

# Effect of Nanoparticle Diameter of Nano-Fluid on Average Nusselt Number in the Chamber

A. Ghafouri, N. Pourmahmoud, I. Mirzaee

**Abstract**—In this numerical study, effects of using  $\text{Al}_2\text{O}_3$ -water nanofluid on the rate of heat transfer have been investigated. Physical model is a square enclosure with insulated top and bottom horizontal walls, while the vertical walls are kept at different constant temperatures. Two appropriate models are used to evaluate the viscosity and thermal conductivity of nanofluid. The governing stream-vorticity equations are solved using a second order central finite difference scheme, coupled to the conservation of mass and energy. The study has been carried out for the nanoparticle diameter 30, 60 and 90 nm and the solid volume fraction 0 to 0.04. Results are presented by average Nusselt number and normalized Nusselt number in different range of  $\phi$  and  $D$  for mixed convection dominated regime. It is found that different heat transfer rate is predicted when the effect of nanoparticle diameter is taken into account.

**Keywords**—Nano-fluid, nanoparticle diameter, heat transfer enhancement, square enclosure, Nusselt number.

## I. INTRODUCTION

THE main restriction of common fluids used for heat transfer application such as water, ethylene glycol, mineral oil or propylene glycol is their low thermal conductivity. Nano-fluid, which is a mixture of nanosize particles suspended in base fluid, has a superior thermal conductivity compared to the base fluid [1]. The nanofluid can be used to various engineering applications such as solar collectors, heat exchangers, materials processing, cooling of electronic devices, crystal growth, metal coating and casting, and nuclear system cooling [2]-[4].

There have been many investigations in the past decade on the natural and combined convective heat transfer and fluid flow in lid-driven square enclosure. Reference [5] considered the buoyancy-driven heat transfer in a two-dimensional chamber with different aspect ratio and filled with different types of nanoparticles for the Rayleigh numbers of  $10^3$  to  $5 \times 10^5$  using the finite volume method. They concluded that the existence of the nanoparticles resulted in an increase in the rate of heat transfer and average Nusselt number for the whole range of Rayleigh number. Reference [6] performed a numerical study of the heat transfer performance of nanofluids inside two-dimensional rectangular enclosures. Their results indicated that increasing the volume fraction of nanoparticles produced a significant enhancement of the average rate of heat transfer. Also their results showed that the variation of average

Nusselt number with increasing the volume fraction of nanoparticles is linear in two studied cases. Reference [7] investigated the laminar natural convection flow through a square inclined enclosure filled with a CuO nanofluid. They found that heat transfer in the cavity increases by adding nanoparticles. Also their results showed that the rate of increase is greater for the enclosures with low Rayleigh number. Reference [8] conducted a study of steady laminar mixed convection flow in single and double lid square cavities filled with water-alumina nanofluid using two viscosity models. Their results showed that a significant heat transfer enhancement could be obtained by increasing the nanoparticle volume fractions at moderate and large Richardson number using both nanofluid models.

Something that is common in most of the cited numerical study on nanofluid combined convection heat transfer problems is the use of the Maxwell-Garnett thermal conductivity model [9] for a nanofluid and hence, predict enhancement of heat transfer because of the presence of nanoparticles and thermal conductivity coefficients of base fluid and nanoparticles. This model does not consider main mechanisms for heat transfer in nanofluids such as Brownian motion and does not consider nanoparticle size or temperature dependence. However, it is evident that the use of the accurate fundamental model for evaluating the thermal conductivity of nanofluids can be very influential. A number of researches have performed in comparative study of variable thermal conductivity models in various configurations. Reference [10] investigated the effects of thermal conductivity of  $\text{Al}_2\text{O}_3$ -water nanofluid on heat transfer enhancement in natural convection. Two different thermal conductivity models namely [11] and [9] evaluated by comparing their predicted results on Nusselt number. He indicated that at  $Ra \geq 10^4$ , the difference in Nusselt number between the Maxwell-Garnett [9] and Chon et al. model [11] prediction was small. However, there was a deviation in prediction at  $Ra = 10^3$  and this deviation becomes more significant at high volume fraction of nanoparticles. Similar comparative investigations with pair of models have been reported by [12]–[17].

The main motivation of this study is to investigate numerically the rate of heat transfer on the hot wall of square enclosure utilized with water-Alumina nanofluid and its effect on combined convections heat transfer. In order to achieve this objective, the model of Masoumi et al. [18] is employed for viscosity of nanofluid while Chon et al. [11] model is used to evaluate the thermal conductivity of nanofluid. The results and enhancement in heat transfer rate will be obtained for a wide

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range of volume fraction and diameter of the solid particles on average Nusselt number and normalized Nusselt number.

## II. PROBLEM DESCRIPTION

Fig. 1 depicts schematics of the square enclosure considered in this paper. The nanofluid is assumed as incompressible and the flow is assumed laminar. It is supposed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The shape of solid particle is furthermore assumed to be uniform. The top and bottom horizontal walls are thermally insulated while the vertical walls are kept at constant but different temperatures. In order to induce the buoyancy effect, the left vertical wall is kept at a relatively high temperature ( $T_H$ ) and right vertical wall is maintained at a relatively low temperature ( $T_C$ ). The top wall is moving rightwards while the bottom wall is moving leftwards with a uniform velocity ( $U_m$ ). The thermo-physical properties of the pure water and the nanoparticle at temperature of 25°C are given in Table I. The thermo-physical properties of the nanofluid are assumed be constant except for the density variation, which is approximated by the Boussinesq model.

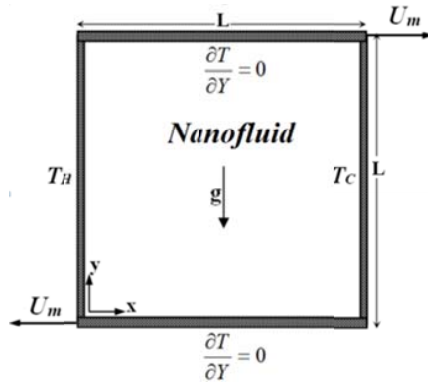


Fig. 1 Schematic of problem

TABLE I  
PHYSICAL PROPERTIES OF BASE FLUID AND NANOPARTICLES

Physical properties	Fluid phase (Water)	$Al_2O_3$
$C_p$ (J/kgK)	4179	765
$\rho$ (kg/m <sup>3</sup> )	997.1	3970
$k$ (W/mK)	0.613	25
$\beta \times 10^{-5}$ (1/K)	21	0.85

## III. GOVERNING EQUATION

The non-dimensional governing equations in the Cartesian coordinate system for the stream function ( $\Psi$ ), vorticity function ( $\Omega$ ) and thermal transport, respectively, can be written as:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega \quad (1)$$

$$U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \frac{\mu_{nf} / \mu_f}{(1-\phi) + \phi \frac{\rho_s}{\rho_f}} \frac{1}{Re} \left( \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + ((1-\phi) + \phi \frac{\beta_s}{\beta_f}) Ri \frac{\partial \theta}{\partial X} \quad (2)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{k_{nf} / k_f}{(1-\phi) + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f}} \frac{1}{Re \cdot Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (3)$$

where, the Reynolds number,  $Re$ , the Prandtl number,  $Pr$ , and the Richardson number,  $Ri$ , are defined as:

$$Re = \frac{U_m L}{\nu_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad Ri = \frac{Ra}{Pr Re^2} \quad (4)$$

The additional dimensionless variables in the above equations are defined as:

$$U = \frac{\partial \Psi}{\partial Y}, \quad V = -\frac{\partial \Psi}{\partial X}, \quad \Psi = \frac{\psi}{U_m L}, \quad \Omega = \frac{\omega L}{U_m} \quad (5)$$

The effective dynamic viscosity of  $Al_2O_3$ -water nanofluid is estimated by [18]:

$$\frac{\mu_{nf}}{\mu_f} = 1 + \frac{\rho_s V_B d_s^2}{72 C \delta \mu_f} \quad (6)$$

The viscosity of the base fluid (water) considered to vary with temperature as:

$$\mu_f = 2.414 \times 10^{-5} \times 10^{247.8/(T-140)} \quad (7)$$

The effective thermal conductivity of the nanofluid is approximated by the Chon et al. [11] model:

$$\frac{k_{nf}}{k_f} = 1 + 64.7 \phi^{0.7640} \left( \frac{d_f}{d_s} \right)^{0.3690} \left( \frac{k_s}{k_f} \right)^{0.7476} Pr_T^{0.9955} Re_T^{1.2321} \quad (8)$$

where,  $Pr_T$ ,  $Re_T$  and  $l_f$  are defined as:

$$Pr_T = \frac{\mu_f}{\rho_f \alpha_f}, \quad Re_T = \frac{\rho_f k_B T}{3 \pi \mu_f^2 l_f}, \quad l_f = \frac{1}{\sqrt{2 m \pi} d_f^2} \quad (9)$$

where,  $k_B$  is the Boltzmann constant ( $k_B = 1.3807 \times 10^{-23}$  J/K). Also,  $l_f$  is the mean path of base fluid particles given as 0.17 nm [19]. This model considers the effect of nanoparticle size and temperature on nanofluids thermal conductivity with a wide range of temperature between 21 and 70°C. Accuracy of this model was confirmed by the experiments of [20].

## IV. NUMERICAL METHOD

The set of nonlinear-coupled governing mass, momentum and energy equations are developed in term of stream function-vorticity formulation. The classical theory of single-phase fluids can be applied, where physical properties of nanofluid are taken as a function of properties of both constituents and their concentrations. In order to numerically

solve, the governing equation for two dimensional velocity and temperature field, (1)–(3) with the corresponding boundary conditions are approximated by second-order central difference scheme. Successive over relaxation (SOR) scheme is used to solve stream function equation. The numerical solution is made by solving sequentially the stream function, vorticity and temperature equations and advanced into a next time level by solving systems of each equation that is numerically modeled in FORTRAN. The convergence criterion is defined by:

$$\varepsilon = \frac{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\zeta^{n+1} - \zeta^n|}{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\zeta^{n+1}|} \leq 10^{-6} \quad (10)$$

where,  $\varepsilon$  represents the tolerance, also  $M$  and  $N$  are the number of grid points in the  $X$  and  $Y$  direction, respectively. The symbol  $\zeta$  denotes any scalar transport quantity namely  $\Psi$ ,  $\Omega$  or  $\theta$ .

The local and average heat transfer rates of the chamber can be presented by means of the local and average Nusselt numbers. The local Nusselt number is calculated along the left heated wall (11) and the average Nusselt number is determined by integrating the local Nusselt number along the heated wall (12):

$$Nu(X) = \frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial X} \Big|_{x=0} \quad (11)$$

$$Nu_{avg} = \int_0^1 Nu(X) dY \quad (12)$$

## V. CODE VALIDATION

The governing equations have been solved for the natural convection flow in a cavity filled by pure fluid, in order to compare the results with those obtained by [21]–[23], [14]. This comparison revealed good agreements between the results, which are shown in Table II.

TABLE II  
COMPARISONS OF THE PRESENT RESULTS FOR THE AVERAGE NUSSELT NUMBER OF THE HOT WALL WITH THE RESULTS OF OTHER INVESTIGATORS

	$Ra=10^4$	$Ra=10^5$	$Ra=10^6$
Present work	2.246	4.521	8.984
Sheikhzadeh et al. [14]	2.242	4.514	8.790
Khanafer et al. [21]	2.245	4.522	8.826
Fusegi et al. [22]	2.302	4.646	9.012
Markatos and Pericleous [23]	2.201	4.430	8.754

## VI. RESULT AND DISCUSSION

Combined convection heat transfer is studied numerically for Alumina-water nanofluid in a square enclosure. Calculations were accomplished for nanoparticle volume fraction of 0.01, 0.02, 0.03, and 0.04 and particle diameter of 30, 60 and 90 nm. Appropriate choices of viscosity models

and thermal conductivity models have strongly influence on predict heat transfer rate and Nusselt number correctly. Results are presented by average Nusselt number and normalized Nusselt number on the hot wall.

Fig. 2 represent variation of the average Nusselt number by nanoparticle volume fraction at  $Re=100$ . As seen from the figures, average Nusselt number increase with nanoparticle volume fraction ( $\phi=0$  to 4%).

At  $\phi=4\%$  Fig. 2 indicates that for  $Ri=1$  the highest average Nusselt number ( $Nu_{ave}=5.75$ ) is observed at  $d_{np}=30$  nm with using the Chon et al. [11] thermal conductivity model by considering the role of Brownian motion, temperature and nanoparticles size.

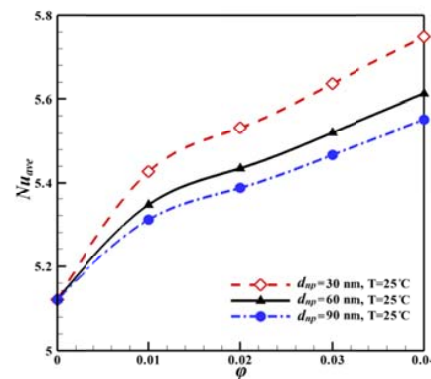


Fig. 2 Effect of different solid volume fractions of nano-fluid on the average Nusselt number

Fig. 3 depicts the Effect of different solid volume fractions of nanofluid on the normalized Nusselt number at different nanoparticle diameter. It is indicated that the highest and lowest assessment are related to  $d_{np}=30$  nm and  $d_{np}=90$  nm by 12% and 8%, respectively.

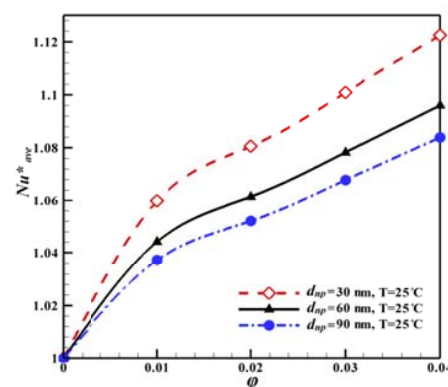


Fig. 3 Effect of different solid volume fractions of nanofluid on the normalized Nusselt number

## VII. CONCLUSIONS

Laminar combined convection in two-dimensional square enclosure filled with a water-Alumina nanofluid was numerically studied. The top wall moved to the right while the bottom wall moved to the left at the same constant speed. Two

horizontal walls of the enclosure were kept insulated while the right and left walls were maintained isothermally, however the temperature of the left wall was higher than the right wall. The governing equations were given in term of the stream function-vorticity formulation in the non-dimensionalized form and then solved numerically by second-order central difference scheme. The model of [18] is employed for viscosity of nanofluid while thermal conductivity of nanofluid is evaluated by Chon et al. [11] model. In these models, the effects of nanofluid bulk temperature, nanoparticles size, nanoparticles volume fraction and Brownian motion were incorporated. The main findings are listed as follows:

- The Nusselt number and thus heat transfer from the left wall of enclosure was enhanced with increasing volume fraction of nanoparticle.
- Heat transfer in square enclosure was enhanced by decreasing the value of nanoparticle diameter keeping constant the other parameters.

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