# Dispenser Longitudinal Movement Control Design Based on Auto - Disturbances – Rejection - Controller

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**Abstract**—Based on the feature of model disturbances and uncertainty being compensated dynamically in auto – disturbances-rejection-controller (ADRC), a new method using ADRC is proposed for the decoupling control of dispenser longitudinal movement in big flight envelope. Developed from nonlinear model directly, ADRC is especially suitable for dynamic model that has big disturbances. Furthermore, without changing the structure and parameters of the controller in big flight envelope, this scheme can simplify the design of flight control system. The simulation results in big flight envelope show that the system achieves high dynamic performance, steady state performance and the controller has strong robustness.

Keywords-ADRC; ESO; nonlinear system.

## I. INTRODUCTION

S the missile dynamics is a coupling, nonlinear system and A the dynamics change with the flight velocity, height, it is difficult to find a control law that can meet the change of the dynamic model to keep the missile has favorable dynamic performances in the whole flight envelope. It is requested to choose different control parameters under different flight conditions if the traditional PID control technology is applied. Then, the Gain Schedule (GS) algorithm is applied to the design. But, GS has a large workload and is time-consuming, inefficient. For the moment, the nonlinear system has no entire and systematic conceptual framework yet, and there is no common method of design and analysis. Although studies have showed that the nonlinear dynamic inversion is an effective method in the area of design of the nonlinear flight control system, it is sensitive to the inaccuracy of model and is difficult to improve the robustness of the controller. Auto-Disturbances-Rejection-Control is a new algorithm of nonlinear control that is developed based on the theory of nonlinear control and can estimate and accomplish the uncertainty [1]. It has been used in the power system, finish turning lathe, chemical process etc.

This paper proposes the ADRC for the dispenser attitude control, which takes into account the nonlinear nature of the system and the coupling among the channels. The control program will increase the anti-interference ability.

The paper is organized as follows. In Section II, the theory of

ADRC is briefly introduced. Then, the missile model and longitudinal movement control problems under consideration are described simply in the Section III. In Section IV, the ADRC based on the techniques of ESO is given. In Section V, an inner loop controller is designed and it comprises the final controller with the ADRC. The simulation results are presented in Section VI and a summary is given in Section VII.

#### II. THE THEORY OF ADRC

Consider the following uncertain system with unknown disturbance:

$$y^{(n)} = f(y, \dot{y}, ..., y^{(n-1)}) + \omega(t) + b_0 u$$
(1)

Where  $f(\cdot)$  and  $\omega(t)$  are unknown, u and y are the control input and the output of the system,  $b_0$  is a known constant. Let $[x_1, x_2, \dots, x_n]^T = [y(t), \dot{y}(t), \dots, y^{(n-1)}]^T$  be the state vector of the system and  $a(t) = f(\cdot) + \omega(t)$  which contains the model uncertainty and the external disturbance is called as Extended State Observer (ESO). Let  $b(t) = \dot{a}(t)$ , then the extended state equation is as follow:

$$\begin{aligned} \dot{x}_{1} &= x_{2}, \dot{x}_{2} = x_{3}, \cdots, \dot{x}_{n-1} = x_{n} \\ \dot{x}_{n} &= a(t) + b_{0}u \\ \dot{x}_{n+1} &= b(t) \\ y &= x_{1} \end{aligned}$$
(2)

Based on the equation (2), the ESO is obtained

$$\begin{cases} \dot{z}_1 = z_2 - b_1 \cdot g_1(z_1 - x_1) \\ \dot{z}_2 = z_3 - b_2 \cdot g_2(z_1 - x_1) \\ \dots \\ \dot{z}_n = z_{n+1} - b_n \cdot g_n(z_1 - x_1) + b_0 u \\ \dot{z}_{n+1} = -b_{n+1} \cdot g_{n+1}(z_1 - x_1) \end{cases}$$
(3)

Where  $g_1, g_2, \dots, g_{n+1}$  are suitable nonlinear functions,  $b_1, b_2, \dots, b_n$  are constant coefficients. By choosing the parameters

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properly, the ESO can track the states stably:  $z_1(t) \rightarrow x_1(t), \dots, z_n(t) \rightarrow x_n(t), z_{n+1}(t) \rightarrow x_{n+1}(t) = a(t)$ .

The ADRC arranges the transient process via the Tracking Differentiator (TD) by which can get the reference input and its approximate tracking values of derivatives:  $v_1(t), v_2(t), \dots v_n(t)$ .

Finally, the control law can be designed as:

$$u(t) = k_1 h_1(v_1 - z_1) + k_2 h_2(v_2 - z_2) + \dots + k_n h_n(v_n - z_n) + z_{n+1} / b_0$$
(4)

Where  $h_1, h_2, \dots h_{n+1}$  are suitable nonlinear functions,  $k_1, k_2, \dots k_n$  are constant coefficients. The feedback quantity  $z_{n+1}/b_0$  can compensate the system disturbance and just because of the compensation makes the system has strong robustness.

The choice of the TD, ESO and the whole structure of the ADRC can refer to the references [2]-[3].

## III. DISPENSER DYNAMIC MODEL

The model for the motion of a dispenser is as follow:

$$\begin{aligned} m(\dot{V}_{x1} + \omega_y V_{z1} - \omega_z V_{y1}) &= F_x \\ m(\dot{V}_{y1} + \omega_z V_{x1} - \omega_x V_{z1}) &= F_y \\ m(\dot{V}_{z1} + \omega_x V_{y1} - \omega_y V_{x1}) &= F_z \end{aligned}$$

$$(5)$$

$$\begin{cases} \dot{\omega}_{x} = \frac{M_{x}}{J_{x}} - \frac{J_{z} - J_{y}}{J_{x}} \omega_{y} \omega_{z} \\ \dot{\omega}_{y} = \frac{M_{y}}{J_{y}} - \frac{J_{x} - J_{z}}{J_{y}} \omega_{x} \omega_{z} \\ \dot{\omega}_{z} = \frac{M_{z}}{J_{z}} - \frac{J_{y} - J_{x}}{J_{z}} \omega_{x} \omega_{y} \end{cases}$$
(6)

$$\begin{cases} \dot{\alpha} = \omega_{z} + (\omega_{y} \tan(\alpha) \tan(\beta) - \omega_{x} \tan(\beta) - \frac{F_{x} \tan(\alpha)}{mV_{x1}} - \frac{F_{y}}{mV_{x1}}) \cos(\alpha) \cos(\alpha) \\ \dot{\beta} = \omega_{y} + (\omega_{z} \tan(\beta) \tan(\alpha) + \omega_{x} \tan(\alpha) - \frac{F_{x} \tan(\beta)}{mV_{x1}} + \frac{F_{z}}{mV_{x1}}) \cos(\beta) \cos(\beta) \end{cases}$$
(7)

Where  $V_{x1}, V_{y1}, V_{z1}$  are velocity components,  $\omega_x, \omega_y, \omega_z$  are angular velocity components,  $\alpha, \beta$  are attack angle and sideslip angle,  $F_x, F_y, F_z$ ,  $M_x, M_y, M_z$  are components of aerodynamic force and aerodynamic moments,  $J_x, J_y, J_z$  are moments of inertia.

In this paper, we study the longitudinal movement only. The pitch aerodynamic moment is  $M_z = M_z^{\alpha} \alpha + M_z^{\omega_z} \omega_z + M_z^{\delta_z} \delta$ , and the aerodynamic forces are

$$F_{x} = -X \cos \alpha \cos \beta + Y \sin \alpha - Z \cos \alpha \sin \beta$$
  

$$F_{y} = X \sin \alpha \cos \beta + Y \cos \alpha + Z \sin \alpha \sin \beta$$
,  
Where  

$$Y = Y^{\alpha} \alpha + Y^{\delta_{z}} \delta_{z} + \Delta Y^{\alpha} \alpha + \Delta Y^{\delta_{z}} \delta_{z}$$
,  

$$Z = Z^{\beta} \beta + Z^{\delta_{y}} \delta_{y} + \Delta Z^{\beta} \beta + \Delta Z^{\delta_{y}} \delta_{y}$$

Ignoring the effects of gravity, considering the motion of short cycle only and supposing the velocity changes slowly, the flight velocity can be seen as constant. Then the longitudinal movement equation is:

$$\begin{cases} \dot{\alpha} = -a_{34}\cos(a)\alpha + \omega_z - \omega_x \tan(\beta)\cos(a)\cos(a) \\ + \omega_y \tan(\beta)\sin(a)\cos(a) - a_{35}\cos(a)\delta_z \\ \dot{\omega}_z = -a_{22}\omega_z - a_{25}\delta_z - a_{24}\alpha + a_{06}\omega_x\omega_y \end{cases}$$
(8)

Where

$$a_{22} = -m_{z}^{\sigma_{z}}QSL^{2}/(2J_{z}V) = M_{z}^{\omega_{z}}/J_{z}$$

$$a_{24} = -57.3m_{z}^{a}QSL^{2}/J_{z} = M_{z}^{a}/J_{z}$$

$$a_{25} = -57.3m_{z}^{\delta_{z}}QSL^{2}/J_{z}$$

$$a_{34} = (57.3C_{y}^{a}QS + P)/mV$$

$$a_{35} = 57.3C_{y}^{\delta_{z}}QS/mV$$

$$a_{06} = -(J_{y} - J_{x})/J_{z}$$

# IV. DESIGN OF ADRC

Type (8) shows that the longitudinal movement of a dispenser can be described as two first-order differential equations of attack angle and pitch angular velocity. Two first-order ADRCs can be designed to meet the control problem. For the first equation of (8), assume  $\alpha$  as the state variable,  $\omega_{z}$ as the control quality and as  $-\omega_x \tan(\beta)\cos(\alpha)\cos(\alpha) + \omega_y \tan(\beta)\sin(\alpha)\cos(\alpha) - a_{35}\cos(\alpha)\delta_z$ the disturbance. The command  $\omega_{rd}$  for the second ADRC can be obtained from the first ADRC designed based on the first equation. Then, assume  $\omega_{z}$  as the state variable,  $\delta_{z}$  as the control quality and  $-a_{24}\alpha + a_{06}\omega_x\omega_y$  as the disturbance for the second equation. Design the second ADRC, and the final control law will be got.

*A.* Design the First ADRC based on the First Equation The TD is:

$$\begin{cases} e_0 = \alpha_{11} - \alpha_d \\ \dot{\alpha}_{11} = -R_1 fal(e_0, a_0, \delta_0) \end{cases}$$
(9)

$$fal(e, a, \delta) = \begin{cases} |e|^{a} \operatorname{sign}(e), & |e| > \delta \\ e / \delta^{1-a}, & |e| \le \delta \end{cases}$$

Where,  $\alpha_{11}$  is the estimation of  $\alpha_d$ ,  $fal(\cdot)$  is a suitably constructed non-smooth function.

To obtain a real-time estimation for  $\alpha$  and the disturbance, an ESO is designed as:

$$\begin{cases} e_{\alpha} = z_{11} - \alpha \\ \dot{z}_{11} = z_{12} - \beta_{01} e_{\alpha} + b_0 \omega_{zd} \\ \dot{z}_{12} = -\beta_{02} fal(e_{\alpha}, a_1, \delta_1) \end{cases}$$
(10)

Where,  $z_{11}$  tracks the state  $\alpha$ ,  $z_{12}$  estimates the model uncertainty and the external disturbance.

Then, design the Nonlinear State Error Feedback (NLSEF) as:

$$\begin{cases} e_{1} = \alpha_{11} - z_{11} \\ \omega_{zd0} = \beta_{1} fal(e_{1}, a_{2}, \delta_{2}) \end{cases}$$
(11)

Therefore, the attract angle ADRC law is

$$\omega_{zd} = \omega_{zd0} - k_{11} z_{12} \tag{12}$$

B. Design the Second ADRC based on the Second Equation Regarding  $\omega_{zd}$  as the control input of the second ADRC,

design the second ADRC base on the second equation of (8). The proper TD is:

$$\begin{cases} e = \omega_{11} - \omega_{zd} \\ \dot{\omega}_{11} = -R_2 fal(e, a_3, \delta_3) \end{cases}$$
(13)

Where,  $\omega_{l1}$  is the estimation of  $\omega_{zd}$ . Then the ESO is designed as:

$$\begin{cases} e_{\omega} = z_{21} - \omega_z \\ \dot{z}_{21} = z_{22} - \beta_{03} * e + a_{25} * \delta_{z0} \\ \dot{z}_{22} = -\beta_{04} fal(e, a_4, \delta_4) \end{cases}$$
(14)

Where  $z_{21}$  tracks the state  $\alpha$ ,  $z_{22}$  estimates the model uncertainty and the external disturbance.

Design the NLSEF as follow:

$$\begin{cases} e_2 = \omega_{zd} - z_{21} \\ \delta_{z0} = \beta_2 fal(e_2, a_5, \delta_5) \end{cases}$$
(15)

Finally, the pitch angular velocity ADRC law is:

$$\delta_z = \delta_{z0} - k_{12} z_{22} \tag{16}$$

For appropriate values of parameters in the equations, the ESO estimates not only the state but also the total disturbance such as the coupling effects, the dynamic uncertainties and unknown factors from the environment.

# V. INNER-LOOP FEEDBACK DESIGN

Usually, the damping of a non-controlled missile is small when it flight in the high-altitude. In order to increase the response speed and reduce the accommodation time, an inner-loop feedback controller is designed to the control object. The feedback control can decrease the uncertainty scope and make the input-output characteristic stay the same under all kinds of flight conditions. Furthermore, it can improve the anti-interference ability of the system. The Linear Quadratic Regulator (LQR) is used to design the inner-loop controller. When the dispenser flight in the BTT mode, the attack angle and sideslip angle are small, then the simplified linear model of (8) is:

$$\begin{cases} \dot{\alpha} = -a_{34}\alpha + \omega_z - \beta\omega_x + \omega_y - a_{35}\delta_z \\ \dot{\omega}_z = -a_{22}\omega_z - a_{25}\delta_z - a_{24}\alpha + a_{06}\omega_x\omega_y \end{cases}$$
(17)

Based on the linear model given above, the controller parameters can be obtained easily. The controller changes while regulating the weighting coefficients Q and R of the target function:

$$J = \frac{1}{2} \int \left( X^{T} Q X + u^{T} R u \right) dt$$

Fig.1 shows the diagram of ADRC and inner-loop feedback controller designed for dispenser longitudinal movement control.



Fig. 1 Structure of the control system

## VI. SIMULATION AND ANALYSIS

A control law for a dispenser is designed using the method proposed in this paper under the flight mode whose constant missile speed is 250m/s and height is 10km. In the simulation, let the attack angle command be 10 degree and the parameters of the ADRC are:

 $R_{1} = 1.5, a_{0} = 0.25, \delta_{0} = 0.01$   $R_{2} = 0.4, a_{3} = 0.15, \delta_{3} = 0.05$   $a_{1} = 0.1, \beta_{01} = 8, \beta_{02} = 0.4, \delta_{1} = 0.05, b_{0} = 1$   $a_{4} = 0.75, \beta_{03} = 5.5, \beta_{04} = 0.01, \delta_{4} = 0.05, b_{1} = a_{25}$   $a_{2} = 0.5, \beta_{1} = 6, \delta_{2} = 0.05, k_{11} = 1$   $a_{5} = 0.5, \beta_{2} = 7, \delta_{5} = 0.05, k_{11} = 0.5 + 1/a_{25}$ The singular product of the second second

The inner-loop controller is [-5.8043 14.8563].

Choosing three different flight conditions in the flight envelop, keeping the control parameters unchanged, and then we can study the dynamic performance of the system and the robustness of the control law. The three flight conditions are as follow: (1) height H=10000m, velocity V=250m/s; (2) height H=4000m, velocity V=250m/s; (3) height H=100m, velocity V=130m/s. In the paper, a PID controller is also designed and simulation results under the three conditions are given.

The simulation results are showed in Fig. 2. Fig. 2(a), (c), (e) are the variation of attack angle of system with ADRC. Fig. 2(b), (d), (f) are the variation of attack angle of system with PID controller.

The simulation results show that, for system with ADRC, the

attack angle can reach the command value quickly and the steady-state error is small. Without changing the control parameters, the ADRC can meet the variable characters of the dispenser under different flight conditions which illustrates that the controller has strong robustness. Fig. 2(g) is the variation of attack angle when there is random disturbance. It shows that the ADRC can accomplish the external disturbance.

The system with PID controller also gets satisfying result under the first flight condition, but there will be larger steady state error in the other flight conditions. Fig. 2(f) is the variation of attack angle when there is random disturbance and it is worse than Fig. 2(g).



Fig. 2 Variation of attack angle

# VII. CONCLUSION

In this paper, two ADRCs are designed for the pitch path of the nonlinear BTT dispenser and an inner-loop controller is designed to improve the flight performances also. The ESO which can estimate the unknown nonlinear dynamics is the key to the ADRC, and the NLSEF improves the performances, such as high speed and high accuracy. So, the ADRC suits the systems with unknown external disturbance and strong coupling. The ADRC provides a new idea to deal with the flight control problem which designed based on the nonlinear model directly. Furthermore, without changing the structure and parameters of controller in big flight envelope, this scheme can simplify the design and will have a broad application prospects. The simulation results also show that system has favorable dynamic and steady state performance which illustrate that the ADRC has strong robustness.

#### REFERENCES

- Han J.Q, "Auto Disturbances Rejection Control Technique," Frontier Science, 2007, pp.24-31.
- [2] Han J.Q, "Auto-Disturbances-Rejection Controller and It's Applications," Control and Decision, 1998, vol.13, pp.19-23.
- [3] Han J.Q, "The 'Extended State Observer' of a Class of Uncertain Systems," Control and Decision, 1995, vol.10, pp.85-88.
- [4] Xiong Z.G, "Auto-Disturbances-Rejection Controller Design of Aircraft Pitch Movement." Information and Control, 2005,vol.34, pp.576-579.
- [5] Huang.Y, XU.K.K, "Flight Control Design Using Extended State Observer and Non-smooth Feedback," IEEE Conference on Decision and Control, Orlando, Florida USA,12.2001.
- [6] Qian X.F, Lin R.X, Zhao.Y.N, *Missile Flight Mechanics*, BeiJing: Beijing Institute of Technology press, 1997.
- [7] Lin B, "Control and Guidance System Study of Bank to Turn Airborne Dispenser," Ph.D. Dissertation of BIT, unpublished.