {T_0}^*)}{\nu\alpha}, \; Sa = \frac{g_0 \beta_C H K({C_1}^* - {C_0}^*)}{\nu D},$$
$$Le = \frac{\alpha}{D}, \; Pe = \frac{{v_0}^* H}{\alpha}, \; Ge = \frac{g\beta H}{c}, \; \gamma = \frac{\beta L^2({C_1}^* - {C_0}^*)}{\alpha({T_1}^* - {T_0}^*)}.$$

Here γ, Ra, Sa, Pe, Le, and Ge are the dimensionless coefficient of internal heat generation induced by radiation absorbing concentrate, thermal Rayleigh number, solutal Rayleigh number, vertical Peclet number, Lewis number, and Gebhart number, respectively.

Basic steady state solution for (6)-(10) is given by

$$u_B = 0, v_B = Pe, w_B = 0,$$
$$C_B = \frac{e^{LePe}}{(e^{LePe} - 1)} - \frac{1}{(e^{LePe} - 1)} e^{LePey},$$
$$\begin{aligned} T_B = {} & A_2 + B_2 e^{Pey} \\ & + \left[\frac{Ge}{Ra} Pey + \frac{\gamma e^{LePe}}{Pe(e^{LePe} - 1)} y - \frac{\gamma e^{LePey}}{LePe^2(e^{LePe} - 1)}\right] \\ & + \left[\frac{Ge}{Ra} + \frac{\gamma e^{LePe}}{Pe^2(e^{LePe} - 1)} + \frac{\gamma e^{LePey}}{Pe^2(Le - 1)(e^{LePe} - 1)}\right], \end{aligned} \tag{11}$$

where

$$\begin{aligned} A_2 = {} & \frac{\gamma e^{LePe}}{(e^{Pe} - 1)(e^{LePe} - 1)Pe^2}\left[1 + \frac{1}{(Le - 1)} - \frac{1}{Le} + Pe^2\right] \\ & - \frac{\gamma e^{Pe}}{(e^{LePe} - 1)Pe^2}\left[e^{LePe} + \frac{1}{(Le - 1)} - \frac{1}{Le}\right] \\ & + \frac{Ge(1 + Pe)}{Ra(e^{Pe} - 1)} - e^{Pe}\left(\frac{Ge}{Ra} - 1\right). \end{aligned}$$

$$\begin{aligned} B_2 = {} & -\frac{1}{(e^{Pe} - 1)}\left[\frac{GePe}{Ra} + 1\right] \\ & - \frac{\gamma}{(e^{Pe} - 1)Pe}\left[\frac{1}{(Le - 1)LePe} + \frac{e^{LePe}}{(e^{LePe} - 1)}\right], \end{aligned}$$

$$\begin{aligned} \frac{dT_B}{dy} = {} & \left[\frac{Ge}{Ra} Pe + \frac{\gamma e^{LePe}}{Pe(e^{LePe} - 1)} - \frac{\gamma e^{LePey}}{Pe(e^{LePe} - 1)}\right] \\ & - \frac{e^{Pey} Pe}{(e^{Pe} - 1)}\left[\frac{-\gamma}{(Le - 1)LePe^2} + 1 + \frac{GePe}{Ra}\right] \\ & + \frac{\gamma}{(e^{LePe} - 1)}\left[-\frac{e^{LePe} e^{Pey}}{(e^{Pe} - 1)} + \frac{e^{LePey} Le}{Pe(Le - 1)}\right]. \end{aligned}$$

$$\frac{dC_B}{dy} = \frac{-LePe}{(e^{LePe} - 1)} e^{LePey}. \tag{12}$$

In (11)-(12), $Pe = 0$ is a singular point. The basic steady state solution when $Pe = 0$ is

$$u_B = 0, \quad v_B = 0, \quad w_B = 0, \quad C_B = 1 - y,$$
$$T_B = \gamma\left[\frac{y^3}{6} - \frac{y^2}{2}\right] + \left[\frac{\gamma}{3} - 1\right] y + 1. \tag{13}$$

III. Linear Stability Analysis

To examine the stability of basic steady state solution, the following perturbations are introduced.

$$u = u_B + \epsilon U, v = v_B + \epsilon V, w = w_B + \epsilon W,$$
$$T = T_B + \epsilon\theta, C = C_B + \epsilon\Phi. \tag{14}$$

Substituting (14) in (6)-(9), and neglecting the terms of order ϵ^2, we get

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0, \tag{15}$$
$$\frac{\partial W}{\partial y} - \frac{\partial V}{\partial z} = -\left[Ra\frac{\partial\theta}{\partial z} + \frac{1}{Le} Sa\frac{\partial\Phi}{\partial z}\right], \tag{16}$$
$$\frac{\partial U}{\partial z} - \frac{\partial W}{\partial x} = 0, \tag{17}$$
$$\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} = \left[Ra\frac{\partial\theta}{\partial x} + \frac{1}{Le} Sa\frac{\partial\Phi}{\partial x}\right], \tag{18}$$
$$\frac{\partial\theta}{\partial t} + V\frac{\partial T_B}{\partial y} + Pe\frac{\partial\theta}{\partial y} = \nabla^2\theta + 2\frac{Ge}{Ra} PeV + \gamma\Phi, \tag{19}$$
$$\frac{\phi}{\sigma}\frac{\partial\Phi}{\partial t} + V\frac{\partial C_B}{\partial y} + Pe\frac{\partial\Phi}{\partial y} = \frac{1}{Le}\nabla^2\Phi, \tag{20}$$
$$y = 0: \quad V = 0, \quad \theta = 0, \quad \Phi = 0,$$
$$y = 1: \quad V = 0, \quad \theta = 0, \quad \Phi = 0. \tag{21}$$

Suppose the disturbances are transverse rolls i.e. z independent. We look for solutions of (15)-(21) such that

$$U = U(x, y, t), \quad V = V(x, y, t), \quad W = 0,$$
$$\theta = \theta(x, y, t), \quad \Phi = \Phi(x, y, t). \tag{22}$$

Introducing stream function such that (15) satisfies

$$U = \frac{\partial \Psi}{\partial y}, V = -\frac{\partial \Psi}{\partial x}, \tag{23}$$

Substituting (23) in to (16)-(21), the following equations are obtained.

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\left[Ra\frac{\partial \theta}{\partial x} + \frac{1}{Le}Sa\frac{\partial \Phi}{\partial x}\right], \tag{24}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \Psi}{\partial x}\frac{\partial T_B}{\partial y} + Pe\frac{\partial \theta}{\partial y} = \nabla^2\theta - 2\frac{Ge}{Ra}Pe\frac{\partial \Psi}{\partial x} + \gamma\Phi, \tag{25}$$

$$\frac{\phi}{\sigma}\frac{\partial \Phi}{\partial t} - \frac{\partial \Psi}{\partial x}\frac{\partial C_B}{\partial y} + Pe\frac{\partial \Phi}{\partial y} = \frac{1}{Le}\nabla^2\Phi, \tag{26}$$

$$y = 0, 1: \quad \Psi = 0, \quad \theta = 0, \quad \Phi = 0. \tag{27}$$

To obtain the solutions of (24)-(27), considering the following plane waves

$$\Psi(x, y, t) = \Psi(y)\exp(\lambda t)\cos(ax),$$
$$\theta(x, y, t) = \theta(y)\exp(\lambda t)\sin(ax),$$
$$\Phi(x, y, t) = \Phi(y)\exp(\lambda t)\sin(ax), \tag{28}$$

where λ is exponential growth rate parameter and a is wave number. To obtain the condition for neutral stability, we set $\lambda = 0$. On substituting (28) in to (24)-(27), the following eigenvalue problem is arrived.

$$(D^2 - a^2)\Psi + a\left[Ra\theta + \frac{1}{Le}Sa\Phi\right], \tag{29}$$

$$(D^2 - a^2)\theta - a\frac{dT_B}{dy}\Psi + 2a\frac{Ge}{Ra}Pe\Psi - PeD\theta + \gamma\Phi = 0, \tag{30}$$

$$(D^2 - a^2)\Phi - aLe\frac{dC_B}{dy}\Psi - LePeD\Phi = 0, \tag{31}$$

$$y = 0, 1: \Psi = \theta = \Phi = 0. \tag{32}$$

where $D = \frac{\partial}{\partial y}$ and (29)-(32) constitute an eigenvalue problem for Ra.

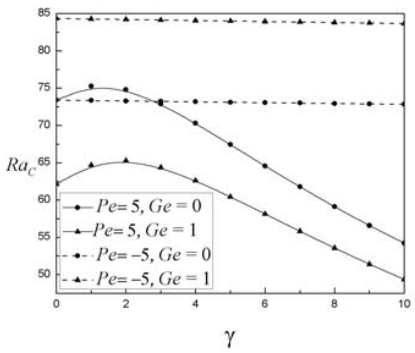
IV. Results and Discussion

Eigenvalue problem (29)-(32) is solved numerically by employing shooting and Runge-kutta methods as reported in [10]. We find eigenvalue Ra for each set of assigned values to the parameters $a, Sa, Le, Ge, \gamma, Pe$. Critical Rayleigh number for this linear stability theory is as follows

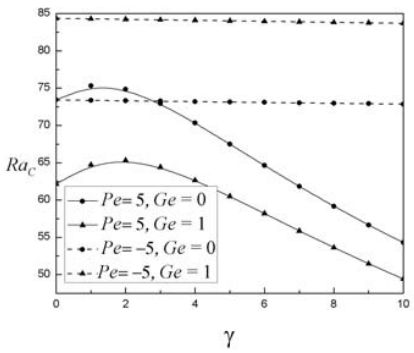
$$Ra_c = \min_a Ra(a, Sa, Le, Ge, \gamma, Pe).$$

In this section the behavior of critical thermal Rayleigh number Ra_c to various input parameters is examined. Ge, γ represents internal heat generation due to viscous dissipation, radiation, respectively. For $Sa > 0$, lower plate is more concentrated than upper plate and it is reverse for $Sa < 0$. When $Pe > 0$ (upward throughflow), porous layer undergo hot fluid input which causes increase in global temperature, and it decreases when $Pe < 0$ (downward throughflow) because of injection of cool fluid. This phenomenon is little complicated when viscous dissipation and concentration based internal heat source are introduced. When $Pe > 0$ (upward throughflow), viscous dissipation and concentration based internal heat source, both causes increase in system heating and encourages thermal convection. But for the case of $Pe < 0$ (downward throughflow), the effects of Ge, γ are competing factors to cooling action of Pe.

Fig. 1a, 1b represent the behavior of Ra_c to γ with $Le = 10$, $Pe = -5, 5$, $Ge = 0, 1$ when $Sa = 5, Sa = -5$. For both values of Sa, response of Ra_c is same. For upward throughflow ($Pe > 0$), for small values of γ has stabilization effect where as γ increases beyond 2, destabilizing effect takes place. But in the case of downward throughflow ($Pe < 0$), destabilization of γ is extremely less. Flow with $Ge = 1$ is more stable than $Ge = 0$ in the case of downward throughflow and it is converse for upward throughflow.



(a) Sa=5

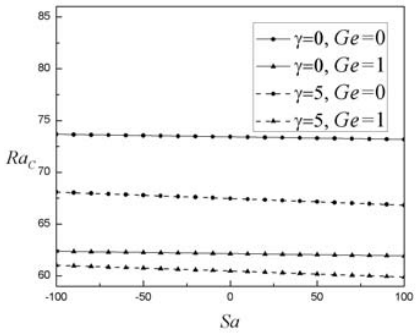


(b) Sa=-5

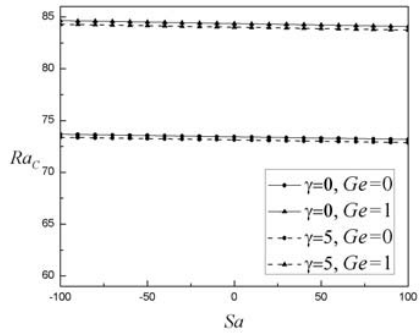
Fig. 1: Variation of Ra_c versus γ for $Le = 10$.

Fig. 2a, 2b show the response of Ra_c to Sa, when $Le = 10$, $Ge = 0, 1$, $\gamma = 0, 5$ for the cases of $Pe = 5$ (upward throughflow) and $Pe = -5$ (downward throughflow). Solutal Raleigh number Sa has insignificant destabilization effect in both cases. Increase of Ge from 0 to 1, causes destabilization

in the upward throughflow and stabilization in the downward throughflow. $\gamma = 5$ has sufficient, less intense to destabilize the flow when $Pe = 5$, $Pe = -5$, respectively.



(a) Pe=5

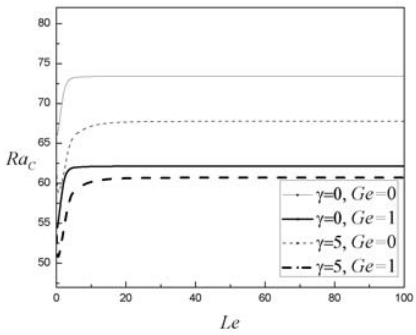


(b) Pe=-5

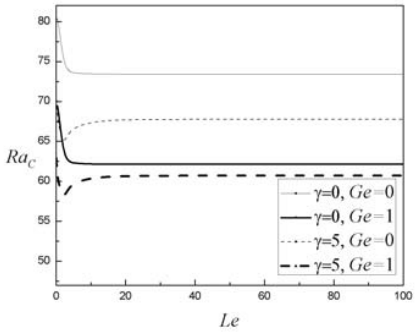
Fig. 2: Plot of Ra_c versus Sa for $Le = 10$.

Fig. 3a, 3b display variation of Ra_c versus Le with $Pe = 5$, $Ge = 0, 1$, $\gamma = 0, 5$, $Sa = 5, -5$. In the absence of γ, for small values of Le which is less than 5, has considerable stabilizing and destabilizing effects for $Sa = 5$ and $Sa = -5$, respectively. These effects becomes insignificant when Le exceeds beyond 10. Flow with $Ge = 0$ is more stable than flow with $Ge = 1$ in both the cases. When γ is present, Le has destabilization effect up to $Le = 0.5$, and significant stabilization effect up to $Le = 20$, and then minor stabilization effect for the values of Le beyond 20.

Fig. 4 exhibits plot of Ra_c versus Pe with $Le = 10, Sa = 0$, $\gamma = 0, 5$, and $Ge = 0, 1$. In the absence of Ge and γ, both upward and downward throughflow has stabilizing effect and plot of Ra_c is symmetric about $Pe = 0$. In the case of upward throughflow, the flow with $Ge = 1$ is more unstable than the flow with $Ge = 0$ and it is reverse for the downward throughflow. For the value of Pe from -5 to 20, the flow with γ=5 is more unstable than the flow with $\gamma = 0$. But when the downward throughflow is strong enough (Pe from -20 to -5), γ has insignificant effect.



(a) Sa=5



(b) Sa=-5

Fig. 3: Graph of Ra_c versus Le for $Pe = 5$.

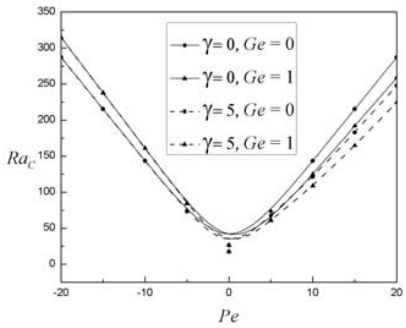


Fig. 4: Variation of Ra_c versus Pe for $Le = 10, Sa = 0$.

V. Conclusion

Linear stability analysis of double diffusive convection in a porous layer has been studied where vertical throughflow, concentration based internal heat source and viscous dissipation effects are present. Flow with $Ge = 0$ is more stable than the flow with $Ge = 1$ for the case of upward throughflow. Flow with $Ge = 1$ is more stable than the flow with $Ge = 0$ for the case of downward throughflow. For upward throughflow, γ has stabilizing effect for small values

of γ, beyond this, it destabilizes the flow. But in the case of downward throughflow, γ has very less destabilizing effect.

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