International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:8, No:11, 2014

Designing Back-stepping Sliding Mode Controller for a Class of 4Y Octorotor

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***Abstract*—**This paper presents a combination of both robust nonlinear controller and nonlinear controller for a class of nonlinear 4Y Octorotor UAV using Back-stepping and sliding mode controller. The robustness against internal and external disturbance and decoupling control are the merits of the proposed paper. The proposed controller decouples the Octorotor dynamical system. The controller is then applied to a 4Y Octortor UAV and its feature will be shown.

***Keywords*—**Backstepping, Decoupling, Octorotor UAV, sliding mode.

I. Introduction

TODAY, UAVs are very popular. UAVs are used in civilian field for aerial drones and Traffic control. Most UAVs are based on piloted configurations, but Quadrotors are different. Unlike conventional helicopters, it has fixed pitch-propellers. Thus control is achieved by varying the speed of rotors [1], [4]. Because of such configurations, it is capable of vertical take-off and landing and it is highly maneuverable. It can be used for monitoring important points of interest, aerial mapping, search and rescue operations and a lot more [2], [5], [9].

In some cases the propellers are enclosed in a cage to insure the safety in indoor flight and also outdoor in case of debries. This can improve the range of applications for these types of UAVs but it is ultimately limited. An important problem of quadrotors is its lack of redundancy. Even if failure strategies have been developed, the quadrotor still depends on all of 4 rotors in order to provide full control. If even one of them is completely inoperative, then stabilization is impossible without reversing the motor or sacrifising the controllability of the yaw state [3], [6], [11], [12].

In this work we will address the payload restrictions of the quadrotor by proposing the use of a 4Y octorotor configuration that an example is shown in Fig. 1. By introducing the additional rotors the reliability of the UAV is increased. That depending on the failures of the rotors and the vehicles configurations the UAV can tolerate 4 rotor malfunctions [6].

In Section II the dynamic modelling of the octorotor UAV will be given. In Section III different control methods are given and discussed. Then in section IV a simulation of the UAV is described with detail and at last the conclusion is given in Section V.

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II. Dynamic Modeling

The dynamics of the 4Y octorotor were derived taking into account the work on quadrotors which is presented in [1], [2], [10], [14]. The following assumptions were made:

1) The structure is rigid and symmetric
2) The center of gravity lies at the center of the device
3) The inertia matrix is diagonal
4) The propellers are hard and don't bend
5) The thrust is proportional to the square of the speed of the rotor
6) The drag is proportional to the square of the speed of the rotor

The 4Y octorotor layout is presented in Fig. 1 along with the chosen coordinate system. The standard definition of a positive rotation is used, and it is defined as a counter-clockwise rotation around the axis as seen from front of the axis line. Also there is two reference frame is used (a body axes frame B fixed at the vehicle's center and an earth fixed frame E).

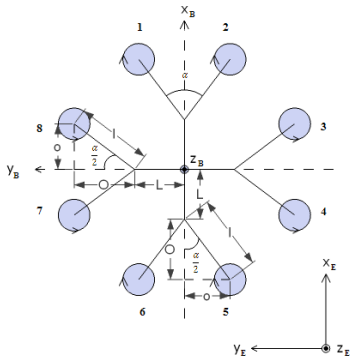

Fig. 1 The 4Y Octorotor

In order to obtain a configuration which is similar to the quadrotor the rotors are paired together two by two in the following design: pair A – 1 with 2 ($\Omega_1 = \Omega_2 = \Omega_A$), pair B – 3 with 4 ($\Omega_3 = \Omega_4 = \Omega_B$), pair C–5 with 6 ($\Omega_5 = \Omega_6 = \Omega_C$) and pair D – 7 with 8 ($\Omega_7 = \Omega_8 = \Omega_D$). Each rotor in the same pair has the same speed and direction. To increase the roll angle, the thrust of pair B is decreased while the thrust of pair D is increased such that overall thrust remains the same. Obtaining a positive pitch angle, the thrust of pair A is decreased while the thrust of pair C is simultaneously increased. The control inputs of the system, $U_1, U_2, U_3 and U_4$, and the disturbance Ω have the following expressions:

$$\begin{cases} U_1 = b\left[O'\left(\Omega_7^2+\Omega_8^2-\Omega_3^2-\Omega_4^2\right)+o\left(\Omega_1^2+\Omega_6^2-\Omega_2^2-\Omega_5^2\right)\right] \\ U_2 = b\left[O'\left(\Omega_5^2+\Omega_6^2-\Omega_1^2-\Omega_2^2\right)+o\left(\Omega_4^2+\Omega_7^2-\Omega_3^2-\Omega_8^2\right)\right] \\ U_3 = d\left(\Omega_3^2+\Omega_4^2+\Omega_7^2+\Omega_8^2-\Omega_1^2-\Omega_2^2-\Omega_5^2-\Omega_6^2\right) \\ U_4 = b\left(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2+\Omega_5^2+\Omega_6^2+\Omega_7^2+\Omega_8^2\right) \\ \Omega = \Omega_3+\Omega_4+\Omega_7+\Omega_8-\Omega_1-\Omega_2-\Omega_5-\Omega_6 \end{cases} \quad (1)$$

where:

$$O = l\cos(\%_2) \quad (2)$$

$$o = l\sin(\%_2) \quad (3)$$

$$O' = O + L \quad (4)$$

The outputs of the system are x, y and z , which denotes the position of the vehicle with respect to the earth fixed frame, and p, q and r , which denote the angular velocity of the vehicle with respect to the body fixed frame. The dynamical systems are as [13], [14]:

$$\begin{cases} \ddot{x} = \frac{1}{m}\left(\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi\right)U_4 \\ \ddot{y} = \frac{1}{m}\left(\cos\phi\sin\theta\cos\psi-\sin\phi\sin\psi\right)U_4 \\ \ddot{z} = -g+\frac{1}{m}\cos\phi\cos\theta U_4 \\ \dot{p} = \frac{1}{I_x}\left(qr(I_y-I_z)-J_r q\Omega+U\right) \\ \dot{q} = \frac{1}{I_y}\left(qr(I_z-I_x)-J_r q\Omega+U_2\right) \\ \dot{r} = \frac{1}{I_z}\left(pq(I_z-I_y)+U_3\right) \end{cases} \quad (5)$$

In order to obtain the angular velocity with respect to the earth fixed frame the following multiplication is performed [15], [16]:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (6)$$

The nominal parameters of the 4Y octorotor and the simplification of the dynamic equations can be found in [17]. Consider the state factors as follow:

$$\begin{aligned} x_1 &= \phi & x_2 &= \dot{x}_1 = \dot{\phi} \\ x_3 &= \theta & x_4 &= \dot{x}_3 = \dot{\theta} \\ x_5 &= \psi \quad , & x_6 &= \dot{x}_5 = \dot{\psi} \\ x_7 &= z & x_8 &= \dot{x}_7 = \dot{z} \\ x_9 &= x & x_{10} &= \dot{x}_9 = \dot{x} \\ x_{11} &= y & x_{12} &= \dot{x}_{11} = \dot{y} \end{aligned} \quad (7)$$

Then with this simplification we can rewrite the dynamic function as follows:

$$f(x) = \begin{cases} x_2 \\ x_4x_6\alpha_1 + x_4\alpha_2\Omega + b_1U_2 \\ x_4 \\ x_2x_6\alpha_3 + x_2\alpha_4\Omega + b_2U_3 \\ x_6 \\ x_4x_2\alpha_5 + b_3U4 \\ x_8 \\ -g+\left(\cos x_1\cos x_3\right)\frac{1}{m}U_1 \\ x_{10} \\ U_x\frac{1}{m}U_1 \\ x_{12} \\ U_y\frac{1}{m}U_1 \end{cases} \quad (8)$$

Here to don't confuse the integers we will use the following parameters:

$$\alpha_1 = \left(I_y-I_z\right)/I_x \ , \ \alpha_2 = -J_R/I_x \ , \ \alpha_3 = \left(I_z-I_x\right)/I_y$$
$$\alpha_4 = -J_R/I_y \ , \ \alpha_5 = \left(I_x-I_y\right)/I_z \quad (9)$$

$$b_1 = l/I_x \ , \ b_2 = l/I_y \ , \ b_3 = l/I_z \quad (10)$$

and;

$$u_x = \left(\cos x_1\sin x_3\cos x_5+\sin x_1\sin x_5\right)$$
$$u_y = \left(\cos x_1\sin x_3\sin x_5-\sin x_1\cos x_5\right) \quad (11)$$

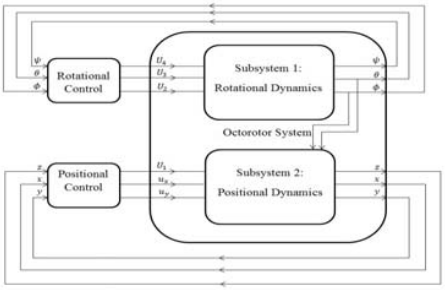

Fig. 2 Two subsystems and their controllers

III. Controller Design Procedure

For designing the controller we have to consider some assumptions in dynamic of the system:

First it can be seen that the rotational model of the octorotor is separate from the Position model, so as can be seen in the Fig. 2 we can consider two sub systems for the model. For the first subsystem we have the Rotational Dynamics, and its controller. And for the second subsystem, i.e. the Position dynamics, we can see from the model that transition on x and y axis depends on U_1, and U_1 is the total thrust of all the motors. Now if we consider u_x and u_y the orientations of U_1 that is responsible for the motion through x and y axis, we can compute the pitch and roll angles necessary to control u_x ,

and u_y [8].

In the end, we can design the controller input as follows:

A. Sliding Controller for the Angular Rotational Dynamics

Consider a nonlinear system:

$$X^{(n)} = f(X) + u + w, |w| \le \varepsilon \tag{12}$$

where f is a known nonlinear part, w is a bounded uncertainty, $X = \begin{bmatrix} x & \dot{x} & \cdots & x^{(n-1)} \end{bmatrix}^T$ is system state and u is the control input. Then sliding variable is

$$S = \left(\frac{d}{dt} + \lambda \right)^{n-1} \hat{X} \tag{13}$$

where λ is a strictly positive constant, $\hat{X} = X - X_d$ and X_d is the desired state. Tracking $X = X_d$ of system is equivalent to $S = 0$. Conventional SMC makes S equal to zero in finite time and then maintain that condition. This controller consists of a reaching mode and a sliding mode, and then we have:

$$u = u_{eq} + u_{reach}, \; u_{reach} = -\tanh(S) \tag{14}$$

In this controller u_{eq} cancels the known terms of the sliding dynamics, and if the uncertainties exist use the Lyapunov stability theorem to get the necessary conditions, as follow:

$$V = \frac{1}{2} S^2, \dot{V} = S\dot{S} \le \eta |S| \tag{15}$$

where η is a positive constant, which implies that [1], [6], [7]:

$$t_{reach} \le \frac{|S(0))|}{\eta} \tag{16}$$

This algorithm has a problem. By introducing linear sliding variable as shown in (4), in sliding phase we have asymptotic stability. In next section we use both terminal sliding mode ideas to cope with these problems. As can be seen in this algorithm tanh(.) is used to eliminate the chattering in reaching phase. Thus tanh(.) is an approximation of sign(.) so deliver a much smoother ending to the system. So now we consider conventional sliding mode, then we have the following equations for both sliding surfaces and also the controller as follow [16], [17]:

$$S_\phi = x_2 + \lambda_1 x_1 \tag{17}$$

$$S_\theta = x_4 + \lambda_2 x_3 \tag{18}$$

$$S_\psi = x_6 + \lambda_3 x_5 \tag{19}$$

and then we have:

$$U_2 = \frac{1}{b_1} \left[-x_4 x_6 \alpha_1 + x_4 \alpha_2 \Omega - \lambda_1 x_2 + k_1 \left(\tanh\left(S_\phi \right) \right) \right] \tag{20}$$

$$U_3 = \frac{1}{b_1} \left[-x_2 x_6 \alpha_3 + x_2 \alpha_4 \Omega - \lambda_2 x_4 + k_2 \left(\tanh\left(S_\theta \right) \right) \right] \tag{21}$$

$$U_4 = \frac{1}{b_1} \left[-x_4 x_2 \alpha_5 - \lambda_3 x_6 + k_3 \left(\tanh\left(S_\psi \right) \right) \right] \tag{22}$$

So with this the angular position of the Octorotor can be controlled.

B. Backstepping Sliding Controller for the Transitional Dynamics

Now for this we consider the second subsystem. For the altitude controller U_1, we use the same sliding controller method to design the controller, therefore we have:

$$S_z = x_8 + \lambda_4 x_7 \tag{23}$$

$$U_1 = \frac{m}{\cos x_1 \cos x_3} \left[g - \lambda_4 x_8 + k_4 \left(\tanh\left(S_z \right) \right) \right] \tag{24}$$

We can see from the model that transition on x and y axis depends on U_1, and U_1 is the total thrust of all the motors. Now if we consider u_x and u_y the orientations of U_1 that is responsible for the motion through x and y axis, we can compute the pitch and roll angles necessary to control u_x and u_y. In the end from back stepping we have the following equations: First consider the tracking error below:

$$z_1 = x_{9d} - x_9 \tag{25}$$

Now consider the following Lyapunov function:

$$V(z_1) = \frac{1}{2} z_1^2 \tag{26}$$

$$\dot{V}(z_1) = z_1 (\dot{x}_{9d} - x_{10}) \tag{27}$$

By considering the following input the stabilization can be obtained:

$$x_2 = x_{9d} + \alpha_1 z_{10} \tag{28}$$

Then, we have:

$$\dot{V}(z_1) = -\alpha_1 z_1 \tag{29}$$

now we make another variable change:

$$z = x_{10} - \dot{x}_{9d} - \alpha_1 z_1 \tag{30}$$

then, the Lyapunov function is:

$$V(z_1, z_2) = \frac{1}{2} \left(z_1^2 + z_2^2 \right) \tag{31}$$

and its derivative is:

$$\dot{V}(z_1,z_2)=z_1\dot{z}_1+z_2\dot{z}_2=-z_1z_2-\alpha_1z_1^2+z_2\left[u_x\frac{U_1}{m}+\alpha_1z_2+\alpha_2^2z_1\right] \quad (32)$$

Then to satisfy the Lyapunov function $\dot{V}(z_1,z_2)<0$, the control input u_x is extracted, and we have:

$$u_x=\left(\frac{m}{U_1}\right)(z_1-\alpha_1(z_2+\alpha_1z_1)-\alpha_1z_1) \quad (33)$$

Also to design u_y we follow the same method and we have:

$$u_y=\left(\frac{m}{U_1}\right)(z_3-\alpha_2(z_4+\alpha_2z_3)-\alpha_2z_4) \quad (34)$$

C. Decoupling of Position Dynamics

Now to have an Innovation in the system of the Octorotor, we can see the Dynamic models that in the altitude dynamics the term $\cos x_1\cos x_3$ is present. This term represents the dependence of the z state with the φ , θ states. To fix this issue we will use the following:

$$U_1=\frac{m}{\cos x_1\cos x_3}\left[g-\lambda_4x_8+k_4(\tanh(S_z))\right]\ ,\ \|\cos x_1\cos x_3\|\le 1 \quad (35)$$

This will give us a controller that is not dependent on other state so we have:

$$U_1=\frac{m}{sign(\cos x_1\cos x_3)}\left[g-\lambda_4x_8+k_4(\tanh(S_z))\right] \quad (36)$$

Now for a much better result we will see the following:

$$\alpha\|\cos x_1\cos x_3\|\le 1 \quad (37)$$

And in this case $\alpha<1$, then we have:

$$U_1=\frac{m}{\alpha\times sign(\cos x_1\cos x_3)}\left[g-\lambda_4x_8+k_4(\tanh(S_z))\right] \quad (38)$$

In the next section all the mentioned controllers will be simulated and the results will be shown.

Fig. 3 A Type of 4Y Octorotor

IV. Simulation Results

We apply the proposed controller on dynamical systems mentioned in (8) on a dynamics of the following Octorotor shown in Fig. 3.

Fig. 4 shows the Rotational Dynamics of the Octorotor states that has been controlled.

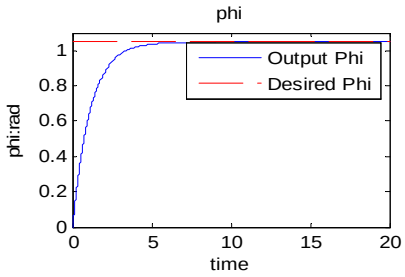

Fig. 4 Output of system ϕ

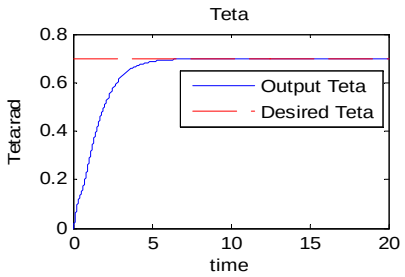

Fig. 5 Output of system θ

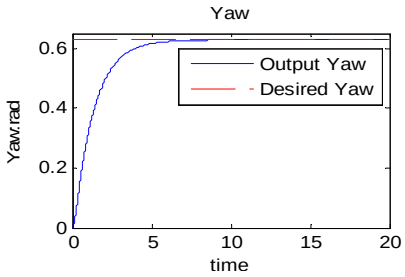

Fig. 6 Output of system ψ

As can be seen the states goes to the desired values with precision. Now we will see the position Dynamics of the Octorotor:

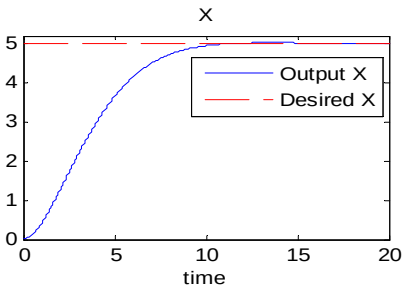

Fig. 7 Output of system x

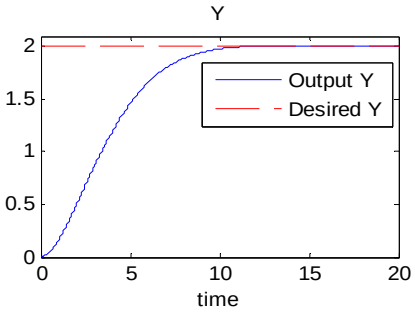

Fig. 8 Output of system *y*

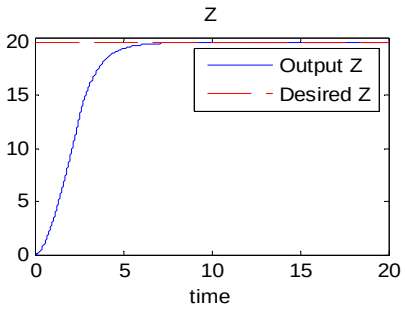

Fig. 9 Output of system z

As it can be seen the system is fully controlled and all the state goes to the desired value. Now here we will see the results for the decoupling controller:

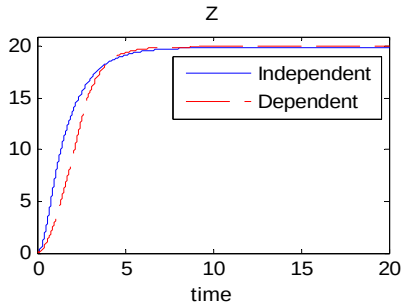

Fig. 10 Output with dependent altitude and independent altitude

Also if we consider the α for the controller we can see a considerable change in the controller:

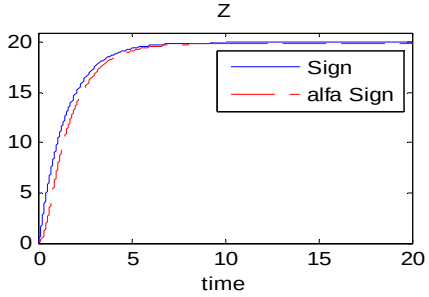

Fig. 11 Output with independent altitude and the α integare

As can be seen the latest equation have a much faster reaction. This change doesn't affect the x or y state, but it can be seen that the system is much faster and goes to the desired states much quicker.

V. Conclusion

The designing robust nonlinear controller is developed in this paper. The combination of both sliding mode and nonlinear controller show the favorable execution. The good response, robustness against uncertainty and disturbance are the main advantages of the proposed controller. The simulation results show the promising performance.

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