# Dissociation of CDS from CVA Valuation under Notation Changes

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***Abstract*—**In this paper the CVA computation of interest rate swap is presented based on its rating. Rating and probability default given by Moody's Investors Service are used to calculate our CVA for a specific swap with different maturities. With this computation the influence of rating variation can be shown on CVA. Application is made to the analysis of Greek CDS variation during the period of Greek crisis between 2008 and 2011. The main point is the determination of correlation between the fluctuation of Greek CDS cumulative value and the variation of swap CVA due to change of rating.

***Keywords*—**CDS, Computation, CVA, Greek Crisis, Interest Rate Swap, Maturity, Rating, Swap.

## I. Introduction

THE Credit Value Adjustment (CVA) is the market value of counterparty risk. This risk has been underestimated for a long time but nowadays after Basel II and III regulation rules, it is becoming a major concern for the authorities. They want the banks to keep some reserve funds in order to survive if they have to face a crisis or at least a default of counterparty. As they did not give a method to evaluate the amount of these funds, it is of crucial importance for banks to find the best correct estimate of funds they have to keep. Even though the concept of default and the financial risk it implies have been established a long time ago, financial authorities have only recently realized the need for banks to have funds to ensure counterparty risk. First minimum capital requirements for banks protection have been set in Basel I 1988 Agreements, in the same period the Value-at-Risk, representing largest losses not to be exceeded for fixed probability and horizon, was created for insurance sector and later extended in 2005 Base II rules for bank sector [1]. However, before 21st century, counterparty risk was not taken into consideration nor controlled, and banks were imposing their own credit limits to guarantee their potential exposure. Also, volatility of banks' credit spread was small so use of CVA was not relevant. After Asian financial crisis and the fears of financial contagion to the rest of the world, Basel II Agreements in 2004-2005 contains obligation for banks to calculate CVA each month for estimating the counterparty risk they are potentially exposed to. New standards have been published by the Financial Accounting Standards Board, completing the International Accounting Standard, and in particular the IAS 39, which is about the fair value and the correction of derivatives position value according to counterparty price [2].

Finally, 2007 financial world crisis led Basel Committee to write new agreements for banks to manage counterparty riskin crisis context when some of them went bankrupt or were about to, forcing governments to help them. Basel III Agreements impose banks to put aside more capital in case of emergency and default of payment [3]. CVA was created in 2010, and Basel III proposed two methods to calculate it, the standardized and the advanced ones, and the constraint to choose and use either way to calculate CVA by 2015. Market volatility experienced during 2007 financial crisis has led many firms to review their accounting methods for counterparty credit risk. Traditional approach has been to set limits against future exposures and to check potential threats against these limits. CVA is a way to dynamically price counterparty credit risk directly into new trades. Many banks already measure CVA in their accounting statements, but the financial crisis has led pioneering banks to invest in systems that assess CVA more accurately, and to integrate CVA into pre-deal pricing and structuring. Their expected return on investment is the ability to support future growth by freeing up more capital and minimizing earnings volatility.

## II. CVA Computation of a Swap

Theoretical calculation rests upon the following hypotheses: 1) no netting i.e. there is only one asset, 2) there is no ''wrong way'' risk, 3) CVA is unilateral: only CVA calculation of one counterparty is required for comparison to the CDS, which covers the risk of one counterparty, 4) the different cash flows are paid at the same time and with same frequency. Advanced CVA capital charge is used to model CVA, and calculation is based on the difference between the pricing of a derivative with risk and without risk. This CVA calculation rests upon four different parts: the loss given default, the discount factor, the expected exposure and the variation of counterparty default probability. The calculation method described below is specific for interest-rate swap. Advanced CVA capital charge expression is [4]:

$$\mathrm{CVA} = \mathrm{LGD} \int_0^T P(t) EE^+(t) dPD(0,t) \quad (1)$$

Swaps are generally used in investment risk restructuring. The interest rate risk can be hedged with a swap. This is why unilateral (risk of one counterparty) CVA is here calculated on a swap in order to compare it with CDS which also measures the default risk of a counterparty to default only one

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counterparty. For CVA calculation, swap maturity period [0,T] is split into a continuous compounding period (with interval Δ between two monitoring times) which will be equally spaced by a step of 0.1 from 0 to maturity T. Continuous compounding can be modeled as infinitesimally small period which is important to compute the CVA and also in the calculation of discount factor.

The loss given default (LGD) and the recovery rate are percentage of derivative nominal value. LGD is defined as the percentage of loss in the event of counterparty bankruptcy. The recovery rate is defined as the percentage of the outstanding claim recovered [5]

$$LGD = 1 - R \tag{2}$$

where R = Recovery Rate. LGD is assumed to be constant, which is implied by the market. It is usually defined as 40% following market basics according to Moody's.

Let $D_t$ the Discount factor, T the actual maturity, ΔT the maturity difference between two steps of calculation, and r the continuously compounded annualized floating interest rate of the swap. The discount factor calculation depends on swap pricing model. $D_t$ is the factor by which a future cash flow must be multiplied in order to obtain the present value. In continuous case it is given by [6]

$$Df_t = Df_{t-1}.e^{-r*\Delta T} \tag{3}$$

Probability default variation $(PD(t_i) - PD\,(t_{i-1}))$ is the probability that counterparty defaults between $t_i$ and $t_{i-1}$. Usual method to estimate probability default is to use CDS spread. To calculate counterparty probability default, data provided by Moody's rating agency will be used. This agency provides Corporate Idealized 10-Year Cumulative Probability of Default (PD) Rates (credit rating) [5] for each rating of a counterparty: Aaa Aaa1 Bbb C … These data have been used to create a model giving counterparty probability default from one to ten years with a thread of 0,1.

It is supposed here that the different cash flows are paid at the same time and with the same frequency. The two swap positions are defined by a floating rate defined in the contract which is the Euribor + "fixed number of base points" and by a fixed rate. These factors will define the calculation of implied fixed swap rate and the different forward swap rates needed for swap pricing valuation. The swap rate is fixed as the fixed swap interest rate such that its value is equal to zero at swap beginning (at time t=0). This swap rate is defined as follow:

$$X = \frac{\sum_{i=1}^{n} Df_{T_i} * L_r(T_i)}{\sum_{i=1}^{n} Df_{T_i}} \tag{4}$$

where $L_r(T_i)$ is the Libor rate, $Df_{Ti}$ the discount factor, and $T_i$ the maturity. The forward swap rate of each cash flow payment is calculated with the same swap rate formula but with different starting maturity. Indeed, for the first swap rate calculation, the maturity of the sum is started at the step after maturity of first cash flow. With this method the swap rate is calculated without considering the first floating cash flow giving the swap forward rate.

The volatility represents the intensity of swap rate variation. This variation reflects expectations about future rate evolution based on swap underlying (Euribor floating leg reference). We use the implied volatility based on curve rates to calculate this future volatility in CVA pricing of derivatives. In present case the historical asset volatility is used: the CVA is calculated during Greek crisis to a horizon of five years from 2008 to 2013. The sigma used by market agents to compute prices is historically around 20 %.

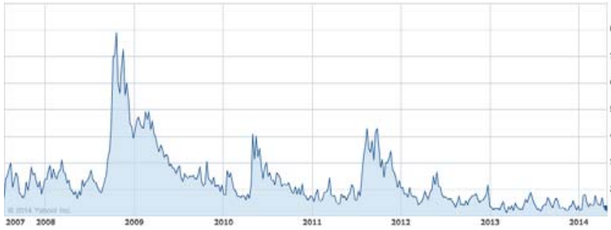


Fig. 1 Volatility S&P 500 (VIX) from 2008 to 2014

The pricing model of European swap with swaption method (Black model) is equivalent to the calculation of expected exposure when performed for each payment. Indeed, the CVA for a swap can be constructed as a function of swaption (reverse swap at each payment date) with different exercises dates for each payment [7]. Intuitively if the counterparty defaults, it is equivalent to cancel the swap trade and taking the "option" of default which means executing the reverse position of the swap. The results are shown on Figs. 2 and 3 when using this method for both parties (payer and receiver):

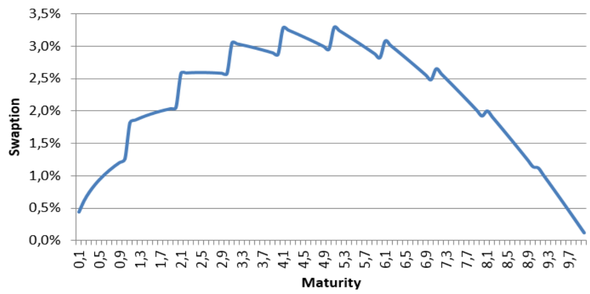


Fig. 2 Expected Swap Exposure (Payer Side)

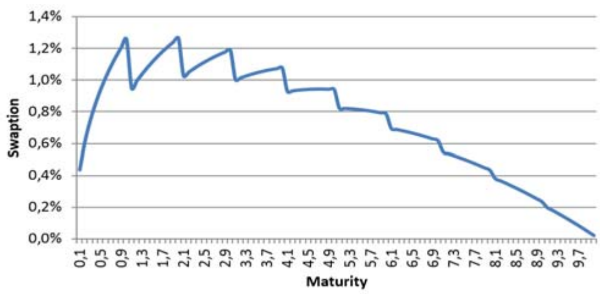


Fig. 3 Expected Swap Exposure (Receiver Side)

Calculation with Black model is performed for each maturity payment date and gives the pricing of swap payoff for each swap expiry date (1 year, 2 years, 3 years…). Then this swaption valuation is divided by the discount factor for each maturity step $T_i$ to get the expected exposure actualized to the date of maturity calculation. Each calculation is based on forward swap rate and implied fixed rate. The variation of maturity used in the calculation is a step of 0.1. The calculation is made within two different positions: the payer (the counterparty concerned on the CVA pays fix interest and receives floating rate) and the receiver (the counterparty concerned on the CVA pays floating rate and receives fixed rate). In consequence there are two different CVAs depending on counterparty position.

In this pricing model of expected exposure (swaption) the different parameters are the actual date of swaption calculation $T_i$, the swap expiry T, the variable leg $i_{sr}$ (forward swap rate at time $T_i$), the fixed leg $i_f$ (fixed swap rate), the Gaussian distribution function $\phi$, and the volatility $\sigma$ of forward swap rate obtained by market data.

Supposing that the payment of both counterparties happens at the same time, and that the swap includes both payments at floating forward swap rate and payments at the implied fixed rate, made at each of the n reset payment dates, the calculation of swaption is compounded by two important factors, the swaption payoff and the underlying swap duration. These two factors increase and decrease monotonically with maturity time, and swaption value peaks somewhere in-between [8]. One gets the expression of valuation of payer swaption position [9]:

$$EE_{Pay(i)} = A * [i_{sr} * \phi\,(d_1) - i_f * \phi\,(d_2)] \quad (5)$$

Of valuation of receiver swaption position:

$$EE_{Rec(i)} = A * [i_f * \phi\,(d_1) - i_{sr} * \phi\,(d_2)] \quad (6)$$

And of underlying swap duration:

$$A = \sum_{i=1}^{n} e^{-iT_i} * (T_i - T_{i-1}) \quad (7)$$

In (5) and (6) the two payoffs spread probability $d_1$and $d_2$ are given by:

$$d_1(i) = \frac{\ln\frac{i_{sr}}{i_f} + 0.5 * \sigma^2 * T_i}{\sigma * \sqrt{T_i}}$$

$$d_2(i) = \frac{\ln\frac{i_{sr}}{i_f} - 0.5*\sigma^2*T_i}{\sigma*\sqrt{T_i}} = d1 - \sigma * \sqrt{T_i} \quad (8)$$

## III. Simulation and Data

Rating agencies are agents or institutions giving financial rating to companies and governments according to criteria defined by market actors. There were 150 registered agencies in 2010, out of which Moody's controls 40% of rating business according to following criteria: objectivity, transparency, independence from government, information of public, sufficient recourse level and credibility. Each rating agency has its own rating system. Schematically, rating degrees are from A to D with twenty-one intermediate notches. Moody's rating is based on transmitter ability to guarantee the payment of a commitment. Moody's did set up a table of idealized Probability of Default rates, which can be used to give a quantitative value to the default risk. The probability default rates are based both on collected data and on perspective for default rate. For a company seeking to finance itself, its rating will determine operation conditions. Through bank financing or by issuing bonds on the market, higher rate will give the company more opportunity to get cheap funds at low interest rates. Conversely, bad rating implies higher interest rates and difficulty to establish adequate funding, a particularly important problem for countries or companies belonging to "speculative" category. [10]

The rating is not fixed and evolves throughout financial products life with consequences on their price. Investors are very sensitive to rating changes. Ratings are not recommendation to buy or sell these products, it is only the credit risk estimation at a given instant and not a guarantee that the issuer is a good investment or is risk-free. Furthermore, there is, sometimes, a lag between the moment where the rating of an investment should be decreased and the moment where the rating agencies actually reduce it. Others critics are made against rating agencies and cast doubt upon the usefulness of current rating.

For CVA calculation, the default probability according to the maturity and the rating are needed. This is why, to create a model for ten years, we used the Corporate Idealized 10-Year Cumulative Probability of Default Rates given by Moody's [5]. Thus, the following created spreadsheet shows the evolution of default probability for a company. Depending on the rating, the default probability will progress differently. The lower the rate, the faster the probability default increases with maturity.

It is now possible to analyze CVA fluctuations dependence on rating fluctuations. From (1) and based on specific considered swap one gets CVA Swap fluctuations vs rating. However as different CVA variation are obtained for different maturity, it is more interesting to use more significant variations between each rating, as displayed on Table I.

TABLE I
CVA Swap Fluctuation Depending on Rating

| Maturity | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| AAA | 0,00% | 0,00% | 0,00% | 0,00% |
| Aa1 | 0,00% | 0,00% | 0,00% | 0,00% |
| Aa2 | 0,00% | 0,00% | 0,00% | 0,01% |
| Aa3 | 0,00% | 0,00% | 0,01% | 0,03% |
| A1 | 0,00% | 0,01% | 0,02% | 0,05% |
| A2 | 0,00% | 0,01% | 0,04% | 0,08% |
| A3 | 0,00% | 0,02% | 0,07% | 0,14% |

TABLE II
IRS CVA FLUCTUATIONS BY DOWNGRADE AND MATURITY

| Maturity | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| AAA | 0 | 0 | 0 | 0 |
| Aa1 | 0,00% | 0,00% | 0,00% | 0,00% |
| Aa2 | 0,00% | 0,00% | 0,00% | 0,01% |
| Aa3 | 0,00% | 0,00% | 0,01% | 0,02% |
| A1 | 0,00% | 0,00% | 0,01% | 0,03% |
| A2 | 0,00% | 0,00% | 0,01% | 0,03% |
| A3 | 0,00% | 0,01% | 0,03% | 0,06% |
| Baa1 | 0,00% | 0,01% | 0,04% | 0,08% |
| Baa2 | 0,00% | 0,02% | 0,05% | 0,11% |
| Baa3 | 0,01% | 0,06% | 0,18% | 0,36% |
| Ba1 | 0,02% | 0,11% | 0,30% | 0,58% |
| Ba2 | 0,03% | 0,16% | 0,42% | 0,81% |
| Ba3 | 0,04% | 0,22% | 0,56% | 1,05% |
| B1 | 0,07% | 0,32% | 0,77% | 1,39% |
| B2 | 0,17% | 0,56% | 1,12% | 1,82% |
| B3 | 0,23% | 0,78% | 1,57% | 2,55% |
| Caa1 | 0,31% | 1,03% | 2,11% | 3,47% |
| Caa2 | 0,46% | 1,51% | 3,00% | 4,84% |
| Caa3 | 1,42% | 4,26% | 7,84% | 11,73% |

## IV. Application to Greek CDS Bonds

Derivatives markets have considerably grown since 1990; in 2000, the total notional principal for outstanding credit derivatives contracts was about $800 billion, and by December 2009, the amount had become $32 trillion [11]. Credit derivatives are contracts where payoff depends on the credit-worthiness of one or more companies or countries. This part shows how credit derivatives CVA fluctuate depending on the rating. Attention is here focused on the most popular credit derivative: the CDS (Credit Default Swap). Furthermore, we are going to study exclusively Greek bonds CDSs, as these ones show intense fluctuations during the crisis period.

A CDS is a contract insuring against default risk of a particular company or country. The company/country is known as the *reference entity* and a default by the company is known as a *credit event*. The insurance buyer obtains the right to sell bonds issued by the company at their face value when a credit event occurs, and the insurance seller agrees to buy the bonds at their face value when a credit event occurs. The CDS buyer makes periodic payments to the seller until the end of CDS life (or until a credit event occurs) [12]. Here, analysis is performed with 5 years Greek bonds CDSs. To model these CDSs fluctuations, a Bloomberg chart [13] has been used. It links CDSs prices and the period ranging from September 2008 to April 2010, which corresponds to the financial crisis and the Greek debt crisis periods. Representative curve exhibiting the main changes of CDSs prices during this period is shown on Fig. 4.

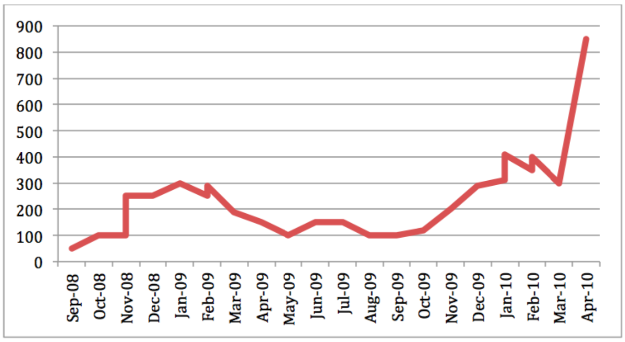


Fig. 4 Greek CDS' variations

It is seen that Greek CDS bonds prices fluctuated strongly from financial crisis collapse (September 08), and particularly from emergence of Greek debt crisis, to 2010. The major causes of these important changes can be listed as follows:

- First, Greek situation starts to deteriorate from the end of 2009, at the explosion of debt crisis. This is why CDS prices are relatively low before this break event (compared to 2010 prices).
- November 09: CDS value begins to rise. In fact, first estimates show a budget deficit of 12.7% of GDP, two times larger than officially announced.
- December 09: Fitch degrades its rating for Greece from A- to BBB+. Standard & Poor's from A- to BBB+, and Moody's from A1 to A2. The CDS value continues to rise, but compared to December 08, it has approximately the same value;
- April 10: Greece formally requests the activation of the bailout plan. It is seen that the CDS value rises considerably from March to April, due to the austerity plan and the large deficit.

In order to distinguish between bullish and bearish periods of CDS value, the curve on Fig. 4 is converted in percentages, see Table III.

TABLE III
CDS FLUCTUATIONS OVER TIME FROM OCT. 2008 TO APRIL 2010

| Month | % |
|---|---|
| October-08 | 6,25% |
| November-08 | 18,75% |
| December-08 | 0,00% |
| January-09 | 6,25% |
| February-09 | -6,25% |
| March-09 | -12,50% |
| April-09 | -5,00% |
| May-09 | -6,25% |
| June-09 | 6,25% |
| July-09 | 0,00% |
| August-09 | -6,25% |
| September-09 | 0,00% |
| October-09 | 2,50% |
| November-09 | 10,00% |
| December-09 | 11,25% |
| January -10 | 12,50% |
| February -10 | -7,50% |
| March -10 | -12,50% |
| April -10 | 62,50% |

- It can then be noticed that, until the end of 2009, CDS prices were relatively stable, as situation in Greece was not a major concern at this time.
- The second period begins at the end of 2009, where the fluctuations are much more pronounced. In fact, this was a full bullish period, with increases ranging from 10% to 62.5%. This is largely due to degradation of Greece by the rating companies and the large deficit of the country.

The CVA computation is based on an estimation of future volatility, which is calculated either by historical method or by implicit method. But even if this volatility is a model of future evolution it does not follow the market: it is an estimation, which is related to the time T when it is made. By taking into account these elements, CVA computation provides the risk calculation of a derivative but is not able to adapt during times of turmoil. However, the expected exposure of CVA computation should estimate the worst probability case of payoff. Present study shows that this is not true in reality. Indeed, CVA fluctuation does not correlate with CDS fluctuation, which is only based on the law of supply and demand. The reason is that volatility calculation at a time T cannot predict future market evolution. With this model of volatility one can only provide an approximation of CVA risk for next period.

## V. Conclusion

From Tab III CDS fluctuations are more important than any CVA fluctuations. The maximum CVA difference calculated was for a swap of maturity 10 years passing from a Caa 2 rating to a Caa 3 rating and it is approximately 26%. The majority of CDS fluctuations are way over this value. This result shows that, in this context of crisis, CDS do not follow the same curve as CVA theoretical calculus. CDS should normally follow the same fluctuations but their intrinsic characteristic as a financial product causes their value to change radically because of speculations and confidence crisis.

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