

# Effect of Mass Transfer on MHD Mixed Convective Flow along Inclined Porous Plate with Thermodiffusion

Md. Nasir Uddin, M. A. Alim, M. M. K. Chowdhury

**Abstract**—The effect of mass transfer on MHD mixed convective flow along inclined porous plate with thermodiffusion have been analyzed on the basis of boundary layer approximations. The fluid is assumed to be incompressible and dense, and a uniform magnetic field is applied normal to the direction of the flow. A Similarity transformation is used to transform the problem under consideration into coupled nonlinear boundary layer equations which are then solved numerically using the Runge-Kutta sixth-order integration scheme together with Nachtsheim-Swigert shooting iteration technique. The behavior of velocity, temperature, concentration, local skin-friction, local Nusselt number and local Sherwood number for different values of parameters have been computed and the results are presented graphically, and analyzed thereafter. The validity of the numerical methodology and the results are questioned by comparing the findings obtained for some specific cases with those available in the literature, and a comparatively good agreement is reached.

**Keywords**—Mass transfer, inclined porous plate, MHD, mixed convection, thermodiffusion.

## I. INTRODUCTION

MHD flow for an electrically conducting fluid has attracted the interest of many researchers due to its important applications in many engineering fields such the magnetic behavior of plasmas in fusion reactors, liquid-metal, cooling of nuclear reactors, electromagnetic casting, petroleum industries, boundary layer control in aerodynamics, MHD generators, crystal growth, Ship propulsion, Jet printers and so on. Moreover the study of MHD is largely concerned with the flow, heat and mass transfer characteristics in various physical situations. An analysis of heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration is investigated by [1]. Reference [2] studied the problem of combined free-forced convection and mass transfer flow over a vertical porous flat plate, in presence of heat generation and thermal diffusion. Reference [3] also investigated the effects of chemical reactions and thermophoresis on magnetohydrodynamics mixed convective heat and mass transfer flow along an inclined plate in the presence of heat generation and (or) absorption with viscous dissipation and joule heating. The author, in [3] considered the chemical reaction but the chemical reaction effect has been neglected

from the momentum equation. Also the similarity solutions were presented neglecting the effects of Grashof number or Richardson number for mixed convection. MHD mixed convective heat transfer about a semi-infinite inclined plate in the presence of magneto and thermal radiation effects has been examined by [4]. Reference [5] analyzed a steady two-dimensional MHD free convection and mass transfer flow past an inclined semi-infinite vertical surface in the presence of heat generation in a porous medium. Reference [6], investigated the effect of conjugate heat and mass transfer on magnetohydrodynamic mixed convective flow past inclined plate in a porous medium.

As mentioned above, it is more reasonable to include the thermophoretic effects on the concentration to explore the impact of the momentum, heat and mass transfer characteristics with a transverse applied magnetic field. Therefore, in the light of above literatures, the aim of the present work is to investigate the effect of mass transfer on MHD mixed convective flow along inclined porous plate.

## II. MODEL AND MATHEMATICAL FORMULATION

The schematic view of flow configuration and coordinates system is shown in Fig. 1.

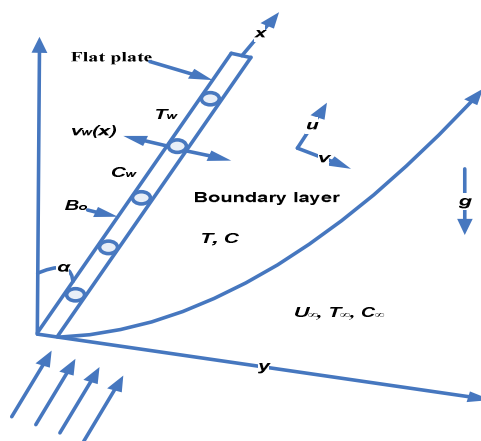


Fig. 1 Schematic view of flow configuration

A steady two-dimensional MHD laminar mixed convective flow of a viscous, incompressible electrically conducting fluid along a semi-infinite inclined porous plate with an acute angle  $\alpha$  to the vertical is considered. The physical coordinates  $(x, y)$  are chosen such that  $x$  is measured from the leading edge in the streamwise direction and  $y$  is measured normal to the surface of the plate. The velocity components in the directions

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of flow and normal to the flow are  $u$  and  $v$  respectively. A magnetic field of uniform strength  $B_0$  is applied normal to the direction of flow. The external flow with a uniform velocity  $U_\infty$  takes place in the direction parallel to the inclined plate. It is assumed that  $T$  and  $C$  are the temperature and concentration of the fluid which are the same, everywhere in the fluid. The surface is maintained at a constant temperature  $T_w$ , which is higher than the constant temperature  $T_\infty$  of the surrounding fluid and the concentration  $C_w$ , is greater than the constant concentration  $C_\infty$ .

Under the usual Boussinesq approximation, the governing equations such as continuity, momentum, energy and concentration equations for steady, two-dimensional, laminar boundary layer flow with the above assumptions can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\alpha + g\beta^*(C - C_\infty)\cos\alpha - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K^*}\right)u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y} \{V_T(C - C_\infty)\} \quad (4)$$

In the above equations,  $\nu$  is the kinematics viscosity,  $g$  is the acceleration due to gravity,  $\beta$  is the volumetric coefficient of thermal expansion,  $\beta^*$  is the volumetric coefficient of expansion with mass fraction,  $\sigma$  is the electrical conductivity,  $K^*$  is permeability of the porous medium,  $\rho$  is the density of the fluid,  $k$  is the thermal conductivity of the fluid,  $c_p$  is the specific heat at constant pressure,  $Q_0$  is the heat generation constant,  $D$  is the mass diffusivity and  $V_T$  is the thermophoretic velocity. The appropriate boundary conditions for the flow field of this investigation are as follows:

$$u = 0, v = \pm v_w(x), T = T_w, C = C_w \quad \text{at } y = 0 \quad (5.1)$$

$$u = U_\infty, T = T_\infty, C = C_\infty \quad \text{as } y \rightarrow \infty \quad (5.2)$$

In addition,  $U_\infty$  is the free stream velocity and  $v_w(x)$  represent the permeability of the porous plate where its sign indicates suction ( $< 0$ ) or blowing ( $> 0$ ), subscripts  $w$  and  $\infty$  refer to the wall and boundary layer edge, respectively. In (4) the thermophoretic velocity  $V_T$  can be expressed in the following form as:

$$V_T = -\frac{\kappa \nu}{T_{ref}} \frac{\partial T}{\partial y} \quad (6)$$

where  $T_{ref}$  is some reference temperature and  $\kappa$  is the thermophoretic coefficient which is defined by [7]. To

facilitate the analysis, the governing differential equations are to be made nondimensional with suitable transformations and the following dimensionless variables defined by [8] are introduced:

$$\eta = y \sqrt{\frac{U_\infty}{\nu x}}, \psi = \sqrt{\nu x U_\infty} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

where,  $\psi(x, y)$  is the stream function defined by  $u = \partial \psi / \partial y$  and  $v = -\partial \psi / \partial x$ , such that the continuity equation (1) is satisfied automatically. In terms of these new variables, the velocity components can be expressed as:

$$u = U_\infty f'(\eta) \quad (8.1)$$

$$v = \frac{1}{2} \sqrt{\frac{\nu U_\infty}{x}} (\eta f' - f) \quad (8.2)$$

Here, the prime stands for ordinary differentiation with respect to similarity variable  $\eta$ . Using dimensionless variables, the transformed momentum, energy, and concentration equations together with the boundary conditions can be written as:

$$f''' + \frac{1}{2} f f'' + Gr_t \theta \cos\alpha + Gr_m \phi \cos\alpha - (M + K) f' = 0 \quad (9)$$

$$\theta'' + \frac{1}{2} Pr f \theta' + Pr Q \theta = 0 \quad (10)$$

$$\phi'' + Sc \left( \frac{1}{2} f - \tau \theta' \right) \phi' - Sc \tau \theta'' \phi = 0 \quad (11)$$

with the boundary conditions:

$$f = f_w, f' = 0, \theta = 1, \phi = 1 \quad \text{at } \eta = 0 \quad (12.1)$$

$$f' \rightarrow 1, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (12.2)$$

where,  $Gr_t$  is the local thermal Grashof number,  $Gr_m$  is the local mass Grashof number,  $M$  is the magnetic field parameter,  $K$  is the permeability parameter,  $Pr$  is the Prandtl number,  $Q$  is the heat generation parameter,  $Sc$  is the Schmidt number,  $\tau$  is the thermophoretic parameter and  $f_w = -v_w(x) \sqrt{x/(\nu U_\infty)}$  is the nondimensional wall mass transfer coefficient such that  $f_w > 0$  indicates wall suction and  $f_w < 0$  indicates wall injection or blowing. The corresponding dimensionless groups that appear in the nondimensional form of governing equations are defined as:

$$Gr_t = \frac{g\beta(T_w - T_\infty)x}{U_\infty^2}, Gr_m = \frac{g\beta^*(C_w - C_\infty)x}{U_\infty^2},$$

$$M = \frac{\sigma B_0^2 x}{\rho U_\infty}, K = \frac{\nu x}{K^* U_\infty}, Pr = \frac{\nu \rho c_p}{k},$$

$$Q = \frac{Q_0 x}{\rho c_p U_\infty}, Sc = \frac{\nu}{D}, \tau = -\frac{\kappa(T_w - T_\infty)}{T_{ref}} \quad (13)$$

By employing definition of wall shear stress  $\tau_w = \mu(\partial u / \partial y)_{y=0}$  along with Fourier's law  $q_w = -k(\partial T / \partial y)_{y=0}$  and Fick's law  $J_s = -D(\partial C / \partial y)_{y=0}$  the nondimensional forms of local skin-friction coefficient is  $C_f = 2Re_x^{-1/2} f''(0)$ , local Nusselt number is  $N_u = -2Re_x^{1/2} \theta'(0)$  and local Sherwood number is  $S_h = -2Re_x^{1/2} \phi'(0)$ , where  $Re_x = xU_\infty / \nu$  is denoting the local Reynolds number.

### III. NUMERICAL PROCEDURE AND COMPARISON

The system of transformed nonlinear ordinary differential equations (9), (10) and (11), together with the boundary conditions (12.1) and (12.2) have been solved numerically using Nachtsheim-Swigert shooting iteration technique along with sixth order Runge-Kutta initial value solver. The numerical methods are described in details, referring to [9]. For the accuracy of the numerical results, the present investigation is compared with the previous investigation by [5] as shown in Fig. 2 while  $Gr_t = 2.0$ ,  $Gr_m = 2.0$ ,  $M = 0.5$ ,  $Q = 0.5$ ,  $K = 0.5$ ,  $\alpha = 30^\circ$ ,  $Pr = 0.71$ ,  $f_w = 0.0$ ,  $Sc = 0.6$  and  $\tau = 0.0$ . It is observed that the present result is in good agreement with that of [5]. This favorable comparison leads confidence in the numerical results to be reported in the next sections.

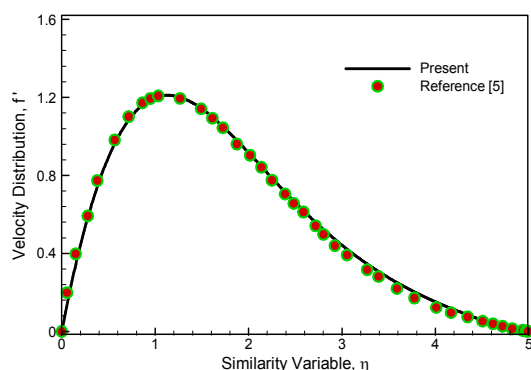


Fig. 2 Comparison of velocity distribution

### IV. RESULTS AND DISCUSSIONS

The effect of various physical parameters on the flow field are examined and discussed in this section. The numerical solutions have been carried out using different values of various physical parameters which are appeared on the nonlinear ordinary differential equations. The following set of

considered values for the key parameters in the numerical solutions were adopted, unless otherwise stated;  $Gr_t = 0.87$ ,  $Gr_m = 0.87$ ,  $M = 0.001$ ,  $Q = 0.50$ ,  $K = 0.01$ ,  $\alpha = 30^\circ$ ,  $Pr = 0.71$ ,  $f_w = 0.50$ ,  $Sc = 0.60$ ,  $\tau = 0.10$  and  $U_\infty / \nu = 1.0$ . Figs. 3 (a)–(c) display the influence of different Schmidt number  $Sc$  on the flow field. The values of Schmidt number  $Sc$  are taken to be 0.22, 0.60 and 1.76 which are corresponding physically to hydrogen ( $H_2$ ), water vapour ( $H_2O$ ) and benzene ( $C_6H_6$ ). In Fig. 3 (a), it is observed that when the Schmidt number  $Sc$  increases, the velocity of the flow field decreases because in presence of heavier diffusing species. The temperature distribution in Fig. 3 (b) changes insignificantly comparing with the velocity on the flow field. As the Schmidt number  $Sc$  increases, the concentration of the flow field decreases which is observed in Fig. 3 (c). This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. As indicated in Figs. 4 (a) and (b), an increase in the Schmidt number  $Sc$  produces a decrease in the local skin friction coefficient  $C_f$  as well as the local Nusselt number  $N_u$ . However, on the basis of Fig. 4 (c), it is found to increase the local Sherwood number  $S_h$  due to increase in the Schmidt number  $Sc$ . The effects of different local thermal Grashof number  $Gr_t$  on the flow field are displayed in Figs. 5 (a) and (b). It can be seen from Fig. 5 (a) that the fluid velocity increases due to the development of the thermal buoyancy force. The increase of the local thermal Grashof number  $Gr_t$ , the peak values of the velocity increases rapidly near the plate and then decreases smoothly to approach the free stream velocity. From the dimensionless temperature distribution in Fig. 5 (b), it is found that an increase in the local thermal Grashof number  $Gr_t$  decrease the temperature of the flow field. This is because of the positive values of local thermal Grashof number  $Gr_t$  correspond to cooling of the porous plate. From the Figs. 6 (a) and (b), it is seen that both of local skin friction coefficient  $C_f$  and local Nusselt number  $N_u$  increase due to increase in the local thermal Grashof number  $Gr_t$  on the flow field. The behavior of the flow field for different values of heat generation parameter  $Q$  and thermophoretic parameter  $\tau$  are shown in Figs. 7 (a) and (b) respectively. As observed in Fig. 7 (a), the dimensionless temperature distribution is found to increase due to increase in the heat generation parameter  $Q$  because the presence of a heat source on the flow field, the thermal state of the fluid increases causing the thermal boundary layer to increase. The velocity and concentration distribution of the flow field for different values of heat generation parameter  $Q$  change insignificantly comparing with the temperature distribution. Therefore the graphical representations of velocity, concentration and local skin friction as well as local Sherwood number are not presented in this paper. It is seen that the concentration distribution in Fig. 7 (b) decreases with increase of the thermophoretic parameter  $\tau$ . The effect of increasing thermophoretic parameter  $\tau$  is limited to increasing the wall slope of the concentration distribution but decreasing the concentration of the flow field. This is true for small values of Schmidt number  $Sc$  for which the Brownian diffusion effect is large compared to the convection effects. As observe in Fig. 8 (a), it is reported that

the local Nusselt number  $N_u$  decreases with increase in heat generation parameter  $Q$ . However, it can be seen in Fig. 8 (b) that the local Sherwood number  $S_h$  increases with the increase of thermophoretic parameter  $\tau$ .

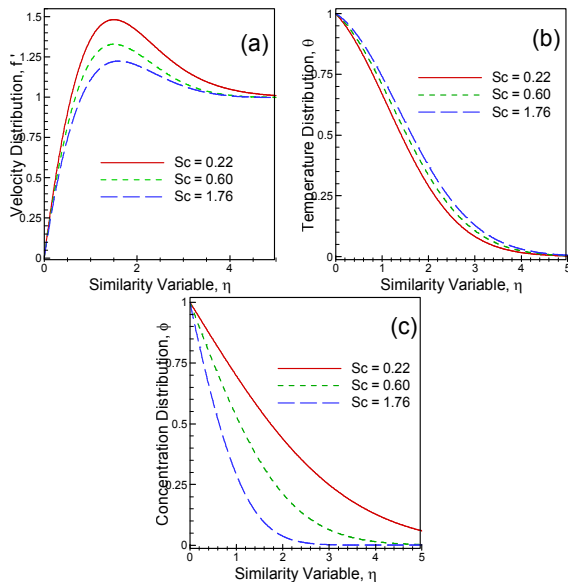


Fig. 3 Representative (a) velocity; (b) temperature; (c) concentration distributions for different values of Schmidt number  $Sc$

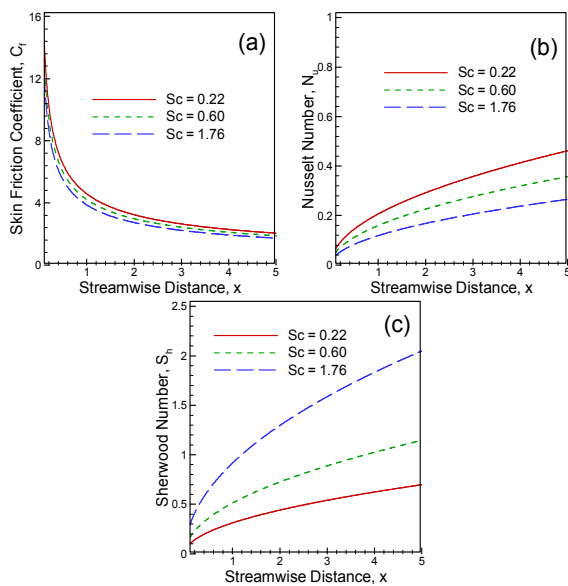


Fig. 4 Effects of  $Sc$  on (a) local skin friction coefficient; (b) local Nusselt number; (c) local Sherwood number against the streamwise distance

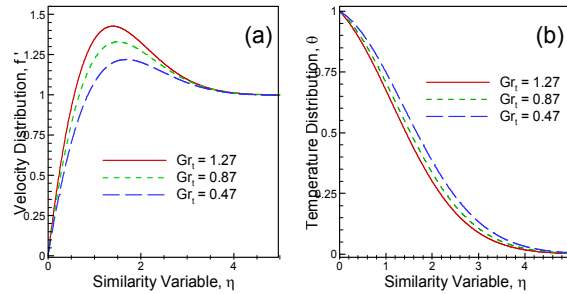


Fig. 5 Representative (a) velocity; (b) temperature distributions for different values of local thermal Grashof number  $Gr_l$

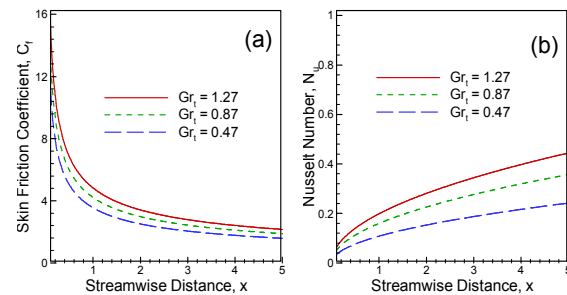


Fig. 6 Effects of  $Gr_l$  on (a) local skin friction coefficient; (b) local Nusselt number against the streamwise distance

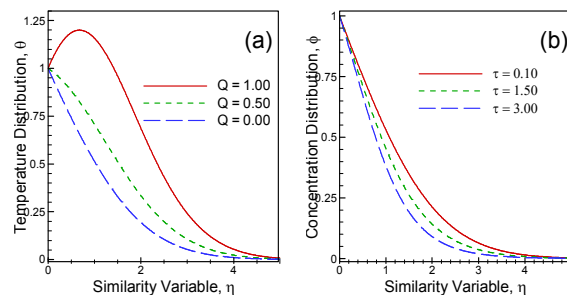
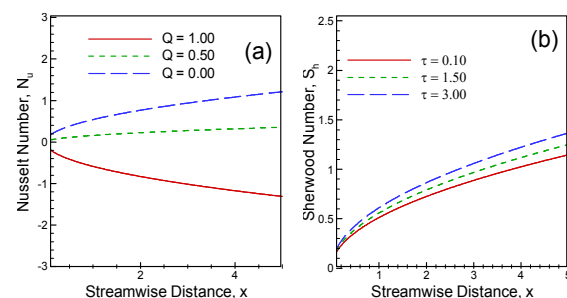


Fig. 7 Representative (a) temperature distribution for different values of  $Q$ ; (b) concentration distributions for different values of  $\tau$



Figs. 8 Effects of (a)  $Q$  on local Nusselt number; (b)  $\tau$  on local Sherwood number against the streamwise distance

## V. CONCLUSION

In this investigation, a mathematical model for MHD mixed convective flow along inclined porous plate has been developed and then solved numerically. The numerical results are presented graphically and excellent agreement is obtained.

This investigation is carried out to find the effects of Schmidt number  $Sc$ , local thermal Grashof number  $Gr_t$ , heat generation parameter  $Q$  and thermophoretic parameter  $\tau$  on the flow field. On the basis of the results, the conclusions can be drawn as; as the local thermal Grashof number  $Gr_t$  increases, the temperature tends to decrease whereas an increase in heat generation parameter  $Q$  and Schmidt number  $Sc$ , increase the temperature of the flow field. Due to increase in local thermal Grashof number  $Gr_t$ , Schmidt number  $Sc$  and thermophoretic parameter  $\tau$ , decrease the concentration of the flow field. In the presence of increasing Schmidt number  $Sc$ , decrease the local skin friction coefficient  $C_f$  while increase the local skin friction coefficient  $C_f$  with the increase of local thermal Grashof number  $Gr_t$ . Increase in the heat generation parameter  $Q$  and Schmidt number  $Sc$  has the effect to decrease the local Nusselt number  $N_u$ , while increase the local Nusselt number  $N_u$  with the increase in local thermal Grashof number  $Gr_t$ . According to the increase in Schmidt number  $Sc$  and thermophoretic parameter  $\tau$ , increase the local Sherwood number  $S_h$ .

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