

$$T = \begin{bmatrix} T_0(1) & T_1(1) & T_2(1) & \dots & T_n(1) \\ T_0(2) & T_1(2) & T_2(2) & \dots & T_n(2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ T_0(L) & T_1(L) & T_2(L) & \dots & T_n(L) \end{bmatrix} \quad (4)$$

The elements of matrix T are calculated by using Chebyshev polynomial [18]

$$T_{n+1}(t) = 2tT_n(t) - T_{n-1}(t) \quad (5)$$

here $T_0(t) = 1$, $T_1(t) = t$.

The vectors C storing the coefficients of the polynomial are obtained by:

$$C = (T^T T)^{-1} T^T Y \quad (6)$$

Equations (5) and (6) are used to solve (3). It will result

$$\hat{Y} = TC = T(T^T T)^{-1} T^T Y = WY \quad (7)$$

here W denotes window's coefficient matrix. Smoothing can be performed by use of different window coefficient matrix.

Riemann-Liouville fractional order derivative can be expressed as:

$${}_0 D_x^\alpha Y(t) = \frac{1}{\Gamma(l-\alpha)} \frac{d^l}{dt^l} \int_0^x (t-x)^{l-\alpha-1} f(x) dx \quad (8)$$

here $0 \leq l-1 < \alpha < l$, and $\Gamma(l-\alpha)$ is the Gamma function of $(l-\alpha)$. α is the positive order of differentiation and its value lies between $l-1$ to l [19].

The fractional order differentiator, corresponding to window coefficient matrix W , can be obtained by (8). Different properties of fractional order differentiation are applied on (7) and we will get

$$\hat{Y}_t^\alpha = T_t^\alpha C = W_t^\alpha Y = c(T^T T)^{-1} T^T Y \quad (9)$$

It is generalized form of (7). Here \hat{Y}_t^α denotes the α^{th} derivative of the t^{th} point in the filtering window, W_t^α denotes the α^{th} derivative coefficient vector of the t^{th} point in the filtering window.

Quadrature mirror filter (QMF) concept is used for design the high pass filter with newly designed digital fractional order differentiator [16], [17]. The low pass and high pass filters are used here $G_0(n)$ and $h_0(n)$ respectively. The high pass filter $h_0(n)$ is a mirror image of low pass filter $G_0(n)$ and can be expressed as

$$h_0(n) = (-1)^n G_0(n) \quad (10)$$

Full algorithm for the design of filter is described in algorithm 1.

Algorithm 1

The algorithm can be expressed as follow

Algorithm (L, n, α,)

Input: L, n, α

L : Length of differentiator

n : Order of polynomial

α : Order of derivative

T : Matrix

a : Constant

W : Window matrix

Γ : Gamma function

Output: W, h_0

begin

for i ← 1 to L

for j ← 0 to n

Calculate matrix T_{ij} .

$T_{ij}(i) = 2iT_j(i) - T_{j-1}(i)$

(Here $T_0(i) = 1$, $T_1(i) = i$)

end for

end for

for i ← 1 to L

for j ← 0 to α

Calculate $a = \left[\frac{\Gamma(n+1)}{\Gamma(n+1-\alpha)} i^{n-\alpha} \right]$

$W_i^{(\alpha)} = a(T^T T)^{-1} T^T$

$G_0(i) \leftarrow W_i^{(\alpha)}$

$h_0(i) = (-1)^L G_0(i)$

end

end

end

III. ALGORITHM FOR EDGE DETECTION OF LOW CONTRAST IMAGES

The preprocessing has been performed on the input image before applying the Laplacian of Gaussian method for edge detection. Normalization operation has been performed on the input image (I). The normalized image, (I_n), has been processed by newly designed low pass and high pass filters. The output of low pass filtering operation (I_{nl}) has been relaxed with the relaxation coefficient. The relaxation coefficient varies from 0 to 2. The output image of high pass filtering operation, I_{nh} and image I_{nl} has been multiplied. The output image of this operation is input image for Laplacian of Gaussian method (LoG). The Laplacian of Gaussian algorithm has been performed on I_l . The algorithm has been described in algorithm II. Fig. 1 has described the flow chart of proposed algorithm.

Algorithm II

Proposed algorithm is as follow

Algorithm ($I_E, I, h_0, G_0, \lambda$)

Input: I, h_0, G_0, λ

I : Input Image

h_0 : High pass filter

G_0 : Low pass filter

λ : Relaxation coefficient

I_n : Normalized image

I_{nl} : Filtered image with low pass filter

I_{nh} : Filtered image with high pass filter

I_1 : Intermediate Image

I' : Pre-processed Image

LoG () : Standard Laplacian of Gaussian edge detection algorithm

$Normalization$ () : Normalization of image

Output: I_E

begin

$I_n = Normalization(I)$

$I_{nh} = h_0(I_n)$

$I_{nl} = G_0(I_n)$

$I_{lr} = \lambda I_{nl}$

$I_1 = I_{lr} \times I_{nh}$

$I_E = LoG(I_1)$

end

IV. EXPERIMENTS

The proposed algorithm has been validated with the two low contrast images shown in Fig. 2. Images have low contrast properties as well as smooth properties. Test images are available in the open domain [20].

V. RESULTS

The proposed algorithm has been tested on two test images. We have taken $\alpha = 0.5$ and order of polynomial $n = 2$. Four different lengths of filters i.e. $L = 3, 4, 5, 7$ have been taken for validation of our proposed algorithm. The effect of length of filters has been also analyzed. The high pass filter coefficients are calculated from (10). These calculated filters have been used for intermediate stage of preprocessing. We have taken the value of relaxation coefficient, $\lambda = 0.1$ corresponding to every filter.

Figs. 3 (a)-(d) show the test results obtained from proposed algorithm for test case 1 (a) for filter size 3, 4, 5 and 7 respectively. Result obtained by proposed algorithm for test case 1 (b) has been shown in Fig. 4. Figs. 4 (a)-(d) have been reported for filter size 3, 4, 5 and 7 respectively. Figs. 5 (a) and (b) show the result obtained by Laplacian of Gaussian (LoG) for test case 1 (a) and test case 1 (b). Results clearly show that the proposed algorithm has been performed better than the Laplacian of Gaussian algorithm. It is difficult to

predict in edges in Fig. 5. Figs. 3 and 4 are more clear visibility. It has been also reported from results that for lower order filter, results are not clearer.

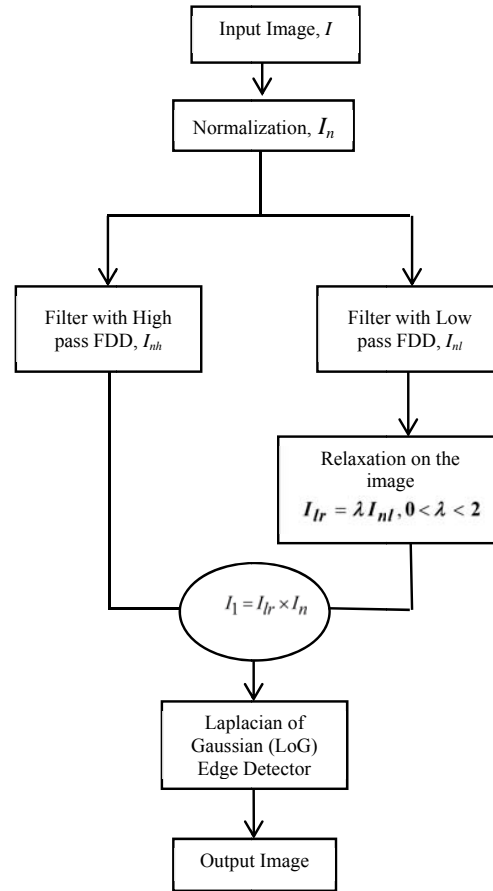


Fig. 1 Flow chart of proposed algorithm

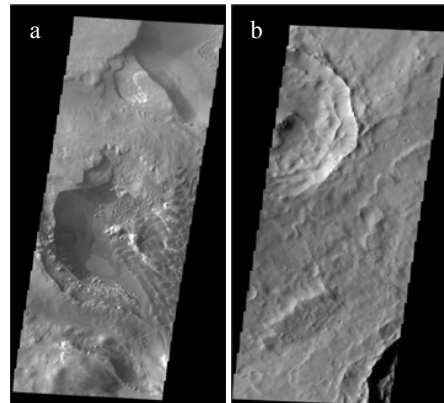


Fig. 2 (a) Test Case I (b) Test case II

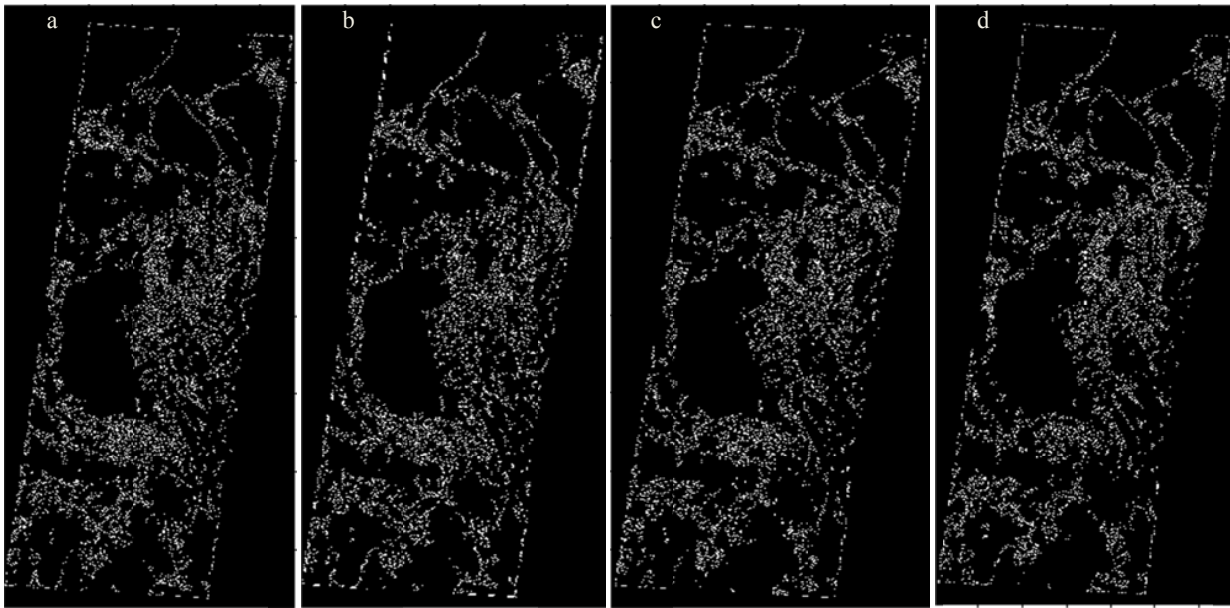


Fig. 3 Edge of proposed image of test case 1 for $L=3, 4, 5$ and 7

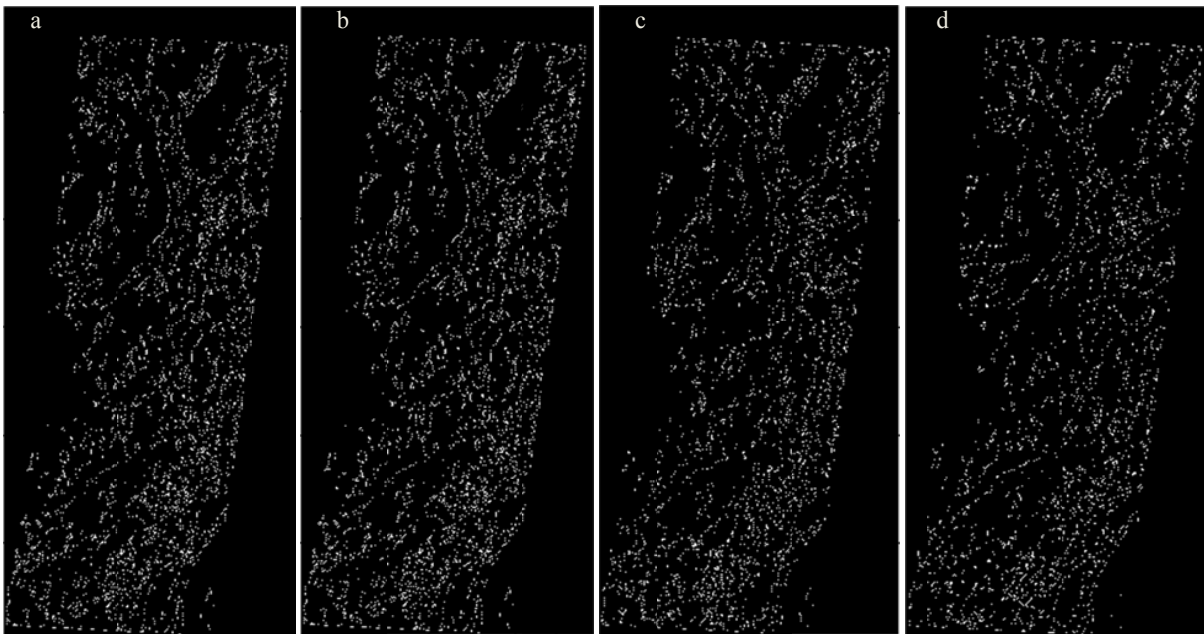


Fig. 4 Edge of proposed image of test case 2 for $L=3, 4, 5$ and 7

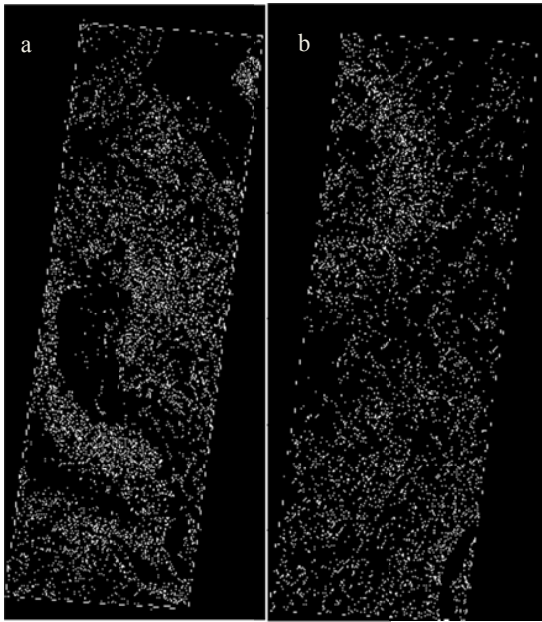


Fig. 5 Edge of image using LoG for test case 1 and 2

VI. CONCLUSION

The major part of algorithm has preprocessing of input image. The preprocessing has been performed by newly designed Chebyshev polynomial based fractional order filter. The Laplacian of Gaussian method has not giving better results in case of low contrast images. Visible results clearly shows that algorithm perform better for higher order of filter.

REFERENCES

- [1] R. C. Gonzalez, Richard E. Woods, Digital Image Processing, Third Edition, Pearson Education, New Delhi, 2009.
- [2] A. Rosenfeld and M. Thurston, "Edge and curve detection for visual scene analysis" IEEE Trans. Comput., vol. C-20, pp. 562-569, 1971.
- [3] D. Man, Vision. San Francisco, CA: Freeman, 1982.
- [4] D. Marr and E. Hildreth, "Theory of edge detection," Proc. Roy. Soc. London, vol. B207, pp. 187-217, 1980.
- [5] A. P. Witkin, "Scale-space filtering" in Proc. IJCAI-8, 1983.
- [6] A. L. Yuille and T. Poggio, "Scaling theorems for zero-crossings" IEEE Trans. Pattern Anal. Machine Intell., vol. PAMI-8, pp. 15-25, 1986.
- [7] J.M.S. Prewitt, "Object Enhancement and Extraction" Picture Processing and Psychopictorics, B. Lipkin and A. Rosenfeld, eds., pp. 75-149. New York: Academic, 1970.
- [8] J. Canny, "A Computational Approach to Edge Detection" IEEE Trans. Pattern Analysis and Machine Intelligence, Vol. 8 (6), pp. 679- 698, Nov. 1986.
- [9] R. Deriche. "Fast algorithms for low-level vision", IEEE Trans, PAMI, Vol. 12 no A pp. 78-87, 1990.
- [10] J. Shen and S. Castan, "An Optimal Linear Operator for Step Edge Detection" Computer Vision Graphics Image Processing, vol. 54 no. 2, pp. 112-13, Mar. 1992.
- [11] S. Lanser and W. Eckstein, "Eine Modification des Deriche-Verfahrens zur Kantendetektion", In B. Radig, ed., Mustererkennung 1991, vol. 290 of Informatik Fachberichte, DAGM Symposium, Munchen, Springer, Berlin, 1991, pp. 151-158.
- [12] B. Jahne, H. Scharr, and S. Korgel, "Principles of filter design", In B. Jahne, H. HauEcker, and P. GeiEJ, eds., Computer Vision and Applications, vol 2, Signal Processing and Pattern Recognition, chapter 6, Academic Press, San Diego. pp 125-151, 1999.
- [13] R.P. Johnson, Contrast based edge detection, Pattern Recognition Vol. 23, pp. 311-318, 1990.

- [14] Koushendra K. Singh, Manish K. Bajpai, Rajesh K. Pandey "A Novel Approach for Enhancement of Geometric and Contrast Resolution Properties of Low Contrast Images" IITDMJ/CSE/2014/PGR0108, 2014.
- [15] D. Chen, Y. Q. Chen, "Digital Fractional Order Savitzky-Golay Differentiator" IEEE trans. on Circuit and System-II: Express Briefs Vol. 58 pp. 758-762, Nov. 2011.
- [16] S. Guillon, P. Baylon, M. Najim, N. Keskes, Adaptive nonlinear filters for 2-D and 3-D image enhancement, Signal Processing, Vol. 67, pp. 237-254, 1998.
- [17] S. K. Mitra, Digital Signal Processing: A computer based approach, McGraw-Hill India edition, 2004.
- [18] T. J. Rivlin, The chebyshev polynomials, John Wiley & Sons press, 1974.
- [19] I. Podlubny, Fractional Differential Equation, Mathematics in Science & Engineering, 198, Academic Press: California, USA.
- [20] <http://photojournal.jpl.nasa.gov/catalog/PIA10300>.

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