

# Dynamics of a Vapour Bubble inside a Vertical Rigid Cylinder with a Deposit Rib

S. Mehran, S. Rouhi, F.Rouzbahani, E. Haghgoo

**Abstract**—In this paper dynamics of a vapour bubble generated due to a local energy input inside a vertical rigid cylinder and in the absence of buoyancy forces is investigated. Different ratios of the diameter of the rigid cylinder to the maximum radius of the bubble are considered. The Boundary Integral Equation Method is employed for numerical simulation of the problem. Results show that during the collapse phase of the bubble inside a vertical rigid cylinder, two liquid micro jets are developed on the top and bottom sides of the vapour bubble and are directed inward. Results also show that existence of a deposit rib inside the vertical rigid cylinder slightly increases the life time of the bubble. It is found that by increasing the ratio of the cylinder diameter to the maximum radius of the bubble, the rate of the growth and collapse phases of the bubble increases and the life time of the bubble decreases.

**Keywords**—Vapour bubble, Vertical rigid cylinder, Boundary element method.

## I. INTRODUCTION

**D**YNAMICS of a vapour bubble generated due to a local energy input in the vicinity of different kinds of surfaces is of significant importance in medicine and industry. Numerical and experimental results have shown that a vapour bubble generated due to a local energy input in the vicinity of a rigid boundary is attracted by the rigid surface. In this case during the collapse phase of the bubble a liquid micro jet is developed on the far side of the bubble from the rigid boundary and is directed towards it [1-3]. Numerical results have also shown that during the growth and collapse of a vapour bubble generated due to a local energy input beneath a free surface, the vapour bubble have a different behaviour. In this case the vapour bubble is repelled by the free surface. During the collapse phase a liquid micro jet is developed on the closest side of the bubble to the free surface and is directed away from it [4-6]. In the case of the pulsation of the vapour bubble near a rigid surface, the impingement of the liquid micro jet to the rigid surface is an important cause of mechanical erosion. Experimental investigations also show

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that at the end of the collapse phase of the vapour bubble and just before its rebound a shock wave is emitted in the liquid domain. The emission of the shock wave inside the liquid domain is also cause of rigid surface destruction [7].

In this paper dynamics of a vapour bubble inside a vertical rigid cylinder with and without a deposit rib generated due to a high local energy input is numerically investigated by employing the Boundary Integral Equation Method. Different ratios of the rigid cylinder diameter to the maximum radius of the bubble and different sizes of the deposit rib are considered. Numerical study on the behaviour of a vapour bubble inside a vertical rigid cylinder is of great importance in medicine and industry.

## II. GEOMETRICAL DEFINITION

The vapour bubble is generated due to a local energy input inside a vertical rigid cylinder with and without a deposit rib. The problem is axisymmetric. The vertical and radial axes are shown in figure 1. The vertical rigid cylinder is assumed as a long pipe.

## III. HYDRODYNAMIC EQUATION

The liquid flow around the vapour bubble is assumed to be inviscid, irrotational and incompressible and the surface tension is neglected. Therefore the liquid flow around the vapour bubble is a potential flow and the Green's integral formula governs the hydrodynamic behaviour of the problem.

$$C(p)\phi(p) + \int_S \phi(q) \frac{\partial}{\partial n} \left[ \frac{1}{|p-q|} \right] dS = \int_S \frac{\partial}{\partial n} [\phi(q)] \left[ \frac{1}{|p-q|} \right] dS \quad (1)$$

where  $\phi$  is velocity potential and  $S$  is the boundary of the liquid domain which includes the bubble boundary and the internal surface of the vertical rigid cylinder.  $p$  is any point on the boundary or in the liquid domain and  $q$  is any point on the boundary.  $C(p)$  is  $2\pi$  when  $p$  is on the boundary and is  $4\pi$  when  $p$  is inside the liquid domain.

The unsteady Bernoulli equation in its Lagrangian form is used for calculating the velocity potential at the successive time steps and is given as

$$\frac{D\phi}{Dt} = \frac{P_\infty - P_b}{\rho} + \frac{1}{2} |\nabla \phi|^2 \quad (2)$$

where  $P_\infty$  is pressure in the far field,  $P_b$  is pressure inside the vapour bubble,  $\rho$  is density and  $t$  is time.

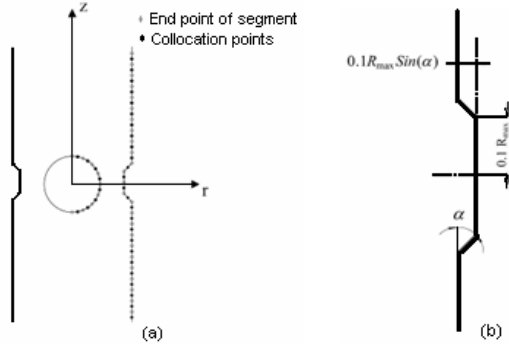


Fig. 1 (a) Discretized representation of the vapour bubble and internal surface of the vertical rigid cylinder (b) Geometrical characteristics of deposit rib inside the vertical rigid cylinder

#### IV. DISCRETIZATION OF THE BOUNDARIES

The boundary of the vapour bubble is discretized into  $M$  cubic spline elements and the internal surface of the vertical rigid cylinder is divided into  $2N$  linear segments as shown in Fig.1.

#### V. DISCRETIZATION OF THE EQUATIONS

Equation (3) is a system of linear equations which represents the discretized form of equation (1)

$$2\pi\phi(p_i) + \sum_{j=1}^{M+2N} \left\{ \phi(q_j) \int_{s_j} \frac{\partial}{\partial n} \left[ \frac{1}{|p_i - q_j|} \right] dS \right\} = \sum_{j=1}^{M+2N} \left\{ \frac{\partial}{\partial n} [\phi(q_j)] \int_{s_j} \left( \frac{1}{|p_i - q_j|} \right) dS \right\} \quad (3)$$

Whereas equation (4) represents the discretized form of unsteady Bernoulli equation and allows the velocity potential to be time marched over a time increment of  $\Delta t$ .

$$[\phi_i]_{t+\Delta t} = [\phi_i]_t + \Delta t \left\{ \frac{P_\infty - P_b}{\rho} + \frac{1}{2} |\nabla \phi|^2 \right\} \quad (4)$$

#### VI. EVOLUTION OF THE VAPOUR BUBBLE

A variable time increment is defined as

$$\Delta t = \min \left| \frac{\Delta \phi}{\frac{P_\infty - P_c}{\rho} + \frac{1}{2} (\psi^2 + \eta^2)} \right| \quad (5)$$

where  $\Delta \phi$  is some constant and represents the maximum increment of the velocity potential between two successive time steps. Also  $P_c$  is saturated vapour pressure,  $\psi$  is normal

velocity on the boundary of the liquid domain and  $\eta$  is tangential velocity.

Mathematical model for predicting the initial minimum radius of the bubble and the corresponding pressure in its inside is based on the Rayleigh equation [8] which is developed by Best [9]. Details of the calculations are given by Shervani-Tabar et. al. [10].

#### VII. NONDIMENSIONAL PARAMETERS

The problem under investigation is non-dimensionalized by employing maximum radius of the bubble,  $R_m$ , diameter of the vertical rigid cylinder,  $D$ , liquid density,  $\rho$ , pressure in the far field,  $P_\infty$  and saturated vapour pressure  $P_c$ . The non-dimensional parameters which are used in this paper are defined as:

$$\lambda = \frac{D}{R_m}, \tau = \frac{t}{R_m} \left( \frac{P_\infty - P_c}{\rho} \right)^{\frac{1}{2}}, \Psi = \psi \left( \frac{\rho}{P_\infty - P_c} \right)^{\frac{1}{2}}. \quad (6)$$

(6)

#### VIII. NUMERICAL RESULTS AND DISCUSSION

The effect of deposit rib inside the vertical rigid cylinder has been studied. Geometrical characteristics of the rib are shown in Fig. 1.

Figures 2, 3, and 4 illustrate the growth and collapse of a vapour bubble inside a vertical rigid cylinder with the deposit rib with different  $\lambda$  s. As it is shown, the vapour bubble during its growth phase elongates in the vertical direction. During the collapse phase two broad liquid micro jets are developed on top and bottom sides of the vapour bubble and directed inward.

Figure 5 shows the variation of the vapour bubble volume with respect to non dimensional time when the bubble is inside a vertical rigid cylinder with different values of  $\lambda$  and in the presence of a deposit rib.

Figure 6 illustrates the variation of non-dimensional velocity of liquid micro jets with respect to non-dimensional time.

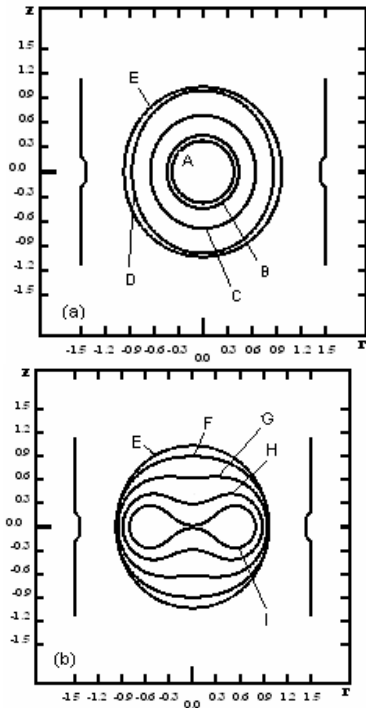


Fig. 2 Growth and collapse of a single vapour bubble,  $\lambda = 3$ , (a) Growth phase: A)0.00364, B)0.14384, C)0.50346, D)1.36254, E)2.46015, (b) Collapse phase: E)2.46015, F)3.09044, G)3.68707, H)4.16801, I)4.49676.

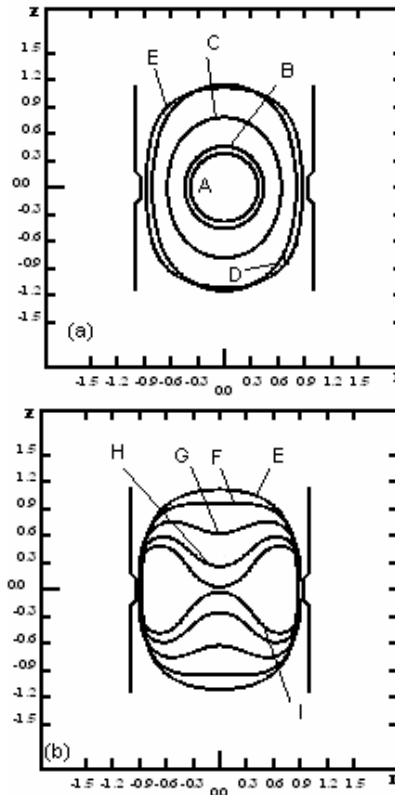


Fig. 4 Growth and collapse of a single vapour bubble inside a vertical rigid cylinder with a deposit rib in the absence of buoyancy forces with  $\lambda = 2$ , (a) Growth phase: A)0.00364, B)0.17767, C)0.66813, D)1.75528, E)2.91754, (b) Collapse phase: E)2.91754, F)3.40211, G)3.98567, H)4.50366, I)4.8100

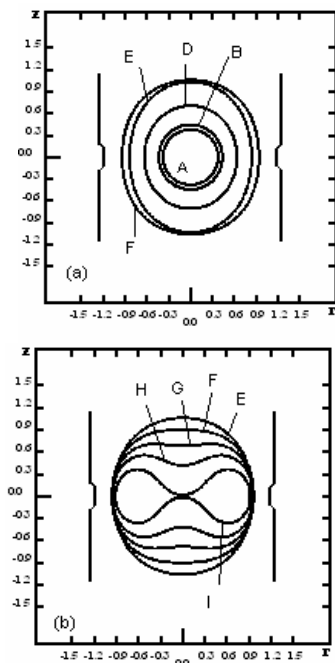


Fig. 3 Growth and collapse of a single vapour bubble,  $\lambda = 2.5$ , (a) Growth phase: A)0.00364, B)0.15202, C)0.54476, D)1.45129, E)2.67847, (b) Collapse phase: E)2.67847, F)3.26957, G)3.71969, H)4.11747, I)4.63179

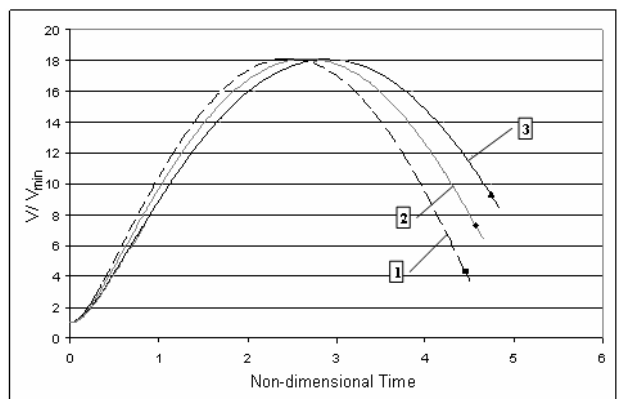


Fig. 5 Variation of rational volume of the bubble against non-dimensional time. (1)  $\lambda = 3$ , (2)  $\lambda = 2.5$ , (3)  $\lambda = 2$

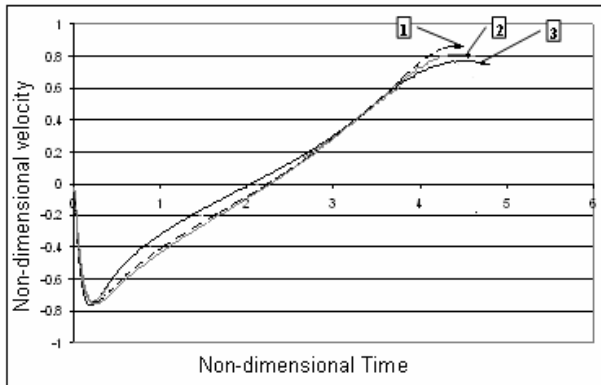


Fig. 6 Variation of non-dimensional velocity of liquid jets against non-dimensional time (1)  $\lambda = 3$ , (2)  $\lambda = 2.5$ , (3)  $\lambda = 2$

#### IX. CONCLUSION

In this paper dynamic behaviour of a vapour bubble generated due to a local energy input in the absence of buoyancy forces is numerically investigated by employing a Boundary Integral Equation Method.

Numerical results show that the vapour bubble during its growth phase elongates in the vertical direction. During the collapse phase of the bubble two broad liquid micro jets are developed on the top and bottom sides of the bubble and are directed inward.

Results also show that by increasing  $\lambda$ , the ratio of the cylinder diameter to the maximum radius of the bubble, the rate of the growth and collapse of the vapour bubble inside a vertical rigid cylinder becomes higher and the life time of the bubble becomes shorter.

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