International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:2, No:12, 2008

Effect of Shear Theories on Free Vibration of Functionally Graded Plates

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***Abstract*—** Analytical solution of the first-order and third-order shear deformation theories are developed to study the free vibration behavior of simply supported functionally graded plates. The material properties of plate are assumed to be graded in the thickness direction as a power law distribution of volume fraction of the constituents. The governing equations of functionally graded plates are established by applying the Hamilton's principle and are solved by using the Navier solution method. The influence of side-to-thickness ratio and constituent of volume fraction on the natural frequencies are studied. The results are validated with the known data in the literature.

***Keywords*—** Free vibration, Functionally graded plate, Navier solution method.

I. Introduction

FUNCTIONALLY graded materials (FGMs) are new inhomogeneous materials which have widely used in many engineering applicants such as nuclear reactors and high-speed spacecraft industries [1]. The mechanical properties of FGMs vary smoothly and continuously from one surface to the other. Typically these materials are made from a mixture of ceramic and metal or from a combination of different materials. The ceramic constituent of the material provides the high-temperature resistance due to its low thermal conductivity. The ductile metal constituent on the other hand, prevents fracture caused by stresses due to the high temperature gradient in a very short period of time. Furthermore a mixture of ceramic and metal with a continuously varying volume fraction can be easily manufactured [2-3]. The analysis of these materials has been considered by many researchers. Reddy [4] investigated the third-order shear deformation theory (TSDT) of plates for the analysis of functionally graded (FG) plates to show the effects of the material distribution on the deflections and stresses. Praveen and Reddy [5] reported the response of FG ceramic-metal plates using a plate finite element formulation. Loy et al. [6] have analyzed the vibration of FG cylindrical shells based on the Love's shell theory. Javaheri and Eslami [7 9] presented the mechanical and thermal buckling of rectangular FG plates based on the classical and high-order plate theories. Birman [10] studied the buckling problem of an FG composite rectangular plate subjected to uniaxial compression.

The mechanical and thermal buckling analyses of FG circular plates are given by Najafizadeh and Eslami [11-13]. A refined theory for thermoelastic stability of FG circular plates based on the first-order shear deformation plate theory is studied by Najafizadeh and Hedayati [14]. Najafizadeh and Heydari [15] have analyzed the thermal buckling of FG circular plates using higher-order shear deformation plate theory. Recently, Batra and Aimmanee [16] employed a higher-order shear and normal deformable plate theory and the finite element method to analyze the free vibrations and stress distribution in a thick isotropic and homogeneous plate. Also, Vel and Batra [17] presented an 3-D exact solution for free and forced vibrations of simply supported FG rectangular plates. They assumed that the plate is made of an isotropic material with material properties varying in the thickness direction only. The effective material properties at a point have estimated from the local volume fractions and the material properties of the phases either by the Mori-Tanaka or the self consistent scheme. Qian et al. [18-19] have used the plate theory for analyzing the static and dynamic deformations of thick plates under different edge conditions. The same authors have employed a higher-order shear and normal deformable plate theory and a Meshless Local Petrov-Galerkin (MLPG) method to analyze the static and dynamic deformations of an FG plate [20-21]. Woo et al. [22] studied the non-linear free vibration behavior of plates made of FGMs using the Von Karman theory for large transverse deflection. Also, Park and Kim [23] investigated the thermal postbuckling and vibration analyses of FG plates. Kim [24] discussed the temperature dependent vibration analysis of FGM rectangular plates. Altay and Dökmeci [25] reported the variational principles and vibrations of an FG plate. They employed a hierarchical system of the two-dimensional approximate equations to derive systematically the vibrations of an FG piezoelectromagnetic plate. Non-linear vibration of a shear deformable FG plate has also studied by Chen [26]. He used the Runge-Kutta method to obtain the non-linear and linear frequencies. The classical plate theory (CPT) neglects shear deformation and is the theory that plane sections normal to the plate axis remain plane and normal after deformation. The first-order shear deformation theory (FSDT) is the simplest theory that accounts for non-zero transverse shear strain and

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ISSN: 2517-9950
Vol:2, No:12, 2008

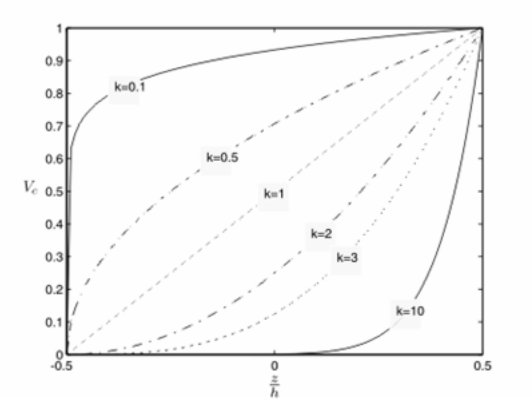


Fig. 1 Variation of the ceramic volume fraction function versus dimensionless thickness with different power k.

incorporates this effect but is often inadequate to account for the distortion of the plane normal to the mid-surface, and the third-order shear deformation theories (TSDT) are those in which the transverse shear stresses are accounted for. Such theories can be used to analyses the mechanical problems with more accuracy [27].

In the present study, free vibration of simply supported functionally graded plates has investigated using the first-order and third-order shear deformation theories (FSDT and TSDT). The governing equations of FG plates are established using the Hamilton's principle. The Navier solution method has used to solve the vibration problem. The material properties are assumed to vary through the thickness direction according to power law distribution of constituent volume fraction. The objective is studying the influence of the shear theories and constituent volume fractions on the natural frequencies. A comparison of fundamental frequencies predicted by the two theories is presented. The results are compared and validated with those are in the literature.

II. FORMULATION

Consider an FG plate with length a, width b, and thickness h. The material properties of plate are assumed to be graded through the thickness direction. The constituent materials are assumed to be ceramic and metal. The volume fractions of the ceramic V_c and metal V_m corresponding to the power law are expressed as [5]

$$V_c = \left(\frac{2z+h}{2h}\right)^k, \qquad V_m = 1 - V_c \tag{1}$$

where subscripts m and c refer to the metal and ceramic constituents, respectively, z is the thickness coordinate $(-h/2 \le z \le h/2)$, and k is the power law index that takes values greater than or equal to zero. The variation of the composition of ceramic and metal is linear for k=1. The value of k equal to zero represents a fully ceramic plate. The variation of the ceramic volume fraction function versus dimensionless thickness with different power k is plotted in Fig. 1. The mechanical properties of FGM are determined from the volume fraction of the material constituents. The Young's modulus, E, and density of material, ρ, are assumed to change in the thickness direction, z, based on the Voigt's rule over the whole range of the volume fraction as [5]

$$\begin{aligned} E(z) &= E_c V_c + E_m V_m \\ \rho(z) &= \rho_c V_c + \rho_m V_m \end{aligned} \tag{2}$$

The Poisson' ratio, ν, is assumed to be constant across the plate thickness. Substituting Eq. (1) into (2), the material properties of the FG plate are determined, which are the same as the equations proposed by Praveen and Reddy [5]

$$\begin{aligned} E(z) &= E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k \\ \rho(z) &= \rho_m + (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^k \end{aligned} \tag{3}$$

The displacement field for FSDT and TSDT can be written as [27]

$$\begin{aligned} u(x,y,z,t) &= u_0 + z\phi_x - c_1 z^3(\phi_x + w_{0,x}) \\ v(x,y,z,t) &= v_0 + z\phi_y - c_1 z^3(\phi_y + w_{0,y}) \\ w(x,y,z,t) &= w_0 \end{aligned} \tag{4}$$

where u, v, and w denote the displacement components in the x, y, and z directions, respectively, $_x$ and $_y$ are the rotations of the transverse normals about y and x axes, respectively. For FSDT c_1=0 and for TSDT c_1=4/3h^2. All of the generalized displacements (u_0, v_0, w_0, $_x$, $_y$) are functions of x, y, and t. From the strain-displacement relations appropriate for infinitesimal deformations, we obtain

$$\begin{aligned} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} &= \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} + z^3 \begin{Bmatrix} \eta_{xx} \\ \eta_{yy} \\ \eta_{xy} \end{Bmatrix} \\ \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} &= \begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} + z^2 \begin{Bmatrix} \xi_{yz} \\ \xi_{xz} \end{Bmatrix} \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\varepsilon^0_{xx} = u_{0,x}, \quad \varepsilon^0_{yy} = v_{0,y}, \quad \gamma^0_{xy} = u_{0,y} + v_{0,x} \\ &\kappa_{xx} = \phi_{x,x}, \quad \kappa_{yy} = \phi_{y,y}, \quad \kappa_{xy} = \phi_{x,y} + \phi_{y,x} \\ &\eta_{xx} = -c_1(\phi_{x,x} + w_{0,xx}), \quad \eta_{yy} = -c_1(\phi_{y,y} + w_{0,yy}) \\ &\eta_{xy} = -c_1(\phi_{x,y} + 2w_{0,xy} + \phi_{y,x}) \\ &\gamma^0_{yz} = \phi_y + w_{0,y}, \quad \gamma^0_{xz} = \phi_x + w_{0,x} \\ &\xi_{yz} = -3c_1(\phi_y + w_{0,y}), \quad \xi_{xz} = -3c_1(\phi_x + w_{0,x}) \end{aligned} \tag{6}$$

The assumed displacement model implies that the transverse normal strain, ε_{zz}, vanishes identically. The stress-strain relations for an FG plate using the assumed displacement model can be written as

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{pmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{11} & 0 & 0 & 0\\ 0 & 0 & Q_{33} & 0 & 0\\ 0 & 0 & 0 & Q_{33} & 0\\ 0 & 0 & 0 & 0 & Q_{33}\end{bmatrix}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{pmatrix} \quad (7)$$

where

$$Q_{11}=\frac{E(z)}{1-\nu^2},\qquad Q_{12}=\nu Q_{11},\qquad Q_{33}=\frac{E(z)}{2(1+\nu)} \quad (8)$$

The force and moment resultants of FG plate are defined by

$$\begin{pmatrix}N_{xx}, M_{xx}, P_{xx}\\ N_{yy}, M_{yy}, P_{yy}\\ N_{xy}, M_{xy}, P_{xy}\end{pmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{pmatrix}(1,z,z^3)\,\mathrm{d}z$$
$$\begin{pmatrix}Q_x, R_x\\ Q_y, R_y\end{pmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}k'\begin{pmatrix}\tau_{xz}\\ \tau_{yz}\end{pmatrix}(1,z^2)\,\mathrm{d}z \quad (9)$$

Here k' denotes the shear correction coefficient. In the terms of displacements, we obtain

$$\begin{aligned}
N_{xx}&=A_{11}\varepsilon_{xx}^0+A_{12}\varepsilon_{yy}^0+B_{11}\kappa_{xx}+B_{12}\kappa_{yy}+E_{11}\eta_{xx}+E_{12}\eta_{yy}\\
M_{xx}&=B_{11}\varepsilon_{xx}^0+B_{12}\varepsilon_{yy}^0+D_{11}\kappa_{xx}+D_{12}\kappa_{yy}+F_{11}\eta_{xx}+F_{12}\eta_{yy}\\
P_{xx}&=E_{11}\varepsilon_{xx}^0+E_{12}\varepsilon_{yy}^0+F_{11}\kappa_{xx}+F_{12}\kappa_{yy}+H_{11}\eta_{xx}+H_{12}\eta_{yy}\\
N_{yy}&=A_{12}\varepsilon_{xx}^0+A_{11}\varepsilon_{yy}^0+B_{12}\kappa_{xx}+B_{11}\kappa_{yy}+E_{12}\eta_{xx}+E_{11}\eta_{yy}\\
M_{yy}&=B_{12}\varepsilon_{xx}^0+B_{11}\varepsilon_{yy}^0+D_{12}\kappa_{xx}+D_{11}\kappa_{yy}+F_{12}\eta_{xx}+F_{11}\eta_{yy}\\
P_{yy}&=E_{12}\varepsilon_{xx}^0+E_{11}\varepsilon_{yy}^0+F_{12}\kappa_{xx}+F_{11}\kappa_{yy}+H_{12}\eta_{xx}+H_{11}\eta_{yy}\\
N_{xy}&=A_{33}\gamma_{xy}^0+B_{33}\kappa_{xy}+E_{33}\eta_{xy}\\
M_{xy}&=B_{33}\gamma_{xy}^0+D_{33}\kappa_{xy}+F_{33}\eta_{xy}\\
P_{xy}&=E_{33}\gamma_{xy}^0+F_{33}\kappa_{xy}+H_{33}\eta_{xy}\\
Q_x&=A_{33}\gamma_{xz}^0+D_{33}\eta_{xz}\\
Q_y&=A_{33}\gamma_{yz}^0+D_{33}\eta_{yz}\\
R_x&=D_{33}\gamma_{xz}^0+F_{33}\eta_{xz}\\
R_y&=D_{33}\gamma_{yz}^0+F_{33}\eta_{yz}
\end{aligned} \quad (10)$$

where

$$(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij})=\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,z,z^2,z^3,z^4,z^6)\,Q_{ij}\,\mathrm{d}z \quad (11)$$

Here A_{ij}, B_{ij}, and D_{ij} are the extensional, bending extensional coupling, and bending stiffnesses, respectively, E_{ij}, F_{ij}, and H_{ij} are the high-order stiffnesses.

The governing equations of motion appropriate for the displacement field, Eq. (4), can be derived using the Hamilton's principle [27]

$$\begin{aligned}
&N_{xx,x}+N_{xy,y}=I_0\ddot{u}_0+\bar{I}_1\ddot{\phi}_x-c_1I_3\ddot{w}_{0,x}\\
&N_{xy,x}+N_{yy,y}=I_0\ddot{v}_0+\bar{I}_1\ddot{\phi}_y-c_1I_3\ddot{w}_{0,y}\\
&\bar{Q}_{x,x}+\bar{Q}_{y,y}+c_1(P_{xx,xx}+2P_{xy,xy}+P_{yy,yy})\\
&\quad+\bar{N}_{xx}w_{0,xx}+\bar{N}_{yy}w_{0,yy}+\bar{N}_{xy}w_{0,xy}\\
&\qquad=-q+I_0\ddot{w}_0-c_1^2I_6(\ddot{w}_{0,xx}+\ddot{w}_{0,yy})\\
&\qquad+c_1\bar{I}_4(\ddot{\phi}_{x,x}+\ddot{\phi}_{y,y})+c_1I_3(\ddot{u}_{0,x}+\ddot{v}_{0,y})\\
&\bar{M}_{xx,x}+\bar{M}_{xy,y}-\bar{Q}_x=\bar{I}_1\ddot{u}_0+\bar{I}_2\ddot{\phi}_x-c_1\bar{I}_4\ddot{w}_{0,x}\\
&\bar{M}_{xy,x}+\bar{M}_{yy,y}-\bar{Q}_y=\bar{I}_1\ddot{v}_0+\bar{I}_2\ddot{\phi}_y-c_1\bar{I}_4\ddot{w}_{0,y}
\end{aligned} \quad (12)$$

where

$$\begin{aligned}
&\bar{M}_{\alpha\beta}=M_{\alpha\beta}-c_1P_{\alpha\beta},\qquad \bar{Q}_\alpha=Q_\alpha-3c_1R_\alpha\\
&\bar{I}_1=I_1-c_1I_3,\qquad \bar{I}_2=I_2-2c_1I_4+c_1^2I_6\\
&\bar{I}_4=I_4-c_1I_6,\qquad I_i=\int_{-\frac{h}{2}}^{\frac{h}{2}}(z)^i\rho(z)\,\mathrm{d}z\\
&\alpha,\beta=x,y\qquad i=0,1,2,..,6
\end{aligned} \quad (13)$$

Here $\bar{N}_{xx}, \bar{N}_{yy}$, and $\bar{N}_{xy}$ are the in-plane edge loads, and q is the distributed transverse load. The superposed dot denotes partial differentiation with respect to time.

III. The Method of Solution

Navier solution is used to analyze the free vibration problem of simply supported FG plates. For free vibration case, we must set q=0. The generalized displacements are expressed as products of undetermined functions and known trigonometric functions so as to satisfy identically the simply supported boundary conditions at x=0,a and y=0,b:

For FSDT:

$$\begin{aligned}
&u=w=\phi_y=N_{xx}=M_{xx}=0 \quad at \quad x=0,a\\
&u=w=\phi_x=N_{yy}=M_{yy}=0 \quad at \quad y=0,b
\end{aligned} \quad (14)$$

For FSDT:

$$\begin{aligned}
&u=w=\phi_y=N_{xx}=\bar{M}_{xx}=0 \quad at \quad x=0,a\\
&u=w=\phi_x=N_{yy}=\bar{M}_{yy}=0 \quad at \quad y=0,b
\end{aligned} \quad (15)$$

and represent the displacement quantities as

$$\begin{aligned}
u_0&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,\mathrm{e}^{-\mathrm{i}\omega t}\\
v_0&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\,\mathrm{e}^{-\mathrm{i}\omega t}\\
w_0&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,\mathrm{e}^{-\mathrm{i}\omega t}\\
\phi_x&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,\mathrm{e}^{-\mathrm{i}\omega t}\\
\phi_y&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\,\mathrm{e}^{-\mathrm{i}\omega t}
\end{aligned} \quad (16)$$

TABLE I
COMPARISON OF NATIONAL FREQUENCIES FOR DIFFERENT MATERIAL DISTRIBUTION

	Theory	k					
		0	0.1	0.5	1	2	10
$b/a=1,\ a/h=10$							
	Exact [28]	0.0932					
	CPT1 [30]	0.0955					
	CPT2 [30]	0.0963					
	FSDT [30]	0.0930					
	TSDT [30]	0.0930					
	FSDT	0.0934	0.0687	0.0431	0.0349	0.0297	0.0242
	TSDT	0.0920	0.0677	0.0425	0.0344	0.0293	0.0238
$b/a=\sqrt{2},\ a/h=10$							
	Exact [29]	0.0704					
	CPT1 [30]	0.0718					
	CPT2 [30]	0.0722					
	FSDT [30]	0.0704					
	TSDT [30]	0.0704					
	FSDT	0.0706	0.0519	0.0326	0.0264	0.0224	0.0183
	TSDT	0.0698	0.0513	0.0322	0.0261	0.0222	0.0181

where ω denotes the natural frequency, and U_{mn}, V_{mn}, W_{mn}, X_{mn} and Y_{mn} are undetermined coefficients. The representation (16) is valid for FSDT and TSDT. Substituting Eq. (16) into (6), results into (10), and then into (12) leads to the following eigen-value problem

$$(\mathbf{C}-\omega^2\mu)\{\lambda\}=0 \tag{17}$$

where **C** is the stiffness matrix, μ is the inertia matrix, and $\{\lambda\}$ is column vector of unknown coefficients. To obtain non-trivial solution, we must set $\left|\mathbf{C}-\omega^2\mu\right|=0$. By solving the achieved equation for ω, the values of natural frequencies of FG plate with simply supported edges will be derived.

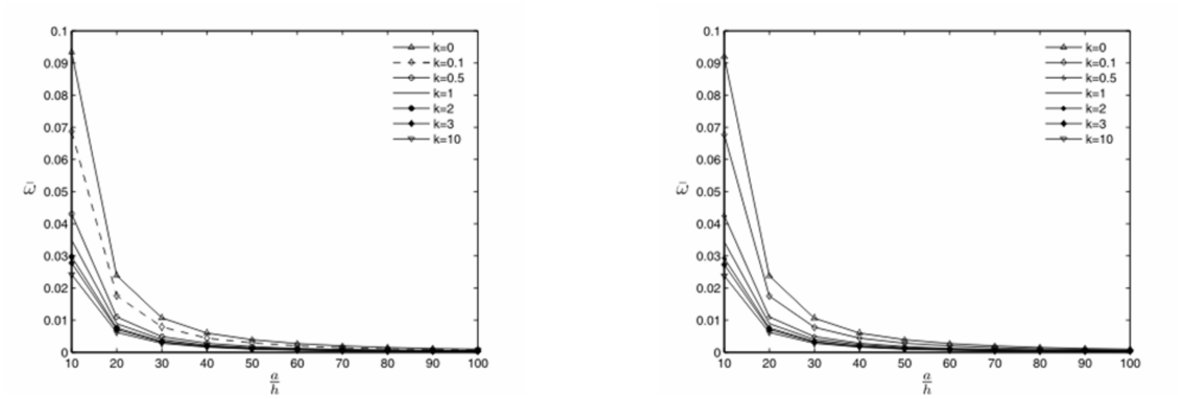
IV. RESULTS

The first-order and third-order shear deformation theories (FSDT and TSDT) have been used to analyze the free vibration of simply supported FG plates for different values of aspect ratios. The material properties are assumed to be graded in the thickness direction as a power law index. The Navier solution procedure developed in the previous section is used to evaluate the natural frequencies. The following material properties are used in the analysis:

Metal: Aluminum E=70 GPa ρ=2707 kg/m^3
Ceramic: Zirconia E=151 GPa ρ=3000 kg/m^3

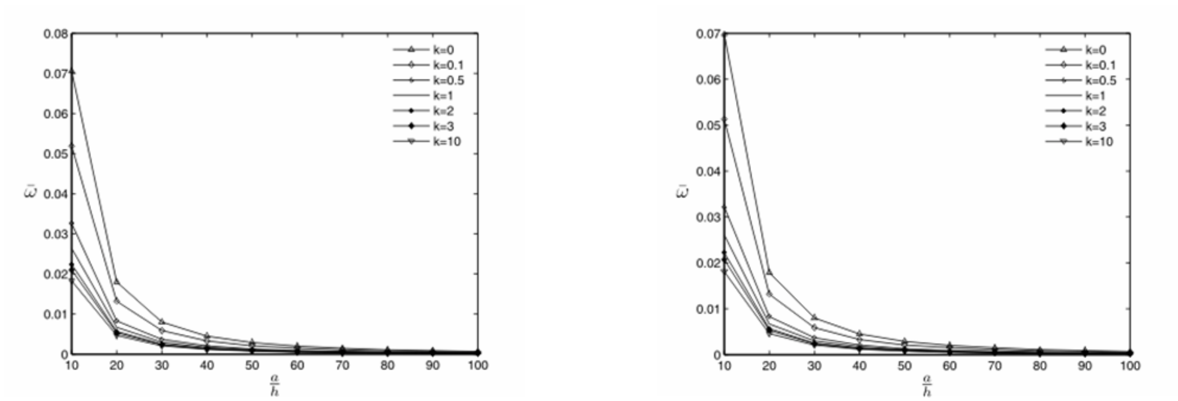
One term, namely n=m=1, is truncated in the trigonometric functions. The Poisson's ratio is chosen to be 0.3. The non dimensionalized natural frequency, , is considered in the form of $\omega h\sqrt{\rho_c/G_c}$ to analysis the results. To investigate the accuracy of the present formulation, comparison with the published data is necessary. The values of dimensionless natural frequencies of simply supported isotropic rectangular plates are presented for two values of width-to-side ratio, b/a, and one value of side-to-thickness ratio, a/h, by Sirinivas and Rao [28] and Reisman and Lee [29] for exact solution and by Reddy [30] for classical theory solution without rotatory inertia (CPT1) and include rotatory inertia (CPT2), first-order, and third-order shear deformation theories (FSDT and TSDT). A comparison between these results and the presented results (FSDT and TSDT) is shown in Table 1. The compression shows that the presented results are in good agreement with those are in the literature. The influence of constituents volume fraction on the natural frequencies of FG plate is studied by varying the value of power index, k. Also, the effects of displacement fields on the natural frequencies of FG plate are presented by considering the FSDT and TSDT. As can be seen, the natural frequencies decreased with increasing the value of power index, k. The natural frequencies of rectangular plate with $b=\sqrt{2}a$ are very smaller than the other one, b=a, and by increasing the value of power index, k, the results of two theories are very close to each other.

The dimensionless natural frequencies of a simply supported FG plate against power index, k for various values of side-to-thickness ratio, a/h and for $b/a=1,\sqrt{2}$ are plotted in Figs. 2 and 3 based on FSDT and TSDT. The frequencies decreased about 75% with increasing the metal percentage of material. It is due to, the Young's modulus of ceramic is higher than metal, and this is the characteristic of FGMs. Note that, as a/h increased, the natural frequencies of both two theories decreased. The decrease between natural frequencies from a/h=10 to a/h=100 is about 99% for various values of k, but this decrease will converge to constant values with more increase of a/h. Also, when the ratio a/h is small, the difference between the results of FSDT and TSDT are more than those for other ones. It is interesting to note that, the thickness of plate in TSDT effects on transverse shear stresses as a coefficient (c_1), but in FSDT the transverse shear stresses



(a) (b)

Fig. 2 Non-dimensionl natural frequencies ( ) as a function of the side-to thickness ratio (*a*/*h*) for FG square plates ($b = a$), (a): FSDT; (b): TSDT.



(c) (d)

Fig. 3 Non-dimensionl natural frequencies ( ) as a function of the side-to thickness ratio (*a*/*h*) for FG rectangular plates ($b = \sqrt{2}a$), (c): FSDT; (d): TSDT.

are constant along the thickness and are independent of thickness. In other words, for a certain value of length, with increasing the thickness of plate, the difference between FSDT and TSDT increases.

V. Conclusion

This study deals with the free vibration of simply supported functionally graded plates. The material properties are assumed to vary in the thickness direction according to power law distribution. The effects of the side-to-thickness ratio, the power index of constituent volume fraction, and shear theories on the natural frequencies are also discussed. The results show that the natural frequencies decrease with increasing the power index as well as side-to-thickness ratio. The first-order and third-order shear deformation theories (FSDT and TSDT) can replaced by each other for thin plates with high accuracy, but TSDT has higher accuracy for thick plates, therefore it is better to use this theory for thick plates.

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