Development of A Jacobean Model for A 4-Axes Indigenously Developed SCARA System

T.C.Manjunath, C.Ardil

Abstract—This paper deals with the development of a Jacobean model for a 4-axes indigenously developed scara robot arm in the laboratory. This model is used to study the relation between the velocities and the forces in the robot while it is doing the pick and place operation.

Keywords—SCARA, Jacobean, Tool Configuration Vector, Computer Control, Visual Basic, Interfacing, Drivers,

I. INTRODUCTION

IMAGINE a day in your life when you wake up in the morning and find a machine walking up to you and saying "GOOD MORNING SIR! Have a cup of tea". How would you respond to such a situation? With so much progress made in the field of science, engineering and technology, this dream is absolutely realizable in the automation age with the advent of robotization [1].

Robotics is an interdisciplinary filed that mixes various engineering disciplines into one. In this, work a unique 4 axes system was designed and fabricated with indigenous components with a brief tool configuration jacobian analysis of the designed robot. The designed robot was used for some PNP operations without human intervention using sensors and was named as a Selective Compliance Assembly Robot Arm (SCARA). The primary motive behind the work was to develop a modular educational robotic system, the CRUST 2002 (Computerized Robotic Unit with Selective Tractability system) with the help of locally available components and sub-systems as shown in Fig. 1.

The paper is organized as follows. First, a introduction to robotics, robots and the design of the mechanical assembly is presented in section 2. In section 3, the review of the jacobian model is presented. Fourthly, the mathematical representation of the jacobian development is presented in section 4. Section 5 gives the TCJM for the developed robot, In Section 6, the algorithm is presented followed by the conclusions and the references.

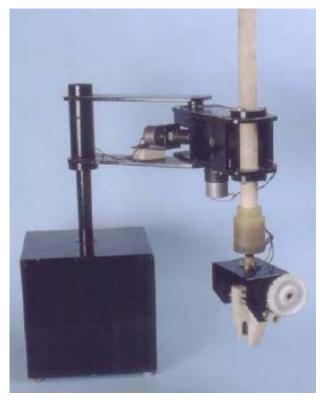


Fig. 1 The designed SCARA robot

II. PHYSICAL SYSTEM DESIGN & A BRIEF REPORT

The mechanical design of the developed system is divided into 3 parts, viz., the base assembly, the arm assembly and the gripper assembly. The designed robot has R-R-P (Rotary-Rotary-Prismatic) type of axes [2].

A four axis / four DOF designed SCARA robot arm as shown in Fig. 1. A SCARA robot is a 4 DOF stationary robot arm having base, elbow, vertical extension and tool roll and consisting of both rotary and prismatic joints. There is no yaw and pitch, only roll. There are 4 joints, 4 axis (three major axes - base, elbow, vertical extension and one minor axis - tool roll) [4]. The 4 DOF's are given by Base, Elbow, Vertical Extension and Roll, i.e., there are three rotary joints and one prismatic joint. Since n=4; 16 KP's are to be obtained and 5 RHOCF's are to be attached to the various joints [3].

T.C. Manjunath is with Electronics and Communications Engineering Dept. of East West Institute of Technology, Bangalore, Karnataka, India.

E-mail: tcmanju@sc.iitb.ac.in, tcmanjunath@gmail.com.

² C.Ardil is with National Academy of Aviation, AZ 1056 Baku, Azerbaijan.



Fig. 2 Roll DOF

The vector of joint variables is a combination of θ and d , i.e., $q = \{\;\theta\;,\,d\;\}^T.$

Vector of joint variables are $q = \{\theta_1, \theta_2, d_3, \theta_4\}^T.$ Vector of joint distances are $d = \{d_1, 0, d_3, d_4\}^T = \{400, 0, d_3, 100\}^T \text{ mm.}$ Vector of link lengths are $a = \{a_1, a_2, 0, 0\}^T = \{250, 200, 0, 0\}^T \text{ mm.}$ Vector of link twist angles are $\alpha = \{\alpha_1, \alpha_2, 0, \alpha_4\}^T = \{\pm \pi, 0, 0, 0\}^T.$

All the 4 joint axes are vertical in nature (all the z-axes can be pointing down or up) as shown in Fig. 3. The first three (B, E, VE) axes are called as the *major axes* and are used for positioning the wrist, while the last one, the *minor axes* (TR) is used to orient the gripper in the direction of the object [5]. The first three major axes determines the shape and size of work envelope. It consists of a L shaped structure, to the end of which the second link is attached. There are two links a_1 and a_2 which move parallel to the work surface; The vertical extension d_3 is variable and moves in a direction \bot^r to the work surface; the length of the gripper or the endeffector (EE) is d_4 [2], [6].

The gripper / EE is permanently pointing down as shown in Fig. 2 and can rotate in a plane \perp^r to the work surface plane $x^0 y^0$. The approach vector r^3 is fixed, i.e., $r^3 \perp^r x^0 y^0$ (work surface) plane; $r^3 = -z^0$. Because of this reason, our designed and kinematically modeled SCARA robot can do robotic manipulation directly from above the

object when exact perpendicularity is required. The SCARA robot is a minimal representation of any robot. Our SCARA robot is a special type of polar / spherical coordinate robot in which the major axes are R R P [10].



Fig. 3 The computer controlled SCARA robot

III. TOOL CONFIGURATION JACOBIAN MATRIX

In this section, we present a review of the jacobians. Jacobian is defined as a multi-dimensional form of partial derivatives and is a matrix of size $(m \times n)$. Jacobian is the study of both velocities and static forces that leads to a matrix entity [1], [9]. The transformation from the joint velocity to the tool configuration velocity is called tool configuration Jacobian matrix V(q). It gives the relation between the speed at which the joint is moving and the speed at which the tool-tip is moving. Tool configuration Jacobian matrix is not a square matrix. Hence, an inverse of it is obtained in order to find the joint velocity in terms of tool configuration velocity [1], [8].

Trajectory planning is always done in TCS, R⁶. Tool configuration gives p, R of tool in 3D space. Since, we cannot control the tool tip directly, indirectly it is controlled by using joint velocity control [7].

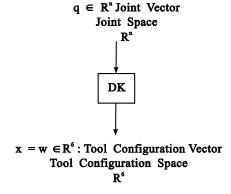


Fig. 4 Relation between TCS and JS

The output of the direct kinematic model of the designed robot is given by [1], [11]

$$T_0^4 = \begin{bmatrix} C_{1-2-4} & S_{1-2-4} & 0 & a_1C_1 + a_2C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_1S_1 + a_2S_{1-2} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^4 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & p_1 \\ R_{21} & R_{22} & R_{23} & p_2 \\ R_{31} & R_{32} & R_{33} & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{w}(\mathbf{q}) = \begin{bmatrix} \mathbf{w}^1 \\ \dots \\ \mathbf{w}^2 \end{bmatrix}$$

$$w(q) = \begin{bmatrix} p \\ \dots \\ \left\{ exp\left(\frac{q_4}{\pi}\right) \right\} r^3 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \dots \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \dots \\ \left\{ exp\left(\frac{q_4}{\pi}\right) \right\} R_{13} \\ \left\{ exp\left(\frac{q_4}{\pi}\right) \right\} R_{23} \\ \left\{ exp\left(\frac{q_4}{\pi}\right) \right\} R_{33} \end{bmatrix}$$

Let the tool configuration vector be represented by [1]

$$\mathbf{x} = \mathbf{w}(\mathbf{q}) = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \mathbf{w}_3 \\ \cdots \\ \mathbf{w}_4 \\ \mathbf{w}_5 \\ \mathbf{w} \end{bmatrix}$$
(1)

The relationship between the joint space variables and tool configuration variables is given by the direct kinematic equations and this relation is given by the Eq. (1) and is shown diagrammatically in Fig. 4 [1], [12].

Given : Trajectory x(t) in TCS.

To find : Trajectory q(t) in Joint Space, JS.

Approach : To invert Eq. (1) and solve

inverse kinematics equations.

Drawback : To get a closed form of solutions

to the inverse kinematics is

difficult [13].

Alternative approach : Differentiate Eq. (1) w.r.t to $q_i\left(q_i\right)$

 $= \theta_i$ or $q_i = d_i$), i^{th} joint variable.

$$\dot{x}(t) = V(q) \dot{q}(t)$$

$$\dot{x} = V(q) \dot{q}$$
(2)

V(q) \rightarrow (6 × n) matrix of partial derivatives of w w.r.t. q, $\frac{\partial w}{\partial q_i}$

 $\frac{\partial w}{\partial q_i}$ \rightarrow Called as TC Jacobian matrix of the arm [1],

Has got 6 rows and n columns, where n is the DOF [1].

 $(6 \times n)$ components of tool configuration jacobian matrix [1], [14].

 $\dot{x} \rightarrow \text{Tool configuration velocity}$.

 $\dot{q} \rightarrow Joint space velocity$.

Note that the differential relationship shown in the Eq. (2) can be used to solve for the joint space trajectory q(t) given the tool configuration trajectory x(t) [1], [15], [40].

IV. MATHEMATICAL REPRESENTATION

The component of V(q) in the k^{th} row and the j^{th} column is the derivative of $w_k(q)$ w.r.t. q_j and is given by [16], [1]

$$V_{ki}(q) = \frac{\partial w_k(q)}{\partial q_i}$$
; $1 \le k \le 6$ and $1 \le i \le n$ (3)

Definition: For each $q \in R^n$, the TC Jacobian matrix V(q) is defined as a linear transformation which maps the instantaneous joint space velocity \dot{q} into the instantaneous tool configuration velocity \dot{x} as shown in the Fig. 5 OR it is a matrix which gives the relation between tool-tip velocity and the joint velocity. Once, a robot starts moving from the pick point to the place point, all the joints moves with a particular velocity [39].

Similarly, the tip of the gripper also starts to move with a particular velocity. The relation between the tip speeds and the joint speeds is given a matrix called as the tool configuration jacobian matrix [17], [1].

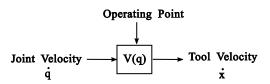


Fig. 5 Relation between the inputs and output of the tool configuration jacobian matrix TCJM

The main use of TC Jacobian Matrix is, it is used to find JS trajectory q(t), given TCS trajectory x(t) [1].

V. Tool Configuration Jacobian Matrix for a 4 Axis Indigenously Developed SCARA Robot

The designed and developed SCARA robot has got 4 DOF, n=4, Joint variables q_1 to q_4 , joints are combination of prismatic and rotary type, 3 revolute and 1 prismatic joint. The TCV w(q) which is obtained from the output of the direct kinematic problem is given by [1], [18], [38]

$$w(q) = \begin{bmatrix} p \\ \dots \\ R \end{bmatrix}$$

$$= \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \dots \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}$$

$$a_1 \cos q_1 + a_2 \cos (q_1 - q_2)$$

$$a_1 \sin q_1 + a_2 \sin (q_1 - q_2)$$

$$d_1 - q_3 - d_4$$

$$\dots$$

$$0$$

$$(4)$$

which can be written in short form as [1]

$$w(q) \; = \; \begin{bmatrix} a_1 C_1 \, + \, a_2 C_{1-2} \\ a_1 S_1 \, + \, a_2 S_{1-2} \\ d_1 \, - \, q_3 \, - \, d_4 \\ \dots \\ 0 \\ 0 \\ -exp \bigg(\frac{q_4}{\pi} \bigg) \; \bigg]_{(6 \, \times \, 1)}$$

Note that $q = \theta$ for a rotary joint and q = d for a prismatic joint. The notations 'p' and 'R' in the TCV is nothing but the position of the tip of the gripper and the orientation of the gripper w.r.t. the base which can be further explained as [19],[20],[1]

 $p \rightarrow position of gripper-tip : Given by the Ist three components of TCV, w(q) [21], [22], [1]$

 $R \rightarrow$ orientation of the gripper : Given by the last three components and TCV, w(q) or approach vector r^3 scaled by an invertible exponential function given by exp $\left(\frac{q_n}{\pi}\right)$, where q_4 is roll angle [23], [24], [1]

Differentiate Eq. (4) w.r.t. q, we get the TC Jacobian matrix; The resultant TC Jacobian matrix V(q) obtained is a (6×4) matrix, since n = 4 [25], [26], [1]

$$V(q) = [v^{1}(q), v^{2}(q), v^{3}(q), v^{4}(q)]$$
 where,

 $v^{1}(q)$ to $v^{4}(q)$ are the 4 columns of V(q) and [36]

 $v^k(q) \rightarrow k^{th}$ column of TC Jacobian matrix and $1 \le k \le 4$ [37], [1]

$$\begin{cases} v^{1}(q) \\ \left\{ \begin{array}{l} \text{differentiate} \\ \text{w.r.t. } q_{1} \text{ only} \end{array} \right\} = \begin{bmatrix} -a_{1}S_{1} - a_{2}S_{1-2} \\ a_{1}C_{1} + a_{2}C_{1-2} \\ 0 \\ \cdots \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
 (6)

$$\begin{cases} v^{2}(q) \\ \text{differentiate} \\ \text{w.r.t. } q_{2} \text{ only} \end{cases} = \begin{bmatrix} a_{2}S_{1-2} \\ -a_{2}C_{1-2} \\ 0 \\ \cdots \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
 (7)

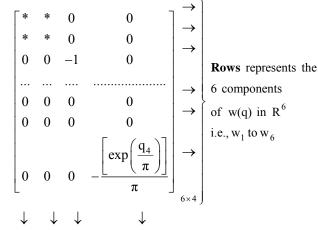
$$\begin{cases} v^{3}(q) \\ \text{differentiate} \\ \text{w.r.t. } q_{3} \text{ only} \end{cases} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ \cdots \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
(8)

$$\begin{cases} v^{4}(q) \\ \text{differentiate} \\ \text{w.r.t. } q_{4} \text{ only} \end{cases} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \\ \\ -\frac{\left\{ \exp\left(\frac{q_{4}}{\pi}\right) \right\}}{\pi} \end{bmatrix}$$
 (9)

Substitute the equations for $v^1(q)$ to $v^4(q)$ in V(q), we get the TC Jacobian Matrix (TCJM) as [27], [28], [1]

Note: Differential of $\sin = \cos$ and that of $\cos = -\sin$. Observe the number of zeros in V(q), [1]

Finally, V(q) is given by [1]



Columns of V(q): Represents the 4 components of joint variables q in R^n , i.e., q_1 to $q_4\lceil 1 \rceil$

Note:

variables $\{q_1 \text{ and } q_2\}$ [30].

 1^{st} two rows of (q) : Gives tool tip position p [29]. x , y components of p : Depends only on the 1^{st} 2 joint

z component of p : Depends only on q_3 [31].

From the Eq. (10), we can come to a conclusion that the matrix is a sparse one and this shows that the kinematics of the designed SCARA robot is very very simple because of the number of zeros in the TCJM, i.e., there are 18 zeros in 24 element matrix [32], [1].

The position of the tip of the gripper depends only on the first two joint variables, i.e., the base and the elbow, whereas the z-axis is entirely dependent on the vertical extension parameter and the orientation of the gripper is finally dependent on the roll angle [33], [1].

The velocities of all the joints and the tip of the gripper was also controlled during the pick and place operation [34], [1].

VI. ALGORITHM

The algorithm for obtaining the joint velocities in terms of tool velocities is shown below in the following form as follows [35]

- Input the pick and place points
- Input the geometric link parameters
- Obtain the DK model
- Obtain the IK model
- Differentiate the IK model w.r.t. q₁ only
- Differentiate the IK model w.r.t. q₂ only
- Differentiate the IK model w.r.t. q₃ only
- Differentiate the IK model w.r.t. q₄ only
- Combine the differentiated IK models w.r.t. the different joint variables into the form of a matrix

- Name that matrix as the tool configuration jacobian matrix.
- MATLAB interfaced with real time and feedback units to plot the velocities of the 4 joints from the pick point to the place point.
- Observe the experimental results

VII. EXPERIMENTAL RESULTS OF THE DESIGNED ROBOT DURING THE PNPO

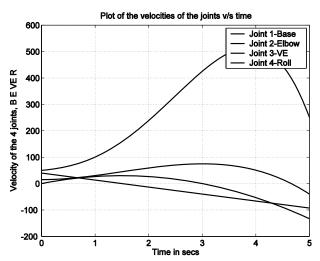


Fig. 6 Velocities of the 4 joints of the robot w.r.t. time

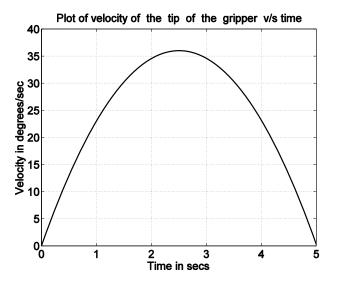


Fig. 7 Velocity of the tip of the gripper w.r.t. time

VIII. CONCLUSIONS

A indigenously designed and fabricated 4-axes robot system was used to obtain the Jacobian model and the same and was used to perform a successful pick and place task using a user-friendly developed graphical user interface and real time implementation. It was seen that the velocities were properly controlled.

REFERENCES

- Robert, J.S., "Fundamentals of Robotics: Analysis and Control", [1]. PHI. New Delhi., 1992
- [2]. Klafter, Thomas and Negin, "Robotic Engineering", PHI, New Delhi, 1990
- Fu, Gonzalez and Lee, "Robotics: Control, Sensing, Vision and [3]. Intelligence", McGraw Hill, Singapore, 1995
- [4]. Ranky, P. G., C. Y. Ho, "Robot Modeling, Control & Applications",
- IFS Publishers, Springer, UK., 1998. T.C.Manjunath, "Fundamentals of Robotics", Nandu Publishers, 5th Revised Edition, Mumbai., 2005.
- [6]. T.C.Manjunath, "Fast Track To Robotics", Nandu Publishers, 3nd Edition, Mumbai, 2005.
- Ranky, P. G., C. Y. Ho, "Robot Modeling, Control & Applications", IFS Publishers, Springer, UK, 2005.
- Groover, Weiss, Nagel and Odrey, Industrial Robotics, McGraw Hill, Singapore, 2000.
- William Burns and Janet Evans, "Practical Robotics Systems,
- Interfacing, Applications", Reston Publishing Co., 2000. Phillip Coiffette, "Robotics Series, Volume I to VIII, Kogan Page,
- London, UK, 1995.
- Luh, C.S.G., M.W. Walker, and R.P.C. Paul, "On-line computational scheme for mechanical manipulators", Journal of Dynamic Systems, Measurement & Control, Vol. 102, pp. 69-76, 1998.
- [12]. Mohsen Shahinpoor, "A Robotic Engineering Text Book", Harper and Row Publishers, UK.
- [13]. Janakiraman, "Robotics and Image Processing", Tata McGraw
- Richard A Paul, "Robotic Manipulators", MIT press, Cambridge. [14]
- Fairhunt, "Computer Vision for Robotic Systems", New Delhi. Yoram Koren, "Robotics for Engineers", McGraw Hill. [15].
- [17]. Bernard Hodges, "Industrial Robotics", Jaico Publishing House, Mumbai, India.
- Tsuneo Yoshikawa, "Foundations of Robotics: Analysis and Control", PHI. [18].
- [19]. Dr. Jain and Dr. Aggarwal, "Robotics: Principles & Practice", Khanna Publishers, Delhi.
- [20]. Lorenzo and Siciliano, "Modeling and Control of Robotic Manipulators", McGraw Hill.
- Dr. Amitabha [21]. Bhattacharva, "Mechanotronics Robotics Systems"
- [22]. S.R. Deb, "Industrial Robotics", Tata McGraw Hill, New Delhi, India.
- Kafrissen and Mark Stephans, "Industrial Robots and T231. Edward Robotics", Prentice Hall Inc., Virginia.
- [24]. Rex Miller, "Fundamentals of Industrial Robots and Robotics, PWS Kent Pub Co., Boston.
- Doughlas R Malcom Jr., "Robotics ... An introduction", Breton Publishing Co., Boston.
- [26]. Wesseley E Synder, "Industrial Robots: Computer Interfacing and Control", Prentice Hall.
- Carl D Crane and Joseph Duffy, "Kinematic Analysis of Robot Manipulators", Cambridge Press, UK.
- C Y Ho and Jen Sriwattamathamma, "Robotic Kinematics Symbolic Automatic and Numeric Synthesis", Alex Publishing
- Corp, New Jersey. Francis N Nagy, "Engineering Foundations of Robotics", Andreas [29]. Siegler, Prentice Hall.
- [30]. William Burns and Janet Evans, "Practical Robotics - Systems, Interfacing, Applications", Reston Publishing Co.
- Robert H Hoekstra, "Robotics and Automated Systems".
- **[32]** Lee CSG, "Robotics, Kinematics and Dynamics"
- [33]. Gonzalez and Woods, "Digital Image Processing", Addison Wesseley.
- Anil K Jain, "Digital Image Processing", PHI.
- [35]. Joseph Engelberger, "Robotics for Practice and for Engineers", PHL USA.
- Yoshikawa T., "Analysis and Control of Robot Manipulators with Redundancy", *Proc. First Int. Symp. on Robotics Research*, Cambridge, MIT Press, pp. 735-748, 1984.
- Whitney DE., "The Mathematics of Coordinated Control of Prosthetic Arms and Manipulators", Trans. ASM J. Dynamic Systems, Measurements and Control, Vol. 122, pp. 303-309, 1972.
- Whitney DE., "Resolved Motion Rate Control of Manipulators and Human Prostheses", *IEEE Trans. Syst. Man, Cybernetics*, Vol. MMS-10, No. 2, pp. 47-53, 1969.

- [39]. Lovass Nagy V, R.J. Schilling, "Control of Kinematically Redundant Robots Using {1}-inverses", IEEE Trans. Syst. Man, Cybernetics, Vol. SMC-17, No. 4, pp. 644-649, 1987.
- [40]. Lovass Nagy V., R J Miller and D L Powers, "An Introduction to the Application of the Simplest Matrix-Generalized Inverse in Systems Science", IEEE Trans. Circuits and Systems, Vol. CAS-25, No. 9, pp. 776, 1978.



T.C.Manjunath, born in Bangalore, Karnataka, India on Feb. 6, 1967 received the B.E. Degree in Electrical Engineering from the University of Bangalore in 1989 in First Class and M.E. in Electrical Engineering with specialization in Automation, Control and Robotics from the University of Gujarat in 1995 in First Class with Distinction and Ph.D. from the Interdisciplinary Programme in Systems and Control Engineering Department of Indian Institute of Technology

Bombay in the year 2007, respectively. He has got a teaching experience of 17 long years in various engineering colleges all over the country (Karnataka, Tamil Nadu, Gujarat, Maharashtra) and is currently working as Professor and Head of the Department of Electronics and Communication Engineering in East West Institute of Technology in Bangalore, Karnataka, India. He also worked as a Research Engineer in the Systems and Control Engineering (IIT Bombay, India) for nearly a year and worked on control of space launch vehicles using FOS feedback technique. He has published 90 papers in the various National, International journals and Conferences and published two textbooks on Robotics, one of which has gone upto the fourth edition, titled, 'Fast Track to Robotics' and the other, which has gone upto the fifth edition, titled, 'Fundamentals of Robotics' in 2 volumes, Vol.-1 and Vol.-2 along with a CD which contains about 150 C / C++ programs for performing various simulations on robotics. He also published a research monograph in the International level from the Springer-Verlag publishers based on his Ph.D. thesis topic titled, "Modeling, Control and Implementation of Smart Structures", Vol. 350, LNCIS, costing 79.95 Euros. He was a student member of IEEE for 6 years, SPIE student member and IOP student member for 4 years, life member of ISSS (India), life member of the ISTE (India), life member of ISOI (India), life member of SSI (India) and life member of the CSI (India) and a fellow of the IETE (India). He has visited Singapore, Russia, United States of America and Australia for the presentation of his research papers in various international conferences. His biography was published in 23rd edition of Marquis's Who's Who in the World in the 2006 issue. He has also guided more than 2 dozen robotic projects. Many of his guided projects, interviews have appeared in various national newspapers and magazines. He has also presented a number of guest lectures and various seminars and participated in more than a dozen CEP / DEP courses, seminars, workshops, symposiums in the various parts of the country in different institutions and also conducted a few courses. His Ph.D. research work was based on the mathematical modeling, control and implementation of smart structures and its applications. His current research interests are in the area of Robotics, Smart Structures, Control systems, Network theory, Mechatronics, Process Control and Instrumentation, Electromagnetic fields and waves, MATLAB, Signals and systems (CT and DT), Industrial automation, Artificial intelligence, Digital signal processing, Digital Image Processing, Periodic output feedback control, Fast output feedback control, Sliding mode control of SISO and multivariable systems and many of the control related subjects and its allied labs and their various applications.