Development of Active Learning Calculus Course for Biomedical Program

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Abstract—The paper reviews design and implementation of a Calculus Course required for the Biomedical Competency Based Program developed as a joint project between The University of Texas Rio Grande Valley, and the University of Texas' Institute for Transformational Learning, from the theoretical perspective as presented in scholarly work on active learning, formative assessment, and on-line teaching. Following a four stage curriculum development process (objective, content, delivery, and assessment), and theoretical recommendations that guarantee effectiveness and efficiency of assessment in active learning, we discuss the practical recommendations on how to incorporate a strong formative assessment component to address disciplines' needs, and students' major needs. In design and implementation of this project, we used Constructivism and Stage-by-Stage Development of Mental Actions Theory recommendations.

Keywords—Active learning, assessment, Calculus, cognitive demand, constructivism, mathematics, Stage-by-Stage Development of Mental Action Theory.

I. INTRODUCTION

THIS paper reviews the development of the Calculus I L Course (the Calculus Course further in the text) for the Biomedical Competency Based program (the Program), that has been developed as a joint project between The University of Texas Rio Grande Valley (UTRGV), and the University of Texas' Institute for Transformational Learning (ITL). The UTRGV's Department of Medicine provided Department of Medicine provided leadership in the general curriculum design of the Program, and development of the components directly related to life science and health sciences of the Program. The author of the paper was responsible for coordinating the development of the science component of the Program (chemistry, physics, mathematical statistics, and Calculus 1), and developed the Calculus Course curriculum in collaboration with ITL experts, having in mind the Mission of the Program, Students' Profile, and the Program Design Principles and procedures established by ITL.

The overarching goal of the paper is to discuss the design and development of a specific general education mathematics course for a particular major. Since we developed this course having in mind first and foremost active learning and flipped classroom environment, the important part of the paper is related to the discussion of organizing and managing various activities and formative assessment of these activities. Though the plan was to teach the course first time mostly in the faceto-face format, the intent to develop the curricula for on-line teaching influenced the design of the Calculus Course activities and assessment strategies.

In the second section of this paper, we will review general information about the Biomedical Program development, i.e., mission of the program, students' profile that affects students' performance, and Program Design Principles. All the above factors have had an impact on the design that we chose while developing the Calculus Course. We will also dwell on the mathematics requirements of the Medical School Acceptance Test (MCAT), and how the developed calculus course addresses these requirements.

In the third section, we will discuss the design of the Course Curriculum, and how general recommendations of the curriculum development [1] defined this process. We will discuss the flipped classroom environment, group work, and various types of activities and assessment strategies.

We start section four with the discussion of the theoretical background of active learning and formative assessment of active learning. Since we argue that the total design (including assessment) of any course is as good as the sound learning theory that supports the design, development, and implementation [2], we will briefly introduce basic concepts and principles of the Stage by Stage Development of Mental Actions theory (SSDMA theory), that our design is based on. In the last section we will discuss in more details the activities and developed assessment strategies and instruments in the light of SSDMA's prescriptions and Bloom's Taxonomy perspective. Particularly, we will discuss Check for Understanding Questions (CFU) - brief pre-class activities quizzes, Practice Problems - in class and post-class assignments, and weekly laboratory activities and assignments, specifically devised for the biomedical major.

II. BIOMEDICAL COMPETENCY BASED PROGRAM

This section is based on the materials that have been developed by the Department of Biomedicine at UTRGV and the Institute for Transformational Leaning at the University of Texas System. Though for the most part it is not related directly to mathematics, the information included to this section has significantly influenced the Calculus Course design. This is the reason why we decided to include this section into our paper.

A. Mission and Students' Profile Figures

"...The BS in Biomedical Science program delivers rigorous learning experiences that inspire students to achieve the highest standards of academic excellence. Students develop the competencies needed for success in

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a wide range of health related professional and research fields such as molecular cell biology, molecular genetics and genomics, as well as anatomy, physiology and neuroscience to human health. By grounding learning in the context and communities of the Rio Grande Valley, the program provides a living laboratory where students will gain the knowledge, experience and cultural awareness needed to take on the most critical challenges of 21st century health care."

Below is the Students' profile with a focus on four major areas, economic, academic, cultural, and aspirations.

1. Economic

- Many students fund their education through full time employment and financial aid.
- Family ties are strong and family responsibilities are taken very seriously.
- Many serve as family caretakers; a percentage are single parents. The time commitment this involves can be a challenge, but the goal of helping to support their family can be a source of motivation as well.
- Students often must resolve transportation and other logistical challenges.
- Some students have challenges with legal issues.

2. Academic

- Academic communication skills are variable.
- College readiness skills and critical path science and math skills are variable.
- Past learning experiences for most students have largely involved the memorization of isolated facts.
- Students' SAT scores mirror state average SAT scores.
- Most students encounter their first opportunity to do research in a scientific environment at the college level.
- Many students require guidance on how to navigate the higher education terrain.
- Given opportunities to develop self-direction, assertiveness, self-confidence, grit, flexibility, and self-esteem, these students will see greater levels of academic success.

3. Cultural

- There is a high proportion of non-traditional students who bring greater maturity and adult motivations to the educational experience
- Many are first in family to attend college and would benefit from greater access to academic and professional role models or mentors.
- Some students have inconsistent support systems; others can rely on strong extended family ties for support.
- Many students are Hispanic and from culturally and linguistically blended families; most "bilingual" students are more comfortable using conversational Spanish than academic Spanish; academic proficiency in English in these students is variable.
- Many students want to stay close to home due to strong family ties and a close bond with their local communities.
- With support, students can begin to find balance between

aspirations and their self-perception and how these can be aligned with their college experience.

- Many students have not yet had much exposure to communities and cultures outside of the Rio Grande Valley.
- Gender disparities exist in retention and persistence.

4. Aspirations

- The end goal for most students is to work in a health sciences field and they would benefit from exposure to career opportunities in the health arena.
- Many students have not yet been exposed to a realistic roadmap (time, effort, and certification) to attain different career goals.
- Students aspire to provide financial stability for themselves and their families.
- Most students want to give back to the community, help others, and make a difference

Below we list Program Design Principles that affected the Calculus Class design. At the end of the listed principles, whenever relevant, we included comments directly related to the Calculus Class design.

- Targeted Program Length: Option to accelerate and finish in three, or take a traditional path and finish in four years.
- *Cohort:* One cohort in the fall semester of the first year with options for more points of entry in future years.
- Comment: The Calculus class is scheduled for the 5th semester. We believe it would have been better to schedule it either for the 1st semester (for calculus-ready students, or for the 3rd semester (for those students who need to pass College Algebra and Pre-calculus prior to taking calculus). However, since all students are expected to take all classes as a cohort, and to make it closer to the time when students take MCAT, the decision was made to schedule it for the 5th semester.
- *Admissions:* Should have a minimal viable competency holistic admissions process.
- Comment: In practice, such relatively low admission requirements result in a big gap in "calculus readiness" within the same cohort. For most students, the main obstacle to master calculus concepts are rooted in the lack of algebra proficiency, though it is typical for calculus class with students' profile outlined above.
- *Delivery Modality*: Flipped program with opportunities for problem solving, research, team-based learning, clinical applications learning communities, etc.
- Comment: Time gap in taking mathematics classes in combination with the mentioned above lack of any other learning experiences, besides memorization of isolated facts, results in significant challenge to design an active learning environment to lean calculus. Students are used to the format where they are passive learners, and are guided step-by-step in any other activities. First time when we taught the Pilot Biomedical Calculus (Fall 2016), we observed a lot of resistance against active learning which resulted in a lower level of students' satisfaction compared to traditional teaching. This

observation is well aligned with the nationwide research, like Machemer and Crawford [3] who found that students' perception of active leaning is not positive.

- *Delivery Device*: At least initially, through an iPad prepopulated with content and apps.
- Schedule and Space: The schedule and instructional space ecosystem must support active, collaborative inquirybased learning experiences.
- Instructional Staffing: Courses will be overseen by faculty members supported by an Instructional Facilitator

The role of Instructional Facilitators is to assist faculty with numerous administrative aspects of running a course and, more importantly, to support students with everything related to the content of the courses. They are subject matter experts and/or individuals who went through the Biomedical Program themselves, and can answer specific questions related to the academic topic at hand. It is the Instructional Facilitator's job to work hand-in-hand with faculty to monitor student progress and help keep students on track.

The TEx app on students' iPads helps students keep track of their schedule of activity, due dates, and let them know when they get off track. Both the Instructor and Instructional Facilitator should monitor students' work and help students. Students are advised to contact the Instructional Facilitator and Faculty immediately if they start to struggle. Students should feel free to reach out to both resources for various issues related to the course.

B. Mathematics Requirements for Medical School Acceptance Test

For the majority of students enrolled into the Biomedical Program, acceptance to Medical School is an ultimate goal of their undergraduate education, thus while developing the Calculus Course we took into account the Medical School Acceptance Test (MCAT) requirements. At the earlier stages of the design process, the intent was to include to every formative assessment element the question "from the MCAT" directly related to the calculus content/objective under the consideration. However, the ITL found out that though for most medical schools Calculus I is in the acceptance criteria, the questions that could be considered "calculus questions" are not included in the MCAT. The questions in the MCAT are directly related to one of the content areas or knowledge/skills listed below.

1. Mathematics Requirements for the MCAT.

- Comprehend linear, exponential, and log scales.
- Calculate slopes from data represented as: Figures, Graphs, and Tables.
- Identify significant digits.
- Calculate estimates.
- Demonstrate knowledge in unit conversion between metric and English units.
- Calculate probability, proportion, ratio, percentage, and square root estimates.
- Comprehend exponents, logarithms, scientific notation, and simultaneous equations.

- Solve trigonometric problems related to: Sine, Cosine, Tangent, Inverse functions, Degrees, Relationship between triangle side lengths
- Comprehend vector addition and subtraction and righthand rule.

Though none of the content areas listed above can be directly attributed to the calculus content, proficiency in almost all of these areas is required to solve calculus problems. Analysis of almost any action to be developed in the Calculus Course indicates direct relevance to the content listed above.

Table I is a two dimensional Bloom's Taxonomy [4] which we extensively used in our design process. "Factual", "Conceptual", "Procedural", "Meta" in Table I stand correspondingly for Factual Knowledge, Conceptual Knowledge, Procedural Knowledge, and Metacognitive Knowledge. By applying the adopted in the course development methodology, we can claim that through Calculus Course problems we develop the concepts/actions from the content of the MCAT at the levels "analyze" and "evaluate".

Two Dimensional Bloom's Taxonomy				
	Factual	Conceptual	Procedural	Met
Remember				
Understand				
Apply				
Analyze				
Evaluate				
Create				

III. DESIGN OF CALCULUS I CLASS FOR THE BIOMEDICAL PROGRAM

Calculus I at UTRGV is a four credit hour class that normally meets three times a week, either Monday and Wednesday or Tuesday and Thursday for one hour and fifteen minutes, and on Friday for one hour and five minutes. For the traditional lecture based instruction, the first two days are normally devoted to formal lectures, though activities on Friday vary. It could be a lectures' continuation, or recital sessions, depending on the week of the semester, and or/and instructor's vision of the class. The adopted by UTRGV Learning Management System is the Blackboard. Though most calculus instructors use the Blackboard, some take advantage of the database of the calculus problems developed within the supported by MAA WeBWorK [5]. The assignment pattern and frequencies vary, from weekly assignments to the end of section assignments.

We chose the TEx platform for the Biomedical Calculus Class. In the last subsection of this section we will discuss in more details the TEx Calculus website that organizes students' activities.

A. Biomedical Calculus Course Content Organization

The Calculus Course content is divided into four more or less traditional for the calculus modules: Module 1 - Limits and Continuity; Module 2- Differentiation; Module 3 -

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Applications of Derivatives; Module 4- Integration. Each week's material (called Unit) is divided into two Subunits (corresponding to Day 1 and Day 2), and Laboratory (Day 3). Though the titles of Labs reflect the calculus topics studied within a particular Unit, the content is directly related to life sciences (list of all Labs is given in Section IV). In Table II below, we give an example of a typical week, the way it is presented in the paper version of the syllabus.

S	TABLE II SAMPLE OF MODULE, UNIT AND SUBUNITS (SYLLABUS)
Week 4	Module 2 Differentiation Unit 1 Concept of derivative and basic rules of differentiation
Monday	2.1.1- Definition of derivative 2.1.1- Check for Understanding (CFU): Definition of derivative
Tuesday	Face-to-face Session 2.1.1- Practice Set: Definition of derivative
Wednesday	2.1.2- Differentiation rules; higher order derivatives 2.1.2- Check for Understanding (CFU): Differentiation rules; higher order derivatives
Thursday Friday	Face-to-face Session 2.1.2- Practice Set: Differentiation rules; higher order derivatives Lab 3: The derivative as a function

B. Program's Competencies and Course Outcomes

In the design of the Calculus Course curriculum we followed Taylor's recommendations that the four components of curriculum development: objectives (learning outcomes), content, strategies, and assessment [1], should be interconnected in the course of instructional design and implementation. In this section we focus mostly on objectives/outcomes, and we will discuss content, strategies, and assessment throughout the paper.

In the State of Texas there are established outcomes for every course approved by the Texas Higher Education Coordinating Board. First and foremost, as with any other calculus course, the Biomedical Calculus course has to be in compliance with course outcomes/objectives, as stated in the Mathematics Low Division Academic Course Manual [6], where the following Calculus 1 objectives are listed.

"Upon successful completion of this course, students will:

- Develop solutions for tangent and area problems using the concepts of limits, derivatives, and integrals.
- *Draw graphs* of algebraic and transcendental functions considering limits, continuity, and differentiability at a point.
- *Determine* whether a function is continuous and/or differentiable at a point using limits.
- Use differentiation rules to differentiate algebraic and transcendental functions.
- Identify appropriate calculus concepts and techniques to provide mathematical models of realworld situations and determine solutions to applied problems.
- *Evaluate definite integrals* using the Fundamental Theorem of Calculus.
- Articulate the relationship between derivatives and integrals using the Fundamental Theorem of

Calculus" [6].

In addition to the state requirements, the Calculus Course, along with other sciences courses in the Program, should contribute to the following proficiencies.

As students complete the activities in this Mission (Program), they work toward demonstrating competence in each of these programmatic objectives:

- A1. Apply knowledge of mathematics, biology, biochemistry, physics, and chemistry to understanding living systems (Level 1);
- A2. When prompted, can adapt and apply knowledge and principles of mathematics and the sciences to draw conclusions about living systems in more complex situations (Level 2);
- A3. Adapt and apply complex knowledge of the language of mathematics and sciences to draw conclusions about living systems in complex and novel situations. (Level 3).

In the competencies listed above, Level 1, Level 2, and Level 3 approximately correspond to the following levels of cognitive demand in Bloom's Taxonomy. Level 1 - Remember and Understand; Level 2 - Apply; Level 3 – Analyze/ Evaluate.

TABLE III (A) SUBUNITS' OUTCOMES (SAMPLE)			
Level 1	Unit	Subunit	Competency: Apply knowledge of mathematics, biology, biochemistry, physics, and chemistry to understanding living systems.
Module 1: Limits and continuity	Unit 1	Subunit 1	Discuss intuitive concept of limit: classify points of a function domain where limit exists and does not exist; estimate a value of a limit; argue why limit does not exist
Module 1: Limits and continuity	Unit 1	Subunit 2	Discuss intuitive concept of one-sided limit: classify points of a function domain where one-sided limit exists and does not exist; estimate a value of a one-sided limit; connect the concept of limit and one-sided limit

Practice			-	
roblems	CFU	Lab	Final	Post Module Assessment (Test)
х	х			
х	х			
х	х		х	х
х		х	х	х
	roblems x x x x x x	x x x x x x x x x x	x x x x x x x x x x x x x x	x x x x x x x x x x x x x x x x x x x x x x

There are two learning cycles in every Unit, corresponding to two Subunits in a week (with rare exceptions because of calendar issues). We will discuss these cycles in the next subsection; here, we would like to mention that course outcomes are defined for every Subunit with assessment strategies for every Subunit's Outcome. Outcomes for Level 1, Level 2, and Level 3, along with assessment strategies that contribute to the Mission's proficiencies, are determined. Below are some examples of outcomes for Level 1. To demonstrate the development process we present them as two separate tables, Tables III (A) and (B), though both of these tables are part of one table that establishes connections

between Program's competencies, Subunits' outcomes, assessment strategies and levels of cognitive demand [7] to be developed and assessed.

C. Course Activities and Assessment Instruments

As we have already stated, during the design process we had in mind 100% flipped on-line teaching. The flow of the students' work and expectations for every Module, Unit and Subunit of every week is currently developed by ITL experts using the TEx platform. Though in order to have a complete picture of the work the designers have accomplished, one needs to teach the course, or at least spend substantial time

reviewing the website. Below are some screenshots of the workflow and content organizer that might give an idea how the students' work is guided on-line. The Website organizer in the Calculus Course design plays a significant role in guiding students' activities, and this role will be increasing with more materials to develop. The front page (Fig. 1) below provides students with links (at the top of the screenshot) to useful information.

Every Module starts with several introductory pages. Fig. 2 is the Module II (Differentiation) Entry Page.



Fig. 1 Calculus Entry Page



Fig. 2 Module II Entry Page

The entire students' coursework throughout the semester is organized and assessed within the activities guided by the Instructor and the Course Website (developed by ITL). Every Unit Plan has the Workflow Page (See Fig. 3), and the Unit Plan Pages (not included here) with recommended time to spend and link to the required activities.



Fig. 3 Unit Workflow Page

After every activity, students need to confirm that they are ready to move on to the next activity; though, there is always an option to revisit and revise previous one(s).

Reading Assignments: There is a direct link to the reading assignment from the Units' website. Though we did not connect the Course objectives with any particular textbook, for

the reading assignments we chose an open source Calculus, Volume 1. OpenStax College Textbook [8]. By clicking on the button students receive the textbook pages to read.

Video Clips. For each reading assignment there is a link to Khan Academy [9] and/or to HippoCampus [10]. See Fig. 5.



Fig. 4 "Read This" Page (Sample)



Fig. 5 "Watch This" Page (Sample)

Checks for Understanding (CFUs) are quiz-like questions with dynamic feedback that allow students and the Instructors to monitor students' progress and understanding of the key concepts. These key concepts are foundational and crucial to students' success in mastering the calculus material. There are 27 CFU quizzes (spring semester), two quizzes every week, (except the first week, as students do not have quizzes prior to the first day of classes), the last week (there is only one meeting day in the last week), and Easter Holiday week (since there are only two meeting days that week). There are nine questions in every CFU quiz. For each quiz students may receive up to 18 points, two points for each question.

Practice Sets consist of problems that require step-by-step solutions, full explanations, and substantiation. There are 28 practice problems sets. Each practice set is worth 20 points, and normally split into five problems (or themes, which may contain two or four sub-problems in a theme).

Exams are designed to be conducted after Module 1, Module 2, and Module 3. These exams are taken in class, the duration is 65 minutes. Each exam is organized the same way as a set of Practice Problems. Exams are administered on Fridays.

Laboratory Assignments consist of problems from life science that require developing mathematical models related to calculus and using calculus methods to solve them. There are 10 labs in a semester. The labs are conducted every Friday, unless there is an exam on this day. There are also no labs during the first and the last week of classes. We included the website Entry Page for the Lab assignments to Section IV, where we discuss labs in more detail.

End of Course Exam is a commprehensive exam that covers all content of the course. Students are admitted to the exam when they have successfully completed all assignments for the course.

As for any Biomedical Program course, students can earn up to 3000 points in the Calculus Course. Distribution and weight of each of the described activities are presented in Table IV.

TABLE IV			
DISTRIBUTION OF ALL ASSESSMENTS IN THE FINAL GRADE			
Assessment	Total Points	Percent	
CFU	486	16.2%	
Practice Set	560	18.7%	
Exam 1	150	5%	
Exam 2	434	14.5%	
Exam 3	450	15%	
Labs	270	9%	
End of Mission Exam	650	21.7%	
	3000	100%	

Since in the flipped classroom environment work and discussion in small groups of peers are essential for success, the entire class is split into small groups/teams of 3-4 students. For each Day 1, Day 2 and Day 3, a team leader is appointed such that all students have an equal opportunity and equal amount of time throughout the semester to serve as team leader for the particular day assignment. For every Unit when the Lab is scheduled, there are three team leaders: Day 1 Subunit team leader, Day 2 Subunit team leader, Day 3 (Lab) team leader. The responsibility of the team leader is to arrange and lead the group discussion for the particular assignment and submit it at the due time. With this arrangement, every student in class assumes a leadership role almost every week of a semester.

D.Activity Pattern with Daily Schedule

To help students develop self-discipline and avoid procrastination, we recommend students to follow some activities and submissions for formative assessments that are due every day (time) of the week. The assumption is that students meet three days a week (either in the classroom or in the online discussion room). We call these Day 1, Day 2 and Day 3 (these days are mandatory for students to meet). Let us assume that these days are Tuesday (Day 1), Thursday (Day 2) and Friday (Day 3) In addition, we recommend students activities for two additional days called Online 1 and Online 2. Assume that these days are Monday (Online 1) and Wednesday (Online 2). We expect students to meet in small groups at least three days a week (Day 1, Day 2 and Day 3) and reserve some time for online (or face-to-face) discussions, (or at least communication) on Online Day 1 and Day 2. This pattern described for the Calculus Course recurs, though with slight variations, throughout all courses in the Program's curriculum. Uniformity of the design helps students to develop study patterns and habits, and not to be confused by the diversity of activities and formative assessment assignments evenly distributed throughout every week of the semester.

Total meeting time (face-to-face or in the on-line meeting room) is four "academic" hours, to be precise 200 minutes per week. We expect students to spend at least 400 minutes (eight "academic" hours) preparing for Day 1, Day 2 and Day 3. Below is a day-by-day schedule that we request/recommend students to follow.

Monday (Online 1): Students can work either from home or the library. Students are expected to:

- Read new material and watch the recommended video.
- Understand solution of the model problems for the new material.
- Talk to team members or the Instructional Facilitator if they have questions about the new material or model problems.
- Complete the Check for Understanding Problems. Do drill-and-practice problems using the computerized system. Students are given the entire set of nine problems at a time with instantaneous feedback. Students can request as many sets as they wish, and repeat the quiz until they are satisfied with the results. They submit the results of the last CFU quiz to be recorded as their CFU grade for the particular subunit.
- The team leader shares with the team the solution of the set of Practice Problems ready for submission (see Day 1 and Day 2 activities for more explanations how and when this work starts). Work on the final submission of the assigned in class practice problems on Day 2. Submit (upload) the file with solutions. The team leader submits the final report.
- Finish Lab Report. The team leader submits the Lab Report.

Tuesday (*Day* 1): Face-to-face meeting or meeting in a meeting/discussion room online (75 minutes).

- The Instructor reviews class activities and answers questions about the reading assignment and the CFU quiz. There is no formal lecture in the classroom, at least no formal lecture on the theory. However, the Instructor is expected to give some helpful hints about how to solve the assigned problems. The Instructors' role in the classroom is to facilitate and guide students' discussion.

- A set of Practice Problems is given to students. Students are expected to work as a team and sketch the solution of each assigned problem in the classroom (online meeting room). The team leader leads the discussion and is responsible for recording the results of the group work.
- The Instructor and Classroom Facilitator spend some time talking to each team addressing the team's individual needs. If necessary, the Instructor makes brief comments to the entire class.
- Students are recommended to meet the same day as a team to finish the solution of the assigned set of problems due the next day (Online 2). The assumption is that after most work is done in the classroom (online meeting room) as a team, and continued the same day after classes, the Practice Problem set is solved, and it is the team leader's responsibility to prepare it for submission. It is expected that the team leader shares the draft ready for the final submission with the entire team for their comments and review.

Wednesday (Online 2): The same as Online 1, with the difference that no Lab Reports are due this day.

Thursday (Day 2): The same as Day 1, with the difference that students are expected to prepare for the next day Lab. The Lab preparation assignment is placed on the Learning Management System (TEx or Blackboard). We are discussing laboratory assignments with the list of all labs in Subsection D of Section IV.

Friday (Day 3): Face-to-face meeting or meeting in the meeting/discussion room online (50 minutes).

- Students come to the classroom (online meeting room) prepared for the Lab.
- Students are recommended to meet the same day as a team to finish the Lab assignment that is due next Monday (Online 1). The assumption is that most work is done in the classroom (online meeting room) as a team. It is expected that the team leader shares the draft ready for the final submission with the entire team for their comments and review.

Saturday and Sunday are not included in the formal schedule. However, we observed that some groups of students meet on Saturday or even Sunday. We believe that this activities schedule gives some flexibility for the students to balance their university/job/family commitments.

One of the challenges to organize these kinds of activities with the recommended day-by-day schedule is that, as we already described in Section II of the paper, most students have at least half-time jobs and family commitments. This situation determines the criteria to split the entire class into small teams based not on students' affiliations and already established ties with their peers, but rather on the availability to meet at the same time on recommended days or days established by the team to work together and submit group assignments (Practice Problems and Labs) at the due time. It is worth noting that group assignments are important for learning as an opportunity to express an action in the speech form, and they also contribute about 28% to the students' final grade.

IV. ACTIVE LEARNING AND FORMATIVE ASSESSMENT INSTRUMENTS

A. Theoretical Background

In our work, we extensively used materials and ideas presented in the technical report [11] and foundations of the Stage- by-Stage Development of Mental Actions as outlined in [12] to develop the methodology to analyze student' activities and assessment instruments aimed at various levels of cognitive demand.

Based on the data analysis presented by Freeman and his co-authors [13], who reviewed more than 300 published and unpublished studies on active learning, there is no doubt that active learning can be very successful and students benefit from it. This is the reason why the Biomedical Calculus is designed to be taught as a flipped classroom with full integration of assignments that play a dual role, guiding and organizing students' activities at a certain level of cognitive demand, and providing assessment of these activities. Lectures are not planned to be delivered during class periods, however, reading assignments, recommended videos are available for students with direct links from the Course's website. A brief instructor's introduction at the beginning of every subunit delivered either face-to-face or pre-recorded is preferred, however it is not necessary, and therefore, is not included in the website.

We believe that to make this model successful, a strong formative assessment component should be integrated and become an essential part of the classroom environment, and this is one of the challenges in designing an active learning environment (including flipped classroom).

In 2010, Gibson and Shaw [14] noticed that the use of active-learning techniques had expanded in previous years, and so had different assessment methods used to evaluate these teaching techniques. The scholarship of assessment, however, is not as comprehensively developed as the teaching methods themselves. Though formative assessment can be organized in many different forms, Piaget argues that formative assessment can employ three main methods for gathering data, namely, observation, test, and clinical interview [15]. By gathering the data "... The teacher needs to learn about performance, thinking, knowledge, learning potential, affect and motivation" [2, p. 111].

The same author argues [2, p. 111] that the above forms of assessment

"...are based on psychological ideas and can be only as good as those ideas The theory should make sense to teacher It need not deal with broad generalities, like constructivism that ... offer little insight into details of students' behavior"

Though both social constructivism and the Stage-by-Stage Development of Mental Actions Theory (SSDMA) recommend very similar specific (not general) strategies for organizing active learning of mathematics at various levels [16], we believe that within the SSDMA we can express our ideas in terms that are easy to adopt for practical application in design and implementation. The active learning instructional strategies, including assessment strategies, that answer fundamental for the formative assessment questions why, what, and when to assess, could be expressed within the SSDMA.

According to the SSDMA theory, a major goal of instruction is developing mental actions with objects of the studied field. This concept of instruction is well aligned with social constructivism that view reflective abstraction as means to construct abstract structures resulting from student reflecting on his/her own activities and the arguments used in social interaction [15]. The perception of the instructor's role is also very similar. For example, Bauersfeld [17] argues that instructors have to adapt to the role of facilitators that help the learner to get to students' own understanding of the content.

Shute [18] argues that efficacy of assessment depends on three factors. First of all, motive: students need it; second, means: students are willing and able to use it; and third, applicability: students receive it in time to use it. We believe we met these requirements and followed the recommendations by Shute in the Calculus Class development by designing elaborate and diverse assessment strategies and instruments (discussed in the second part of this this section), and we managed to overcome most shortcomings of traditional lecture based teaching.

B. Check for Understanding Questions

Our observations of teaching calculus provide a lot of evidence that in a typical calculus teaching and homework assignment the problems at the level "remember" are rarely assigned for homework. The assumption is that "memorization" will occur during reading the assigned materials and will be reinforced by assigning as homework "apply" and "analyze" level problems. The situation is almost the same with "understand" level problems. Though often the problems at the level "apply" are claimed to be the ones at the level "understand", and it is not hard to demonstrate (though it is material for a different paper) that:

- Students are not assigned to do any problems specifically aimed at "memorization" of concepts, facts and basic techniques;
- Students are rarely assigned to do problems that could be attributed to the level of "understand".
- The instructor does not provide formative assessment for students' activities related to the problems that could be attributed to levels "remember" and "understand", and therefore, students do not have any immediate incentives to memorize and understand the concept under development prior to the time when they are exposed to the problems at levels "apply" and "analyze".

Designing student' work with the CFU we had two didactic goals in mind. First of all, to give students incentives to do drill-and-practice exercises at the level of "remember" and "understand". One of the flaws of the traditional homework assignments is that if a student gets stuck with one or two assigned problems in a row, he/she gets discouraged to move forward. The problems our students are offered as the CFU can definitely be solved by any student who has read the assigned text and/or watched the recommended video.

The second goal is to perform a formative assessment with instantaneous feedback at the time when students need it most, which helps to avoid developing misconceptions and mistakes in executing mathematical tasks.

Unlike in traditional lecture based teaching of calculus, the assignments at "memorize" and "understand" levels practice problems were designed according to Bloom's Taxonomy.

To streamline the design process, we developed the CFU Template that contains the following fields: Title of the CFU Set field (also includes Module #, Unit #, Subunit #, Set #). Objective/Outcome field; Bloom's Taxonomy Fields (these fields give information at what level the question is in the Bloom's Taxonomy, and level of difficulty), the Stem of the Question Field, Choice of Correct and not Correct Answers, Response to the Correct Answers Field; Response to the Wrong Answer Field. Unfortunately, since the CFU Development Template is very large and contains many fields, we will introduce some it its elements while giving examples.

QUESTION STEM		
$Q(x) = \frac{a_{\pi}x^{\pi} + \dots + a_{1}x + a_{0}}{b_{\pi}x^{\pi} + \dots + b_{1}x + b_{0}}$ mark all correct answers about the limit of this function at positive infinity		
CONTENT: Limits and Continuity Module1 Unit 3_2 SUPPORT OUTCOME: Find Limits of Rational Functions (image, video)		
Bloom's Taxonomy: Factual/Mer	Level 1	
ANSWER CHOICES CORRECT		FEEDBACK
If m=n, then $\lim_{x \to +\infty} Q(x) = \frac{a_n}{b_m}$	Yes	
If m <n, <math="" then="">\lim_{x \to +\infty} Q(x) = \pm \infty Yes</n,>		
If m>n, then $\lim_{x\to+\infty} Q(x) = 0$ Yes		
Limit is undetermined	No	

Fig. 6 Sample of CFU Questions

QUESTION STEM			
Find $\lim_{x \to +\infty} \frac{6x-1}{-2x+5}$			
CONTENT: Limits and Continuity, Module1 Unit 2_2 SUPPORT OUTCOME: Recognize Continuous Functions (image, video			
Bloom's Taxonomy: Conceptual/Understand	Level 1		
ANSWER CHOICES	CORRECT	FEEDBACK	
-3	Yes		
-1 No			
-1/5 No			
5 No			

Fig. 7 Sample of CFU Questions

At the beginning of every cycle, we try to avoid questions at the level "apply". However, some of the "understand" questions, with a difficulty of two, could be attributed to simple questions at the level "apply". Questions from Figs. 6 and 7 are from the same CFU quiz, the first one is aimed at assessment of factual knowledge of the concept of limits, while the second one is aimed at assessing the same concept at the level "understand".

Irrespective of what answer is selected, the feedback not only confirms/corrects response, but also gives general information about the concept/techniques under discussion. In the example above, the feedback is "Correct/Incorrect. The stem of the question as a premise and possible answers 1-3 as conclusion constitute Theorem Limit of Rational Function at Infinity that you are expected to remember and be able to apply".

The Feedback to any students' response makes a reference to the mentioned above theorem "Limit of Rational Function at Infinity" and shows how to apply it ("since in the question m=n=1, according to the theorem "Limit of Rational Function at Infinity", limit is equal to $a_n/b_m=6/(-2)=-3$. By including "factual" and "understanding" questions aimed at the same object, we not only provide assessment with instantaneous feedback, but also reinforce theory with applications of the theory. The next question can also be attributed to Conceptual/Understand.

QUESTION STEM		
Which of the following statements are true for $f(x)=(x^2+2x-1)/(x^2+1)$		
CONTENT: Limits and Continuity, Modulel Unit 3_2 SUPPORT OUTCOME: Recognize Continuous Functions (image, video)		
Bloom's Taxonomy: Conceptual/Understand		Level 2
ANSWER CHOICES CORRECT		FEEDBACK
f(x) is continuous at $x = 1$	Yes	
f(x) has a removable discontinuity at $x = 1$	No	
f(x) has an infinite discontinuity at $x = 1$	No	
f(x) has a jump discontinuity at $x = 1$	No	

Fig.8 Sample of CFU Questions

Irrespective of which answer is selected, the feedback not only confirms/corrects response, but also gives general information about the concept/techniques under discussion. The feedback to the response above (Fig. 8) is: "Correct/Incorrect. Rational functions are continuous at all points of their domains. x = 1 belongs to the domain of f(x)because the denominator is not equal to zero for this value". For most questions, like the one we just discussed, we give the same information whether the Correct/Incorrect answer is selected. We have to admit that because of the cost to develop the course, we did not have the resources to develop a precise response to every single mistake made by students.

C. Practice Problems and Group Discussions

Though the most challenging task, from our point of view,

is organizing and guiding students' activities at the initial level of a concept/action development, the next step is to develop that action at the level "apply" and "analyze". For this purpose, in our design for every class period we organize group activities and present some problems for students to do as a group. About 70% of class time is devoted to team work. Our expectation is that at the first stage students create a plan how to do every assigned problem, and make a draft of the plan without performing the executive part of action which may require more time than the class period allows. By talking to peers, making assumptions, asking questions, finding substantiated path to the solution, and making notes students develop an action in the speech form. They meet after class to discuss how to finalize the solution of each problem, they also create a draft of the report to be submitted to the Instructor for grading. It is the team leaders' responsibility (every class period there is a team leader for the given subunit or lab assignment) to make all records and prepare that final draft of the report. Team leaders share final report with their groups for the group's input, and only after that the report is submitted.

With this organization of students' activity we achieve not only an outcome for every specific class period, but also provide a path for every student to achieve all course objectives. Twice a week all students receive feedback on their assignment, every week one of them serves as a group leader which requires developing math writing skills, leadership, and commitment. Below are some examples of Practice Problems for two different modules of the course. We have chosen both examples from the same units as examples for CFU to demonstrate the development of the same action/concept in progress. For the solution of each of the problems below, we expect full explanation and substantiation of every step.

Module 2 Unit 1_1

- Use the definition of a derivative to find the derivative of the function at the given value.
- a. $f(x) = -x^3 + 4x^2 4$ find f'(4).

b.
$$f(x) = \frac{20}{2x^2}$$
 find $f'(3)$

- Find the derivative function by the definition of derivative.
- a. $f(x) = x^3 12x$

b. $f(x) = \sqrt{x+4}$

- Determine at what points the function continues/discontinues and why. If the function discontinues at point "a", determine the type of discontinuity. If the function has a removable discontinuity, "remove" it (e.g. define the function at the point to make it continue).

a.
$$f(x) = \frac{1}{x}$$

b.
$$g(x) = \frac{x^2 - 1}{x - 1}$$

- c. $h(x) = (\cos x 1)/x$
- d. $s(x) = \sin x/x$
- State the interval(s) over which the function is continuous.

Explain why.
a.
$$f(x) = \sqrt{49 - x^2}$$

b. $f(x) = \frac{x^2}{2x+4}$

c.
$$f(x) = 3x - \cos x$$

D. Laboratories in Biomed Calculus Course Curriculum

In the process of the Biomedical Calculus Course development we use the results (materials, findings, recommendations) of the Project "A Discipline Specialized Laboratory Component to the Calculus course" (see Acknowledgement Section) that was reported and discussed in [19] in a broader context than this paper. In this section we list Labs as they pertain to Life and Biomedical Science. It was part of the college's effort to gradually move traditional lecture based teaching to the strategies that are either 100% active learning, or are incorporated into active learning as an essential part of teaching.

Though developed and tested within a different project, Lab Component became an essential part of the Biomedical Calculus Course for at least three reasons. First of all, the content of the lab is directly related to Life and Biomedical Science, and therefore is highly motivational for biomedical students. It would be no exaggeration to say that for the first time in students' experience, mathematics is applied to their own field of study. The connections established through labs between students' own field of study and calculus increase the motivation to study calculus and students retention.

Second, the entire course is designed as an active learning course. Learning is considered to be active if students are engaged in meaningful learning activities that require higherorder thinking, rather than just listening, and are provided a learning environment that enables the development of skills, rather than just absorption of information [20], [21]. Integration of Laboratory Component into Biomedical Science definitely moves students' thinking to higher-order thinking, i.e., ability to find an appropriate mathematical model to solve a problem in other than mathematics field of expertise using mathematical methods.

And last but not least, major related Labs as an important students' activity within the course need to be assessed at the level "analyze-evaluate" according to Bloom's Taxonomy. Unlike traditional means of assessments in mathematics, (e.g. tests/exams, and homework), which assess mostly performance and learning potentials, laboratories, the way they were designed in our study, assess also thinking /knowledge at the level "analyze-evaluate" as well as affect/motivation. We have already used Piaget's argument that formative assessment can employ three main methods for gathering data, namely, observation, test, and clinical interview [15]. Traditional teaching utilizes almost 100% test/exam method. The integration of the laboratory component allowed us to also use observation, and to some extent, interviews with students.

As we have already briefly discussed in Section III, the Calculus Course design Labs take place every Friday for 65 minutes with some exceptions. There are no labs in the first

and the last weeks of classes, and there are no labs when students have midterm exams (three midterm exams). The day before a lab, students are expected to review the Introduction to Lab, as well as the calculus concepts that need to be used to accomplish the Lab task. All labs are designed as projects that students may be involved in their future professional life. Below is a list of all Labs in the Calculus Course that are directly related to Life Sciences.

Each specialized Lab project was designed and described according to the template which consisted of major objectives of the lab, brief definitions and concepts from the area, recommended calculus concepts, Lab assignment and Solution to assignment. The descriptions of the projects were presented in the form of booklets to make them usable by other instructors teaching Calculus. Below is an example of one set of Lab materials. Analysing the function describing the weight growth for male yellow baboon shows how useful Calculus methods are for finding valuable information about their growth spurt. In this project, the students were given a function modelling the weight growth of a baboon which was linear for the first three years and nonlinear starting at age three and lasting four years. The analysis of the function showed that male baboons hit a pronounced growth spurt at around 3-years of age. The students start the lab work with Lab Entry Page (Fig. 9) at the website.

TABLE V	
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LABS IN THE CALCULUS COURSE		
Calculus Content	Lab's Title	
Derivative as Function	Amount of Ozone in atmosphere	
Derivative as Rate of Change	Weight Growth for Male Yellow Baboons	
Chain rule	Blood Pressure	
Related Rates	Increasing blood pressure	
Linearization and Differentials	Chlorine oxide and the depletion of ozone	
Antiderivative	Body mass of Adolescent Yellow Baboons	
Area and Estimating with Finite Sums	Oil spills and adding it all up	
Substitution and Area Between Curves	Ozone and the Chapman Cycle	



Fig. 9 Lab Entry Page (Sample)

Below we included the Lab "Derivative as Rate of Change" the way it is introduced to students in the Calculus Course.

Derivative as a Rate of Change. Lab: Growth Rates of Young Yellow Baboons

- Major objectives of the lab
- Reinforce that the derivative is the slope of the tangent line.
- Reinforce that the derivative is a function in its own right.
- Introduce the connection between increasing functions and positive derivatives.
- Introduce the concept of tangent line approximation.
- First introduction to second derivatives, concavity, and the relationship with the tangent line.
- Brief definitions and concepts from the area
- The growth rate (in body mass) of the yellow baboon of central Africa is essentially linear for the first three years,

but decidedly non-linear for the following four years.

- Recommended to review mathematical concepts.
- Notations and meanings of the derivative.
- Domain and Range of Functions.
- Point-slope formula for finding the equation of a straight line.
- Tangent lines
- Student Introduction to the Lab

The yellow baboon (*Papio cynocephalus*) is natively found in many African nations ranging from Angola to Ethiopia though they have been mainly studied in southern Kenya at the Amboseli National Park. The scientific name for the yellow baboon is motivated by their appearance and is derived from the Greek words for dog and head. Likewise, the origin for their common name is obvious from the yellowish fur that covers a significant portion of their bodies.



Fig. 10 Photos by: Kathryn L. Rasmussen

Understanding the nature and physiology of baboons, is helpful in understanding the same within human beings. For such reasons, baboons have long been studied to understand their similarities and differences to humankind.

Baboons appear to be more similar to man than are the other monkey species, which makes them a desirable model for use in experimental research to approximate human health conditions. Baboons were most intensively used in surgery, then in hematology, cardiology, and endocrinology (list of references for the Lab is provided for students).

Baboons are sexually dimorphic. That means the size of the yellow baboon varies greatly based upon their gender. Male yellow baboons, on average, will weigh more than twice that of a typical female. Understanding where these similar creatures differ from human beings is very important when one considers their use in research based upon that degree of similarity. In this lab, we will discuss the growth rate in young male yellow baboons as they mature.

If you are interested in reading more about yellow baboons and the benefits to humanity from studying them, please check out the following articles (the website listed is also the source for the wonderful pictures above):

The list of references to scholarly articles is given to students.

- Lab assignments

Weight growth for male yellow baboons was found to be linear for the first three years of their life, and then modeled

by the following $\frac{dW}{dt} = \frac{49}{2W(t)}$ formula for the subsequent

four years. Where the time, t, is measured in years and the weight at that time, W(t), was measured in kilograms.

- 1) What is the domain for the function W(t) described above?
- 2) If a 3-year-old male baboon weighs 7 kg, then what would be his rate of growth by the model?
- 3) Write the equation of the line T, that is tangent to the graph of W(t) at the point (3,7).
- 4) Sketch a graph of the tangent line T within our domain.
- 5) Is the weight of the animal increasing or decreasing as time goes on? Why?
- 6) How about the rate of change? Will the rate increase or decrease as the time in our domain gets larger?
- 7) What is the point of intersection of that tangent line T with the line t = 4?
- 8) Using this, what estimate can you give for the weight of that 3-year old baboon when he turns four?
- 9) Do you expect this estimate be higher or lower than the

actual value one year from now? Why?

- Follow up Lab questions
- If the baboon in our example above was a typical male, can you say anything about baboons as they become around 3-years old?
- 2) What can you say and why?
- Answers to lab assignments

We do not include answers to the Lab's questions here, however, they are available for students after they submit Lab Reports.

- References to Scholarly Articles for Labs are included at the end of each Lab.

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