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Design Parameters Selection and Optimization of Weld Zone Development in Resistance Spot Welding

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***Abstract*—**This paper investigates the development of weld zone in Resistance Spot Welding (RSW) which focuses on weld nugget and Heat Affected Zone (HAZ). The effects of four factors namely weld current, weld time, electrode force and hold time were studied using a general 2^4 factorial design augmented by five centre points. The results of the analysis showed that all selected factors except hold time exhibit significant effect on weld nugget radius and HAZ size. Optimization of the welding parameters (weld current, weld time and electrode force) to normalize weld nugget and to minimize HAZ size was then conducted using Central Composite Design (CCD) in Response Surface Methodology (RSM) and the optimum parameters were determined. A regression model for radius of weld nugget and HAZ size was developed and its adequacy was evaluated. The experimental results obtained under optimum operating conditions were then compared with the predicted values and were found to agree satisfactorily with each other.

***Keywords*—**Factorial design, Optimization, Resistance Spot Welding (RSW), Response Surface Methodology (RSM).

I. Introduction

RESISTANCE Spot Welding (RSW) is one of the most important manufacturing process in automotive industry for assembling bodies. Quality and strength of the welds are defined by the quality of the weld nuggets [1]. The quality is best judged by the nugget size, Heat Affected Zone (HAZ) and joint strength [2]. Therefore, it is of important to select the welding process parameters for obtaining optimal size of the weld nugget. Welding input parameters play a very significant role in determining the quality of weld nugget and HAZ formation. Usually, the desired welding process parameters are determined based on experience or from handbooks. However, this does not ensure that the selected welding process parameters can produce an optimal weld nugget for that particular welding machine and environment.

In order to overcome this problem, various optimization methods can be applied to define the desired output variables through developing mathematical models to specify the relationship between the input parameters and output variables. One of the optimization methods is by using Design of Experiment (DoE). DoE is a scientific method for identifying the parameters associated with a process and thereby determining the optimal settings for the process parameters for enhanced performance and capability. To predict the welding parameters accurately without consuming time, materials and labour effort, there are various methods of obtaining the desired output variables through models development. Using appropriate statistical technique such as Factorial design, Response Surface Methodology (RSM) and Taguchi Method (TM), the number of necessary experiments can be reduced and the statistical significance of parameters can be safely identified.

In general, optimization is the process of estimating the potential minimum value of machining performance at the optimal point of process parameters. In welding process, literature reports that work has been done on various aspects of modeling and optimization in order to determine the welding input parameters that lead to the desired weld quality. RSM was applied by Koleva [3] to establish the relationship between performance characteristics and their influencing factors. A new statistical approach was proposed to choose the focus position at a condition of maximum thermal efficiency and welding depth. Elangovan *et al*, developed a mathematical model to predict the tensile strength of friction stir welded AA6061 aluminum alloy joint by incorporating welding parameters and tool profiles using RSM [4]. The developed mathematical model can be effectively used to predict the tensile strength of Friction Stir Welding (FSW) joints at 95% confidence level.

The investigation on the optimization and the effect of welding parameters on the tensile shear strength of spot welded galvanized steel sheet was presented by Thakur *et al*. [5]. The authors found that it is possible to increase tensile shear strength significantly using the proposed statistical technique. A mathematical model for predicting the nugget diameter and tensile shear strength of galvanized steel was developed by Luo *et al.* [6] using nonlinear multiple regression analysis and Artificial Neural Network (ANN) approach. According to the prediction models, the predicted systems of welding process parameters were formulated in order to obtain the desired welding quality. Another approach is using TM to investigate on the optimization and effect of welding parameters on the tensile shear strength of spot welded SAE 1010 steel sheet which was reported by Esme [7]. The author concluded that TM can be effectively used for optimization of spot welding parameters. The use of Taguchi's loss function analysis to a spot welding process in order to discover the key process parameters which influence the tensile strength of welded joints was investigated by Rowlands and Antony [8].The purpose of this research was to illustrate

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an application of DoE to a spot welding process.

In this work, the main objective is to find the optimum parameter to optimize the radius of weld nugget and HAZ which can improve the welding quality and performance of RSW. The operating parameters were optimized using Central Composited Design (CCD). It started with using a general 2^4 factorial design to determine which of the various parameters were important in the response surface study. Next, an experimental design was selected to evaluate the relations existing between the important parameters and the responses (radius of weld nugget and HAZ). Experiments were conducted according to the selected experimental design, followed by data analysis which included regression analysis, model adequacy checking and determination of optimum conditions.

II. Experimental Set-up and Procedures

A. Experimental Set-up

The workpiece material used in this study was 1.21 mm thick uncoated low carbon steel as the base metal. The chemical composition and mechanical properties of this material is shown in Table I. The experiment involved joining of two sheets layer using RSW machine model NDN-50-M10-F. Welding was performed using a 45-deg truncated cone Class 2 electrode with 6 mm face diameter.

Samples for the metallographic examination were prepared using standard metallography procedure. Nital etching reagent was used to reveal the macrostructure of the samples. Radius of weld nugget and HAZ were measured for all the samples on the metallographic cross-sections of the welds. A schematic illustration of the weld zone is shown in Fig. 1.

TABLE I
Chemical Composition and Mechanical Properties of Workpieces

Percent composition (%)		Yield strength (MPa)	Tensile strength (MPa)
C	0.186		
Mn	0.146		
Si	0.011		
S	0.0011	144	209
P	0.001		
Cr	0.035		
Ni	0.032		

B. Factorial Designs

Factorial designs are most efficient designs to study the joint effect of two or more factors on a response. A general 2^4 factorial design augmented by 5 centre points was used to study the effect of factors namely weld current, weld time, electrode force and hold time on the development of weld zone. Replicates runs at centre point of the design allow the experimenter to check for the quadratic effect as well as an independent estimate of error to be obtained. Table II shows the parameters and levels applied in the 2^4 factorial designs. The low and high levels of these parameters are coded as -1 and +1 respectively. Other parameter such as squeezing cycles was set to be constant throughout the study. This design experiment is to determine the most likely important operating parameters in the response surface study. The statistical significance of each parameter and their combinations were then evaluated using the Minitab software at 5% significance level.

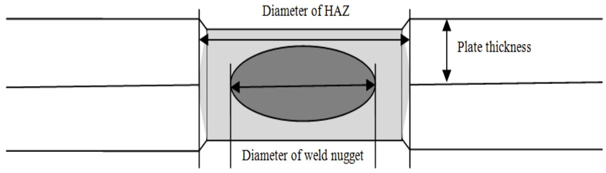

Fig. 1 Schematic illustration of the weld zone

TABLE II
Parameters and Levels Applied in 2^4 Factorial Design

Parameters	Symbols	Units	Levels	
			-1	+1
Weld current	A	kA	8	12
Weld time	B	Cycles	10	14
Electrode force	C	kN	1.5	3.0
Hold time	D	Cycles	2	4

C. Optimization Experiment

Optimization of the significant parameters affecting the development of weld zone obtained from the factorial design was carried out using CCD. By expanding the design, several points are evaluated which increases the chances of detecting the response at which the optimum for a factor occurs. Table III summarizes the parameters studied (-α, -1, 0, +1, α) in CCD, where levels -1 and +1 represent the low and high values, -α and α indicate the low and high extreme values, and 0 is the centre value of each parameter. Since the hold time did not influence the development of weld zone significantly, it was fixed at 2 cycles [9] throughout the optimization experiment.

TABLE III
Parameters and Levels Applied in CCD

Parameters	Symbols	Units	Levels				
			-α	-1	0	+1	α
Weld current	A	kA	6.6	8	10	12	13.4
Weld time	B	Cycles	9	10	12	14	15
Force	C	kN	1.0	1.5	2.25	3.0	3.5

D. Regression Analysis

A multiple regression analysis was performed on a regression model which corresponds to the following second-order response function [10]:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum\sum_{j<i} \beta_j x_i x_j + \varepsilon \tag{1}$$

where β_0, β_i, β_{ii} and β_{ij} are the coefficients of intercept, linear, quadratic and interaction variables respectively, y is the dependent variable or the response, x_i and x_j are the independent variables in coded unit, and ε is the error term that accounts for the effects of excluded parameters. The equation used for coding is [10]:

$$x = \frac{X - \left(X_{high} + X_{low}\right)/2}{\left(X_{high} - X_{low}\right)/2} \tag{2}$$

where x is the coded variable, X is the natural variable, while X_{high} and X_{low} are the high and low values of the natural variables respectively. During the analysis, coefficients that caused the model (Eq. 1) to best fit a set of collected response variable data obtained from the optimization experiments were determined by the least squares method with the aid of the Minitab software.

E. Optimum Operating Parameters

To determine the optimum operating parameters for the development of weld zone, the optimization feature of the Minitab software was used. The software searched for a combination of parameters that simultaneously satisfy the ultimate goals and limits placed on the response and each of the parameters. The optimum values of the parameters obtained were then assessed by composite desirability, which carries a value from 0 to 1, to determine the degree of satisfaction of the optimum values for the ultimate goal of response.

III. Results and Discussion

A. Factorial Design of Operating Parameters

The design matrix and results of 2^4 factorial design augmented by five centre points with four parameters (weld current, weld time, electrode force and hold time) and two responses (radius of weld nugget and HAZ) are given in Table IV. The experimental sequence (Std Order) was randomized in order to minimize the unexpected variability in the observed response. The significant effect of each parameter was evaluated by a normal probability plot of standardized effect at 5% significance level as shown in Fig. 2 and Fig. 3 for radius of weld nugget and radius of HAZ respectively. From Figs. 2 & 3, it was observed that the main effects such as the weld current (A), weld time (B) and electrode force (C), as well as the interaction effect of (weld current x weld time), (weld current x force) and (weld time x force) are the influential parameters for the development of weld zone, thus these factors were selected for the next step.

B. Optimization of Operating Parameters

RSM was used to fit a second-order polynomial model (Eq. 1) where a CCD with 20 runs was required to cover all possible combination of the factor levels which are composed of 8 factorial points (standard order 1 – 8), 6 star points (standard order 9 – 14) and 6 replicates of the centre point (standard order 15 – 20). The centre point is very important since it represents a set of experimental conditions at which six independent replicates were run. The variation between them reflects the variability of all design. It was used to estimate the standard deviation. All optimization experiments were conducted randomly in one block of measurement. The design matrix and results of CCD with three parameters (weld current, weld time and electrode force) and the responses (radius of weld nugget and HAZ) are tabulated in Table V.

TABLE IV
Design Matrix of 2^4 Factorial Design and Experimental Results

Standard order	Run order	Parameters				Radius of weld nugget (mm)	Radius of HAZ (mm)
		A	B	C	D		
14	1	12	10	3	4	3.050	3.670
18	2	10	12	2.25	3	2.610	3.430
1	3	8	10	1.5	2	1.830	2.750
13	4	8	10	3	4	2.300	3.110
3	5	8	14	1.5	2	1.950	3.200
20	6	10	12	2.25	3	2.620	3.444
17	7	10	12	2.25	3	2.630	3.450
5	8	8	10	3	2	2.400	3.100
6	9	12	10	3	2	2.930	3.610
4	10	12	14	1.5	2	3.030	3.560
2	11	12	10	1.5	2	3.150	3.800
8	12	12	14	3	2	3.245	3.860
7	13	8	14	3	2	2.750	3.410
9	14	8	10	1.5	4	1.880	2.800
15	15	8	14	3	4	2.810	3.350
21	16	10	12	2.25	3	2.630	3.455
19	17	10	12	2.25	3	2.610	3.440
11	18	8	14	1.5	4	1.930	3.120
12	19	12	14	1.5	4	3.055	3.780
16	20	12	14	3	4	3.315	3.940
10	21	12	10	1.5	4	3.000	3.630

C. Regression Model

Table VI shows the estimation coefficient (Coef) of each variable term in a regression model for weld nugget and HAZ radius along with the corresponding standard deviation (SDcoef), t-statistics (t-Stat) and probability (P) values determined at 5% significance level. Variable terms with $P < 0.05$, are A, B, C, A^2, AC and BC which are considered statistically significant for radius of weld nugget, while for the radius of HAZ, the significant variables are A, B, C, A^2, B^2 and AB. Therefore, a second-order model was built to describe the behavior of each response, followed by the optimization stage to find the best setting for each factor. The second-order models for radius of weld nugget and radius of HAZ in terms of coded variables with all significant terms are given in Eqs. (3) and (4), respectively.

$$\begin{aligned} Y_1 &= 2.65893 + 0.46611A + 0.12760B + 0.15197C \\ &-0.08532A^2 + 0.04245B^2 + 0.02114C^2 - 0.03250AB \\ &-0.14750AC + 0.0775BC \end{aligned} \tag{3}$$

$$\begin{aligned} Y_2 &= 3.59991 + 0.33316A + 0.13760B + 0.10525C \\ &-0.07814A^2 - 0.07285B^2 + 0.1395C^2 - 0.095AB \\ &-0.02250AC - 0.01250BC \end{aligned} \tag{4}$$

where Y_1, Y_2, A, B and C are radius of weld nugget, radius of HAZ, weld current, weld time and electrode force respectively.

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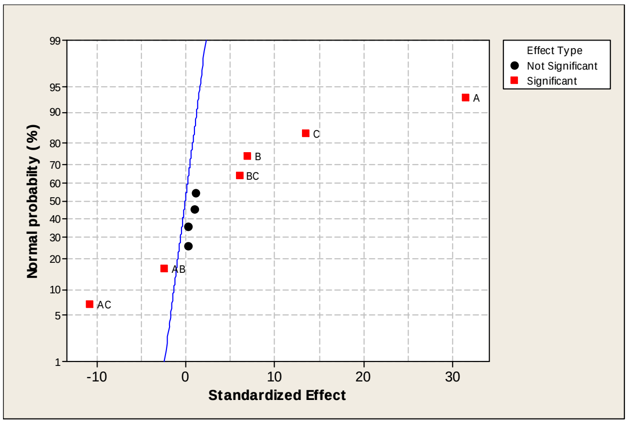

Fig. 2 Normal probability plot of standardized effects for radius of weld nugget

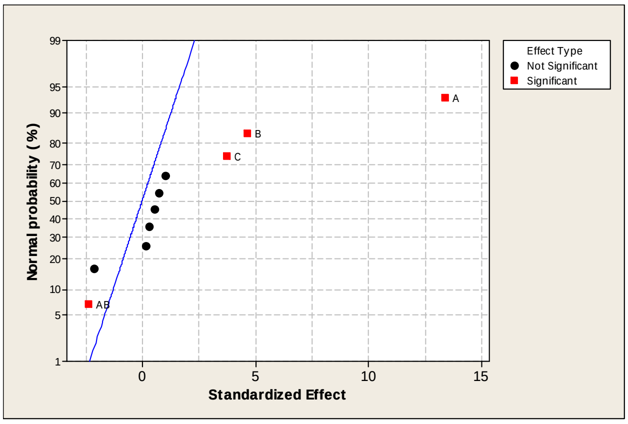

Fig. 3 Normal probability plot of standardized effects for radius of HAZ

D. Model Adequacy Checking

Table VII shows the analysis of variance (ANOVA) of the regression model for the weld nugget (Eq. 3) and HAZ radius (Eq. 4). The regression model and each variable term (linear, square and interaction) in the model show P values less than 0.05 thus are statistically significant. The high P value ($P > 0.05$) of lack-of-fit indicates that the model is adequate for predicting the weld nugget and HAZ radius. To test the global fit of the model, the coefficient of determination (R^2) were evaluated. The R^2 for radius of weld nugget was 0.981 and for the radius of HAZ was 0.971. The second-order model obtained for the radius of weld nugget and HAZ are satisfied since the values of R^2 are high and closed to 1 [11]. The high R^2 values indicate that the model is highly significant and provides a good estimate of the response within the experimental domain studied. Therefore, the data for each response are considerably well-fitted in the developed model. The adequacy of model was also examined from the normal probability plot of standardized residuals as shown in Fig. 4 and Fig. 5 for the weld nugget and HAZ radius respectively. From the figures, all points cluster along the straight line, which indicates that the underlying regression assumptions are satisfied [12].

The positive coefficients of variable terms in Eqs. (3) and (4) indicate their synergistic effect, whereas negative sign indicates antagonistic effect. For example in Eq. (3), 0.46611A indicates that with the increase of weld current (A), the sizes of weld nugget are increased. The fusion zone size increases with the welding current, as stated by Pouranvari [13]. In RSW, welding current and contact surface have the greatest effect on the growth of weld nugget [14-16].

TABLE V
DESIGN MATRIX OF CCD AND EXPERIMENTAL RESULTS

Standard order	Run order	Parameters			Radius of weld nugget (mm)	Radius of HAZ (mm)
		A	B	C		
5	1	8	10	3	2.30	3.09
3	2	8	14	1.5	1.95	3.25
1	3	8	10	1.5	1.83	2.80
10	4	13.4	12	2.25	3.30	4.00
14	5	10	12	3.5	2.89	3.85
11	6	10	9	2.25	2.54	3.15
19	7	10	12	2.25	2.62	3.55
15	8	10	12	2.25	2.71	3.67
20	9	10	12	2.25	2.63	3.57
7	10	8	14	3	2.80	3.43
6	11	12	10	3	2.95	3.83
2	12	12	10	1.5	3.00	3.69
18	13	10	12	2.25	2.62	3.55
16	14	10	12	2.25	2.75	3.70
8	15	12	14	3	3.25	3.85
4	16	12	14	1.5	3.06	3.70
13	17	10	12	1	2.53	3.45
17	18	10	12	2.25	2.63	3.55
9	19	6.6	12	2.25	1.51	2.77
12	20	10	15	2.25	2.95	3.75

E. Determination of Optimum Parameters

An optimization study is required to determine the optimal conditions for the development of weld nugget and HAZ simultaneously. In fact, once the model has been developed and checked for adequacy, the optimization criteria can be set to find the optimum conditions. Response optimizer was used to identify the combination of input variables settings that jointly optimize a set of responses (radius of weld nugget and radius of HAZ).This research is aimed to normalize the radius of weld nugget and minimize the radius of HAZ. Therefore, the predicted radius of weld nugget value was 3.00 mm and radius of HAZ was 3.40 mm. The optimum parameters obtained in uncoded units were (weld current at 11.37 kA, weld time of 9 cycles and electrode force of 1257 N) which give the highest composite desirability (1.00) as shown in Fig. 6.

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TABLE VI
ESTIMATED COEFFICIENTS OF THE REGRESSION MODEL

Term	Coef	SD_{coef}	t-Stat	P
Radius of weld nugget				
Constant	2.6893	0.03516	75.632	0.000
A	0.46611	0.02322	20.075	0.000
B	0.12760	0.02322	5.241	0.000
C	0.15197	0.02341	6.492	0.000
A x A	-0.08532	0.02232	-3.741	0.003
B x B	0.04245	0.02232	1.530	0.144
C x C	0.02114	0.02305	1.003	0.377
A x B	-0.03250	0.03047	-1.067	0.310
A x C	-0.14750	0.03047	-4.840	0.001
B x C	0.07750	0.03047	2.543	0.029
Radius of HAZ				
Constant	3.59991	0.03107	115.849	0.000
A	0.33316	0.02052	16.239	0.000
B	0.13760	0.02052	6.508	0.000
C	0.10525	0.02069	5.088	0.001
A x A	-0.07814	0.01972	-4.121	0.004
B x B	-0.07285	0.01972	-2.981	0.015
C x C	0.1395	0.02037	0.517	0.528
A x B	-0.09500	0.02693	-3.528	0.007
A x C	-0.02250	0.02693	-0.836	0.445
B x C	-0.01250	0.02693	-0.464	0.668

A: weld current; B: weld time; C: electrode force

TABLE VII
ANOVA OF THE REGRESSION MODEL

Term	DF	Seq SS	F	P
Radius of weld nugget				
Regression	9	3.88505	58.54	0.000
Linear	3	3.51040	158.70	0.000
Square	3	0.14409	6.51	0.010
Interaction	3	0.23055	10.42	0.002
Residual error	10	0.07373		
Lack-of-fit	5	0.05813	3.73	0.088
Pure error	5	0.01560		
Total	19	3.95878		
Radius of HAZ				
Regression	9	2.14115	37.14	0.000
Linear	3	1.91638	99.73	0.000
Square	3	0.14727	7.66	0.006
Interaction	3	0.07750	4.03	0.040
Residual error	10	0.06405		
Lack-of-fit	5	0.04077	1.75	0277
Pure error	5	0.02328		
Total	19	2.20520		

DF = degree of freedom; Seq SS = sequential sum of squares; F = F values from Fisher's statistical test

The final step is to conduct the confirmation test experiment which will then be compared to the predicted value. Experiments under the optimum operating conditions were repeated three times and the results are tabulated in Table VIII. The percentage error between confirmation experiment and prediction is 1.01% for the weld nugget and 2.86% for the HAZ. It shows that the optimum operating condition agrees well with the predicted one.

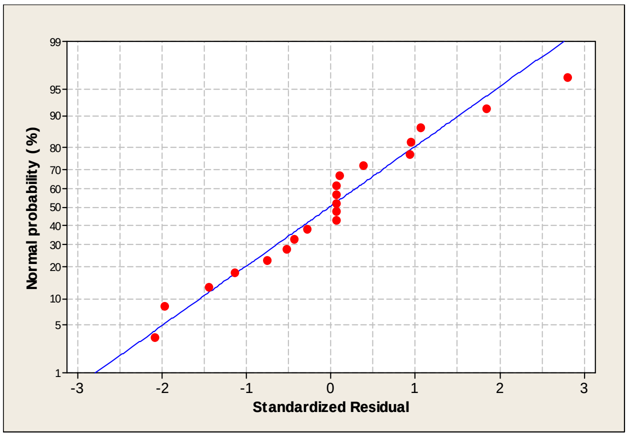

Fig. 4 Normal probability plot of standardized residuals for radius of weld nugget

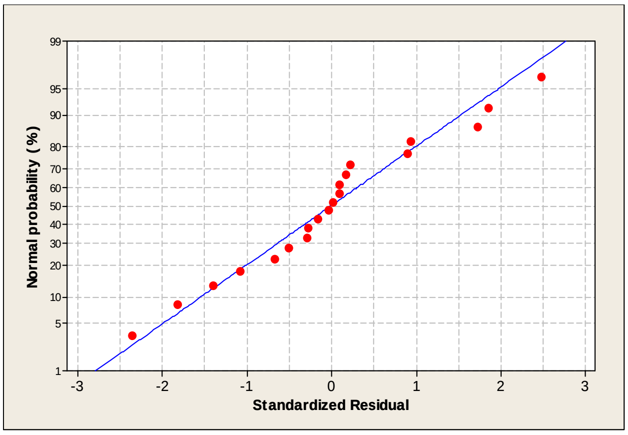

Fig. 5 Normal probability plot of standardized residuals for radius of HAZ

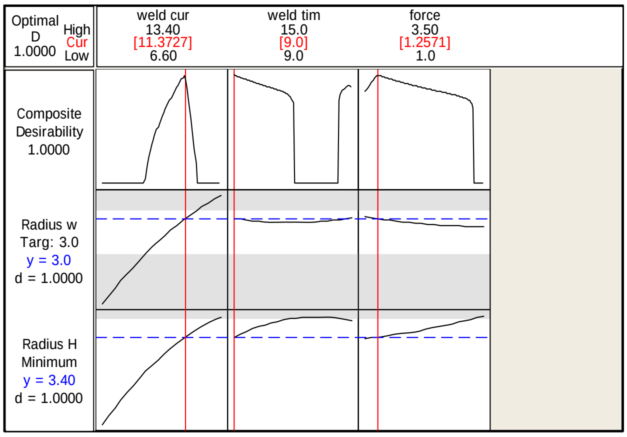

Fig. 6 Response optimization plot

TABLE VIII
CONFIRMATION EXPERIMENTAL RESULTS

Response	Prediction (mm)	Experiment (mm)				Error (%)
		1	2	3	Average	
Radius of weld nugget	3.00	2.90	2.95	3.05	2.97	1.01
Radius of HAZ	3.40	3.50	3.45	3.55	3.50	2.86

IV. CONCLUSION

An experimental design was used to determine the effects of welding parameters (weld current, weld time, electrode force and hold time) on the development of the weld zone. All the selected factors except hold time affected the radius of weld nugget and HAZ significantly and thus were optimized using the Central Composite Design by RSM. A quadratic model for radius of weld nugget and radius of HAZ as a function of the significant parameters were developed. The second-order model obtained was satisfied since the values of R^2 for the radius of weld nugget (0.981) and HAZ (0.971) were high and closed to 1. The optimum operating parameters for predicting the size of weld zone was determined as follows: weld current at 11.37 kA, weld time of 9cycles and electrode force of 1257 N. The experimental obtained under the optimum operating conditions and the predicted one was found to agree satisfactorily with each other.

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REFERENCES

[1] H.Eisazadeh, M.Hamedi, A.Halvaee,"New parametric study of nugget size in resistance spot welding process using Finite element method", Materials and Design, vol.31, pp.149-157, 2010.
[2] A.G.Thakur, T.E.Rao, M.S.Mukhedkar, V.M.Nandedkar, "Application of Taguchi method for resistance spot welding of galvanized steel", ARPN J of Eng and Applied Sci, vol.5(11), pp.22-26, 2010.
[3] E. Koleva, "Elecro beam weld parameters and thermal efficiency improvement", Vacuum Journal, vol.77, pp.413-421, 2005.
[4] K.Elangovan, V.Balasubramaniam, S.Babu, "Predicting tensile strength of friction stir welded AA6061 aluminum alloys joints by a mathematical model", Mat and Design, vol.30, pp.188-193, 2009.
[5] A.G.Thakur and V.M.Nandedkar, "Application of Taguchi method to determine resistance spot welding conditions of austenitic stainless steel AISI 304", Journal of Scientific & Industrial Research, vol.69, pp.680-683, 2010.
[6] Y. Luo, C.Li, H.Xu, "Modeling of resistance spot welding process using nonlinear regression analysis and neural network approach on galvanized steel sheet", Advanced Material Research, vol.291-294, pp.823-828, 2011.
[7] U. Esme, "Application of Taguchi method for the optimization of resistance spot welding process", The Arabian Journal for Science and Engineering, vol.34, pp.519-528, 2009.
[8] H. Rowlands and J. Antony, "Application of design of experiments to a spot welding process", Assembly Automation, vol.23, pp.273-279, 2003.
[9] N.Muhammad, Y.H.P.Manurung, M.Hafidzi, S.K.Abas, G.Tham and M.R.A.Rahim, "A quality improvement approach for resistance spot welding using multi-objective Taguchi method and response surface methodology", Int. Journal on Adv. Sci. Eng. Inform. Tech., vol. 2, pp. 17-22, 2012.
[10] R.H. Myers, D.C. Montgomery, C.M.Anderson-Cook, Response Surface Methodology, 3rd ed., Wiley, New York, 2009.
[11] L.W. Low, T.T. Teng, A.F.M. Alkarkhi, A. Ahmad and N. Morad, "Optimization of the adsorption conditions for the decolorization and COD reduction of methylene blue aqueous solution using low-cost adsorbent", Water Air Soil Pollut, vol. 214, pp.185–195, 2011.
[12] W.D. Berry, Understanding Regression Assumption, Sage Publication Inc., The United States of America, 1993.
[13] M. Pouranvari, "Effect of fusion zone size on the mechanical response of DQSK steel resistance spot welds", Australian Journal of Basic and Applied Science, vol. 5 (12), pp.573-577, 2011.
[14] S.M.Darwish and S.D.Al-Dekhial, "Statistical models for spot welding of commercial aluminium sheets", Int Journal of Machine Tools & Manufacture, vol.39, pp.1589-1610, 1999.
[15] Y.H.P.Manurung, N.Muhammad, E.Haruman, S.K.Abas, G.Tham, K.M.Salleh and C.Y.Chau, "Investigation on weld nugget and HAZ development of resistance spot welding using SYSWELD's customized electrode meshing and experimental verification", Asian Journal of Industrial Engineering, vol.2, pp.63-71, 2010.
[16] M.Hamedi and H.Pashazadeh, "Numerical study of nugget formation in resistance spot welding", International Journal of Mechanis, vol.2, pp.11-15, 2008.