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Determination of the Proper Quality Costs Parameters via Variable Step Size Steepest Descent Algorithm

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Abstract**—**This paper presents the determination of the proper quality costs parameters which provide the optimum return. The system dynamics simulation was applied. The simulation model was constructed by the real data from a case of the electronic devices manufacturer in Thailand. The Steepest Descent algorithm was employed to optimise. The experimental results show that the company should spend on prevention and appraisal activities for 850 and 10 Baht/day respectively. It provides minimum cumulative total quality cost, which is 258,000 Baht in twelve months. The effect of the step size in the stage of improving the variables to the optimum was also investigated. It can be stated that the smaller step size provided a better result with more experimental runs. However, the different yield in this case is not significant in practice. Therefore, the greater step size is recommended because the region of optima could be reached more easily and rapidly.

Keywords**—**Quality costs, Steepest Descent Algorithm, Step Size, System Dynamics Simulation

I. Introduction

HISTORICALLY, many researchers have attempted to determine the optima for prevention and appraisal spending which provide the optimum return from the quality improvement. Some works such as [1] – [6] determined an economic quality level by balancing the quality costs which consist of prevention cost, appraisal cost, internal failure cost, and external failure cost. The conceptual models which illustrate the relationship between the total quality cost and the quality level were proposed. The economic quality level is presented at the minimum point of the total quality cost curve in those models.Reference [2] proposed an investigation of the economic quality level by using the mathematics. In that work, the quality costs equation was formulated. The optimum spending on prevention and appraisal which provide the minimum total quality cost was determined by using the mathematics. Nevertheless, this method can be misleading if the turning points of the yield are more than one, and this method is not able to provide continuous monitoring of the process at the stage of moving toward the optimal.

References [7] – [9] employed the Factorial design as a technique to optimise the quality costs of a case study in automotive industry. This technique is the method of Steepest Ascent / Descent suggested by [10]. It is a process of variable development by moving sequentially from the operating condition to the optimal in the response surface with linear regression analysis. In those works, the prevention and appraisal spending were defined as the independent variables, whereas the profit was a yield to optimise. It can be stated that the optimisation with this technique can be explained by statistical theory or the context of response surface methodology, so the results have a high reliability, and the useful information in building up process knowledge toward the optimal can be also displayed.

However, for finding the optimum yield in the response surface precisely, economically, and efficiently by using the combination of factorial design and the first-order model with various levels of the step size for moving sequentially along the part of Steepest Descent or Steepest Ascent to the optimum response may influence. Normally, the step size always base on process knowledge or practical consideration. A greater step size regularly reaches the region of the optimum rapidly and simply, but roughly. This paper describes a quantitative investigation of an effect of the step size in a stage of moving from the current operating condition to the optimum. The rapidity and the accuracy between the different step sizes were considered.

Response Surface Methodology (RSM) is a bundle of statistical and mathematical strategies that are very helpful for modeling and analysing industrial problems. A system response is affected by several variables. An objective is to optimise this system response. For example, suppose that a process engineer wishes to find the levels of pressure (x_1) and temperature (x_2) that minimise the yield (y) of a process. The process yield is a function of levels of pressure and temperature; $y = f(x_1, x_2) + \varepsilon$ as in [11]. Where ε represents the level of signal noise (standard deviation) or error observed in the system response y. If we denote the expected system response by $E(y) = f(x_1, x_2) = \eta$, then the surface represented by; $\eta = f(x_1, x_2)$. So, it is named as a response surface.

A response surface above explains how the process yield varies with changes in k independent variables. Estimation of such surfaces, and hence identification of near optimal settings for variables is a practical issue with interesting theoretical

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aspects. Many iterative processes for making an efficient empirical investigation of such surfaces have been proposed in the last fifty years. These are generally referred to as evolutionary operation (EVOP). RSM is used to enhance the current operating conditions until the optimal conditions are satisfied. In most RSM problems, a form of the relationship between the system response and the variables are unknown. Thus, the first step in RSM is to find a proper approximation for the true functional relationship between the system response of y and the set of its variables via design of experiments. Usually, a low-order polynomial in some region of the variables is applied [12]. If the system response is well modeled by a linear function of the variables, then the approximating function is the first-order model.

This paper is organised as follows. Section II describes the system dynamics simulation in this experiment. Sections III and IV are briefing about algorithms of a conventional design of experiment and steepest descent algorithm, respectively. Section V illustrates computational results and analysis. The conclusions and discussions are also summarised and it is followed by the references.

II. System Dynamics

The experiment in this study employed the system dynamics simulation as an approach because it is able to display the relationships and impacts between each variable in the form of digraph and the behavior of all variables over time can be examined with quantitative analysis. Fig. 1 shows a system dynamics quality costs model of a case study company which is the leading electronic device manufacturer in Thailand. The real data was gathered to construct the model which the operation process and the occurrence of the quality costs are described.In the model, the products are produced for 33,000 pieces per day in average. The proportion of nonconformance (D) depends on the spending on prevention activities (P) whereas the capability of the quality inspection process to detect the nonconformance (DD) depends on the spending on appraisal activities (A). These relationships were constructed by using real data gathered from a case study company. The internal failure cost can be determined from the repairing cost of the detectable nonconformance, whereas the external failure cost are determined from the cost when then undetectable nonconformance are found by the customer. Finally, the total quality cost is the summation of the prevention cost, appraisal cost, internal failure cost, and external failure cost.The experiment was based on a simulation of a quality improvement scenario which spent on prevention and appraisal activities for 100 and 1,000 Baht per day respectively as a current operating condition. The outcome from the simulation along twelve months time scale was considered in term of the cumulative total quality cost. An optimisation technique, the method of Steepest Descent was applied to improve the prevention and appraisal spending sequentially along the direction of the maximum decrease in the response from the current operating condition to the region of the minimum yield. The different step sizes in the stage of moving toward the optimum were experimented. Finally, the proper quality costs parameters were determined, and the effect of step size in the process of optimisation could be investigated.

III. Conventional Design of Experiments (DOE)

Design of Experiment (DOE) is an organized, and structured process that is used to determine the relationship between the different variables affecting a process and the output of that process or system response. This method was first proposed in the 1920s and 1930, by Sir Ronald A. Fisher, the renowned mathematician and geneticist. DOE involves designing a set of ten to twenty experiments, in which all relevant variables are systematically varied. When the results of these factorial experiments are analysed, they help to identify optimal process conditions, the variables that most influence the results, and those that do not, as well as details such as the existence of interactions and synergies between variables.DOE strategies require well-structured data matrices. When applied to a well-structured matrix, analysis of variance delivers accurate results, even when the matrix that is analysed is quite small. Today, Fisher's methods of design and analysis are international standards in applied science and also business. Design of experimental is a strategy to gather empirical knowledge, i.e. knowledge based on the analysis of experimental data and not on theoretical models. It can be carried out whenever you intend to investigate a phenomenon in order to gain understanding or improve performance. Building a design means, carefully choosing a small number of factorial experiments that are to be performed under controlled operating conditions. There are four interrelated steps in building a design:

1. Define an objective to the investigation, e.g. better understand or sort out important variables or find the optimum.
2. Define the variables that will be controlled during the experiment (design variables), and their feasible levels or ranges of variation.
3. Define the variables that will be measured to describe the outcome of the experimental runs (system responses), and examine their precision.
4. Among the available standard factorial designs, choose the one that is compatible with the objective, number of design variables and precision of measurements, and has a reasonable related cost.

Standard designs are well-known classes of experimental designs. They can be generated automatically as soon as you have decided on the objective, the number and nature of design variables, the nature of the system responses and the number of experimental runs you can afford. Generating such a design will provide you with a list of all experiments you must perform, to gather enough purposed information. DOE is widely used in study, research and development, where a large proportion of the resources go towards solving optimisation problems. The key to minimising optimisation costs is to perform as few experiments as possible. DOE requires only a small set of factorial experiments and thus helps to reduce related costs.

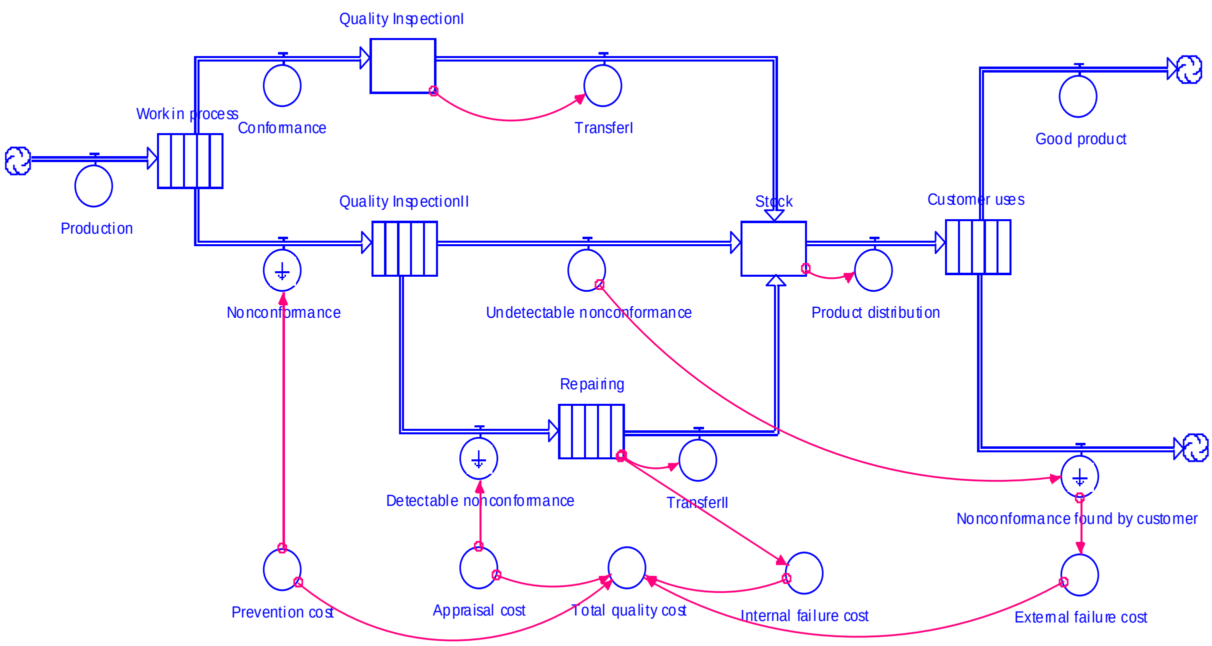

Fig. 1 System dynamics quality costs model

IV. Steepest Descent Algorithm (SDA)

The procedure of SDA is that a hyperplane is fitted to the initial 2^k factorial design points. The data from these points are then analysed. If there is an evidence of main effect(s), at some preset level of statistical significance and no curvature evidence, at the same level of significance, the direction of steepest descent on the hyperplane ($\hat{y}$) is then determined by using the least square principle. It is possible, although rather unlikely, that this design point is the centre preceding factorial design. If there is no evidence curvature or of a main effect the factorial design is replicated. The process repeats until the stopping rule is at the preset state. Whilst continually checking stopping criteria, following steps below would be performed:

1. Random starting factorial design points.
2. Calculate a system response (y) for each point of the factorial design which compose of a center and peripheral point. Then measure a first order model.
3. Determine β_0, β_1, β_2, …, β_k by the least square method from the first order model or a linear multiple regression.
   $Y = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k$
4. Review the significance of the first order model by looking at each of linear regression coefficient (β_i). If none of linear regression coefficient is zero, all factors are significant to the model.
5. Redo the same process; otherwise determine a quadratic effect, in case of an improper equation.
6. 6.1 If model is appropriate, move a center coordinate (x_1, x_2, …, x_k) to a new coordinate (x_1^N, x_2^N, ….x_k^N) by calculating a step size (ΔX_i) which is related to the following equation:

   $\Delta X_i = \beta_i / (\beta_{Largest} / \Delta X_{Largest})$

   Then calculate a new condition from $X_i^N = X_i + \Delta X_i$

   6.2 Scale with a multiplication of 'n' where n = 1, 2, … until a system response (Y_n) could not achieve a better value then termination.

   $Y_n = Origin + n\Delta$
7. Repeat 3-6 to calculate the system responses.
8. Compare each iterative responses and keep the best so far value for a solution.
9. End the algorithm when the stopping criteria is met.

V. Computational Results and Analyses

In this paper, the study was conducted by applying the SDA to determine the proper levels of quality costs parameters. For the computational procedures described above a computer simulation was implemented in a System Dynamics program. The relationship of the parameters on the model was determined. Scatter plots in Fig. 2 and Fig. 3 show the natures of the D versus P and DD versus A, respectively.

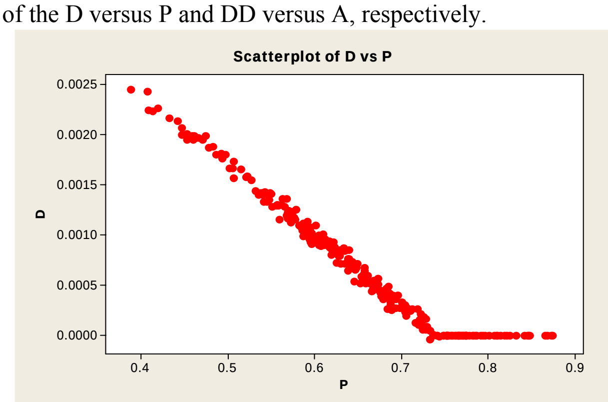

Fig. 2 Scatter Plot of D versus P

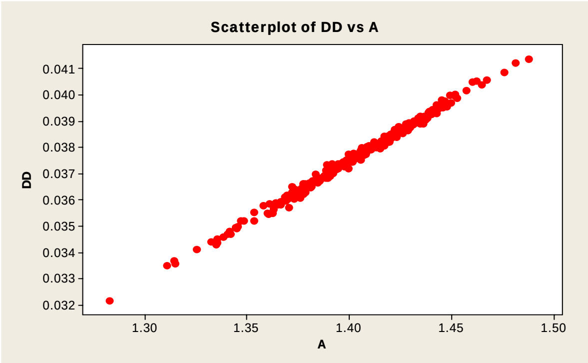

Fig. 3 Scatter Plot of DD versus A

The first order model or a linear regression is then calculated via the least square method. If none of linear regression coefficients, categorized by D and DD, are equal to zero, all factors are significant to the model (Tables 1 and 2).

TABLE I
ANALYSIS OF VARIANCE (ANOVA) AND REGRESSION COEFFICIENTS AND THEIR SIGNIFICANCE FOR D

Source	DF	SS	MS	F	P-value
Regression	1	0.00010465	0.00010465	5251	0.000
Residual Error	298	0.00000594	0.00000002		
Total	299	0.00011059			

Parameters	Coef	T-Stat	P-value
Constant	0.00460519	85.05	0.000
A	-0.00600106	-72.47	0.000

TABLE II
ANALYSIS OF VARIANCE (ANOVA) AND REGRESSION COEFFICIENTS AND THEIR SIGNIFICANCE FOR DD

Source	DF	SS	MS	F	P-value
Regression	1	0.00061965	0.00061965	49655	0.000
Residual Error	298	0.00000372	0.00000001		
Total	299	0.00062337			

Parameters	Coef	T-Stat	P-value
Constant	-0.0269770	-93.10	0.000
A	0.0460557	222.83	0.000

The next experimental procedures to determine the proper levels of quality costs parameters are presents as follow.

1. At the current operating condition, it could be extended to four points around it by changing ± 5% of prevention and appraisal spending in the simulation. Table 3 shows the simulation result which is the cumulative total quality cost in twelve months.

TABLE III
SIMULATION RESULTS AROUND THE CURRENT OPERATING CONDITION

		P = 100 Baht/day	
		P + 5% = 105 Baht/day	P - 5% = 95 Baht/day
A = 1000 Baht/day	A + 5% = 1050 Baht/day	11,093,590 Baht	11,373,660 Baht
	A - 5% = 950 Baht/day	7,419,240 Baht	7,693,600 Baht

From these four points of yield, a first-order model can be formulated as

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i$$

2. Employing a partial statistical t test with the following hypotheses.

$$H_0: \beta_1 = 0$$
$$H_1: \beta_1 \neq 0$$

and

$$H_0 : \beta_2 = 0$$
$$H_1 : \beta_2 \neq 0$$

TABLE IV
ANALYSIS OF VARIANCE (ANOVA) AND REGRESSION COEFFICIENTS AND THEIR SIGNIFICANCE

Source	DF	SS	MS	F	P-value
Regression	2	13598684	6799342	834170	0.0007
Residual Error	1	8.151025	8		
Total	3	13598692			

Parameters	Coef	T-Stat	P-value
Constant	-24604.8775	-609.0161819	0.001045324
P	-27721.5	-97.09807356	0.00655623
A	36772.05	1287.987741	0.000494275

3. Improving the variables, the step of moving along a part of Steepest Descent to the optimum point were determined, a ratio between the coefficients of variables in a first order model was calculated as D. It represents a slope of a part of the Steepest Descent. For the step for climbing to a new potential optimum point, the step size for this case was 0.05 as show in table 5. This step size should be identified as an amount of spending which is a suitable for a company's consideration, and it can provide a significantly different profit.

For the variable x_1, the step is

$$D_{x_1} = 1$$

Whereas for the variable x_2, the step is

$$D_{x_2} = \text{coefficient of } x_2 \,/\, \text{coefficient of } x_1$$

TABLE X
IMPROVING THE VARIABLES ALONG A PART OF STEEPEST DESCENT

Steps	Prevention cost (1,000 Baht)	Appraisal cost (1,000 Baht)	Cumulative Total Quality Cost (1,000 Baht)
Origin	0.10	1.00	
D = 0.05	0.05	-0.07	
Origin + D	0.15	0.93	564.60
Origin + 2D	0.20	0.87	523.42
Origin + 3D	0.25	0.80	499.52
Origin + 4D	0.30	0.73	475.92
Origin + 5D	0.35	0.67	452.27
Origin + 6D	0.40	0.60	428.79
Origin + 7D	0.45	0.54	405.46
Origin + 8D	0.50	0.47	382.15
Origin + 9D	0.55	0.40	359.26
Origin + 10D	0.60	0.34	336.26
Origin + 11D	0.65	0.27	313.35
Origin + 12D	0.70	0.20	290.21
Origin + 13D	0.75	0.14	267.00
Origin + 14D	0.80	0.07	261.00
Origin + 15D	0.85	0.01	258.00
Origin + 16D	0.90	-0.06	270.00
Origin + 17D	0.95	-0.13	285.00

The table 5 shows that the response decreases along the path of Steepest Descent until an increase in response is noted.

4. The graphs of simulating repetition to reach the region of the optimum were plotted for different step sizes e.g. 0.05, 0.075, and 0.1 as show in fig. 4. The rapidity and the accuracy between the different step sizes were clarified.

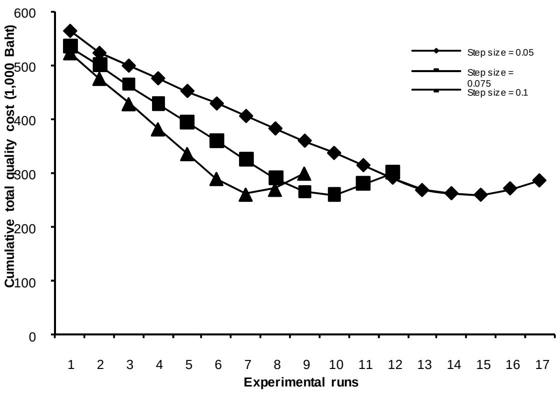

Fig. 4 Simulating repetition to reach the region of the optimum

VI. Conclusions And Discussions

This work shows that the response surface methodology can be applied to improve the variables in this case which are the spending on prevention and appraisal activities to the optima which is the minimum quality cost systemically and gradually. The experiment shows that the method of Steepest Descent can be applied effectively through a first-order response surface analysis. This method is not too complicated and can be explained by statistical theory. It can also provide the useful information for the process analysis such as the trend, the effect, or the variance between the variables and the yield from the continuous monitoring. It was also investigated that the step size could affect the efficiency of the variable improving from the current operating condition to the optimal.

The experimental results indicate as follows:-

1. At the current operating condition which the case study company spends on prevention activities for 100 Baht/day and spends on appraisal activities for 1,000 Baht/day is not the optimum quality level. Therefore, these two variables are needed to be improved. The step size which is 0.05 provided the optimum yield as 258,000 Baht. This point was reached at the fifteenth experimental run.
2. The improvement of the variables along the direction of the maximum decrease in the response surface from the current operating condition with the greater step size, which are 0.075 and 0.10, they provided the optimum yields which are the minimum cumulative total quality cost in twelve months for 258,000 Baht and 261,000 Baht respectively. These optimum points were reached at the tenth experimental run when step size is 0.075, and reached at the seventh experimental run when the step size is 0.1. In this experiment, these three scenarios for testing the step size toward the optimum, each scenario ended when the new design point deteriorated when compared to the previous condition. The new factorial design to determine the path of steepest Descent had no significance on the regression model. The procedure ended and determined the operating condition from the best so far solution from the previous path. In the next phase, the second order design could be done to go further the study and detect the optimum.
3. It can be stated that in this study the step size did affect the efficiency of reaching the optimum yield. The greater step size provided a worse optimum result rapidly, whereas the smaller step size provided a better result with more experimental runs. However, these could not be applied for other problems. In the context of response surface methodology, the response contour plot depends on its own system. The preliminary study could be carried out in each problem to guarantee the optimum or near optimum.
4. In this case, the different step size provided a different yield for 3,000 Baht in a year. This amount of cost does not present a significant different yield in practice for a large company's consideration. Therefore, the greater step size is more suitable for this case because the region of optima could be reached easily and rapidly. However, the other case which the different step size provides a

significant different yield, or it is a very important difference to the company, the small step size seems to be recommended. The shorter step size does no serious problem when compared. It can be only the neighbourhood or close to current operating condition of the problem. The larger step size has more benefit when the current operating condition is very far from the optimum.

5. In practice, if the optima of variables are needed to reach with a high accuracy, the greater step size should be applied for a rough yield in response surface. Focusing on this region of this rough optimal, a smaller step size can be then applied to reach the more precise optima, or the second-order analysis can be also employed for the exact result. These techniques are the shortcut to reach the optima with fewer experimental runs.

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