

# Design and Analysis of Gauge R&R Studies: Making Decisions Based on ANOVA Method

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**Abstract**—In a competitive production environment, critical decision making are based on data resulted by random sampling of product units. Efficiency of these decisions depends on data quality and also their reliability scale. This point leads to the necessity of a reliable measurement system. Therefore, the conjecture process and analysing the errors contributes to a measurement system known as Measurement System Analysis (MSA). The aim of this research is on determining the necessity and assurance of extensive development in analysing measurement systems, particularly with the use of Repeatability and Reproducibility Gages (GR&R) to improve physical measurements. Nowadays in productive industries, repeatability and reproducibility gages released so well but they are not applicable as well as other measurement system analysis methods. To get familiar with this method and gain a feedback in improving measurement systems, this survey would be on “ANOVA” method as the most widespread way of calculating Repeatability and Reproducibility (R&R).

**Keywords**—Analysis of Variance (ANOVA), Measurement System Analysis (MSA), Part-Operator interaction effect, Repeatability and Reproducibility.

## I. INTRODUCTION

THE manufacturing environment, by its very nature, relies on two types of measurements to verify quality and to quantify performance: (1) measurement of its products, and (2) measurement of its processes. Therefore, product evaluation and process improvement require accurate and precise measurement techniques. Due to the fact that all measurements contain error, and in keeping with the basic mathematical expression:

Observed value = True value + Measurement Error, understanding and managing “measurement error”, generally called Measurement Systems Analysis (MSA), is an extremely important function in process improvement [1].

Measurement System Analysis (MSA) is a comprehensive collection of tools for the measurement, acceptance, and data or errors analysis which consist of topics such as Statistical Process Control (SPC), capability analysis, and Gage Repeatability and Reproducibility (GR&R). Measurement System Analysis (MSA) is used in order to discern simple and complex measurements of products with physical instruments or even visual inspection. This has a strong relation on human

judgments of product specifications and attributes.

Despite the comprehensive aspect of MSA and the importance of gage control, specialists in productive industries, express their anxiety about the measurement reliability which is used in decision making. Quality of a measurement instrument or method is effective for the validity of a production process and statistical methods based on these data. For making correct decisions, relying on a measurement system and the same for the statistical method used, is necessary as it could be said, when the data quality are low the benefits of a measurement system is also low; likewise when the data quality is high, the benefit is high too [2].

## II. PROVENANCE AND REASONS OF MEASUREMENT ERRORS

A simple division for measurement errors resources are the two categories: regular errors and accidental errors. Measuring an instrument’s regular errors usually might be done by its constituent or the calibration lab.

On the other hand, accidental errors are those which occur because of the differences between measurement instruments, operators, not a permanent bias in instrument, environmental changes and different adjustments.

Therefore, it would be necessary to analyse measurement capacity by accuracy and precision definitions:

A. Accuracy: Shows difference between the average values of the observed measurement and the actual measurement. As much as this difference is smaller, the measurement system will have the better accuracy and the probable average of distribution measurement would be adapted to the real value.

Accuracy could be divided into three aspects:

1. Stability: Total measurement consistency resulted from a measurement system, while the specification of a part is measured over times.
2. Bias: Differentiation between averages and actual value of the measured part.
3. Linearity: Measuring bias values through the expected range of measurements.

B. Precision: Variation that occurs while measuring the same part with same instrument. The measurement system’s precision might become higher, if the variation becomes lower.

Precision could be divided into two aspects:

1. Repeatability (equipment variability): Variation in repetitious measurements of a part using unique

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equipment under exact condition.

In fact, this is an index shows capability of a measurement system to obtain similar results from repetitious measurements (Fig. 1).

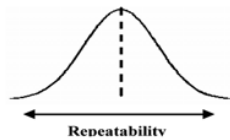


Fig. 1 Repeatability

2. Reproducibility (appraiser variation): Variation in the average of measurements when different operators measure the same part with unique equipment. Reproducibility shows the variation of human factors in using one unique tool and method (Fig. 2).

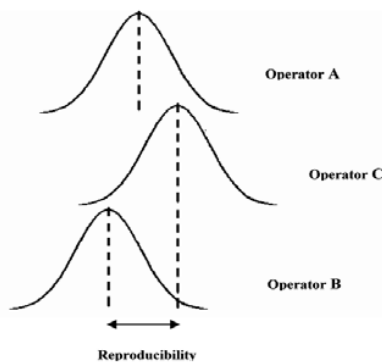


Fig. 2 Reproducibility

In an ideal measurement system, repetitive values gained from a single part, shouldn't show any differences. Many reasons could be found for these changes that some contributes to the environmental performance and some others to the variation of measurement system itself. It could be concluded that repeatability and reproducibility are variations seen by repetitious measurements of a unique part specification.

Regarding to the Raffaldi and Kappel (2004): "If measurement variation can be reduced and gauge repeatability and reproducibility ratios improved, it is easier to differentiate between parts that are in or out of specification, allowing parts to be accepted or rejected with greater confidence" [1]

Hence, the R&R gage (GR&R) could use as an auditing tool which gives feedbacks on the improvement of measurement methods.

For the importance and of necessity the R&R index, as an estimation of a measurement system impact in process improvement surveys ratiocinations, it will be investigated more.

### III. R&R INDEX AND ITS COMPUTATION TECHNIQUES

A measurement system calibration is the most reliable way for making certainty of the measurement system accuracy. Although, calibration only study an instrument or system's measurement errors and not the reliability and reproducibility

errors made by different operators. To identify accuracy and reach to the variation range, standard deviation of the system has to be calculated.

A measurement system could be divided into two main aspects: reliability and reproducibility. Relation between these two can be shown by Fig. 3.

$$\begin{aligned} \sigma^2_{\text{Total}} &= \sigma^2_{\text{Measurement System}} + \sigma^2_{\text{Process Output}} \\ \sigma^2_{\text{Total}} &= \sigma^2_{\text{Repeatability}} + \sigma^2_{\text{Reproducibility}} + \sigma^2_{\text{Process Output}} \\ \sigma^2_{\text{Total}} &= \sigma^2_{\text{Repeatability}} + \sigma^2_{\text{operator}} + \sigma^2_{\text{operator: Process Output}} + \sigma^2_{\text{Process Output}} \end{aligned}$$

Fig. 3 Reliability and Reproducibility Relations [3]

In the GR&R studies, reliability and reproducibility observations illustrate how much of the production process variations belong to the measurement system dispersion.

Various methods could calculate an instrument's R&R index. In continue, some of them are compared.

1. Range Method: Used to determine a quick estimate in measurement system variation. But this method couldn't divide these variations to the two paraphrases: repeatability and reproducibility.
2. Average and Range Method: A computational way for representing a system repeatability and reproducibility estimate. Comparing to the range method, this one divides variation to its paraphrases.
3. Average and Standard Deviation: This method has the same characteristics as the previous one.
4. Analysis of Variance (ANOVA) Method: This method could segregate repeatability and reproducibility paraphrases. These segregations are between variations related to operators and instruments.

From all these methods, "Average and Range" and "ANOVA" methods are the most prevalent and important. Although all of them contain information on variation reasons but "ANOVA" method has a more widespread usage. This function leads to its comparison with "Average and Range" method.

### IV. ANOVA METHOD AND ITS PREFERENCES TO THE OTHER METHODS

Analysis of variance (ANOVA) is a general statistical technique to analysis deviation and identifies other variation's sources in a measurement system. In the "Average and Range" method, dispersion could be divided into three components: part variation, repeatability, and reproducibility, but in "ANOVA" it is divided into four components. In other words, in this technique, variation of repeatability is also divided into operator variation and operator-part variation.

These four variations are defined as below:

1. Part variation
2. Operator variation
3. Operator-part variation (operator stability)

4. Instrument/tool variation (repeatability)

Because of considering operator-part variation, “ANOVA” method is known as a more accurate technique than “Average and Range”.

In continue, some of the “ANOVA” method’s advantages are mentioned:

- Analysis could be done in any circumstances
  - Estimates variations more accurate
  - More information such as relation between parts variations and the operators’ role would be gained
- “ANOVA” method, based on differences in entry data, divides into these categories [4]:

1. One-Way Between Groups

- Diversity between groups is under attention
- Only one operative used to identify each group
- The simplest technique
- Either might be used to compare variables between different groups

2. One-Way Repeated Measures

- Used when a part measured many times
- Help to define meaningful changes during time
- Known in Excel software as single factor ANOVA function (Table I)

TABLE I  
ANOVA: SINGLE FACTOR

Source of Variation	SS	DF	MS	F
Operator	SSO	k-1	SSO / k-1	MSO / MSO*P
Operator-Part	SSO*P	k*(n-1)	SSO*P / K(N-1)	-
Total	SS	k*n-1	-	-

3. Two-Way Between Groups

- Useful in complex groupings
- Examination of two different elements and their internal effects

• Each of the main effects is “one-way tests”. The interaction effect is simply asking "is there any significant difference in performance when you consider two factors acting together" [4].

• Known, in Excel software, as two factors without replication ANOVA function.

4. Two-Way Repeated Measures

• Uses the repeated measures structure of the “One-way repeated measures” method while includes the interaction effect of the “Two-way between groups” method.

• Known in Excel software as two factors with replication ANOVA function (Table II).

TABLE II  
ANOVA: TWO-FACTOR WITH REPLICATION

Source of Variation	SS	DF	MS	F
Operator	SSO	k-1	SSO / k-1	MSO / MSO*P
Part	SSP	n-1	SSP / n-1	MSP / MSO*P
Operator-Part	SSO*P	(k-1)*(n-1)	SSO*P / (k-1)*(n-1)	MSO*P / MSE
Equipment	SSE	n*k*(r-1)	SSE / n*k*(r-1)	-
Total	SS	n*k*r-1	-	-

Regarding to the ANOVA tables (Table I, II, and III), the below definitions could be presented:

TABLE III  
ANOVA TABLE

Source of Variation	SS	df	MS	F
Between	$SS_{between} = \sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2$	j-1	$MS_{between} = \frac{SS_{between}}{j-1}$	$\frac{MS_{between}}{MS_{within}}$
Within	$SS_{within} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2$	n-j	$MS_{within} = \frac{SS_{within}}{n-j}$	
Total	$SS_{total} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2$	n-1		

A. Sum of Squares (SS)

This is the measurement of the variation around the mean. SS<sub>O</sub>: Variation around operators’ average (Between Groups)

SS<sub>P</sub>: Variation around parts’ average (Between Groups)

SS<sub>O\*P</sub>: Variation around operators and parts’ average (Within Groups)

SS<sub>E</sub>: Variation around measurement instrument

B. Degree of Freedom (DF)

This is the factor that considers number of groups and adjusts how large the groups could be.

C. Mean Square (MS)

This is like a standard deviation. Its numerator is the sum of squared deviations (SS), divided by the appropriate number of degrees of freedom.

MS<sub>O</sub>: Operator discordance

MS<sub>P</sub>: Part discordance

MS<sub>O\*P</sub>: Discordance between parts and operators

MS<sub>E</sub>: Measurement instrument discordance

D. F-Statistic or F-Ratio

This factor shows proportion of “between-group variation” compared to “within-group variation”.

In general, the larger this value is, the more significant “between-group variation” would be.

The level of significance for the samples is determined by comparing it to the F-Critical value. If the F-Statistic is larger than F-Critical, then the variation between the groups is statistically significant.

E. P-Value

The most important part in the ANOVA table is the P-Value amount which is shown by P. Its related factor would be more important and significant when this value is lower. Whether P>0.5 or in other words α=0.05, the outcome demonstrates a significant differentia between analysis results.

$$F_{Value} > F_{Critical-95\%-Confidence\ Level\ Between\ the\ 2\ degree\ of\ freedom}$$

Analyze of variance is a statistical method that often used in Design of Errors (DOE) to analyze variable data from multiple groups in order to compare averages and these variable sources.

This method’s functions can be described as:

- Detailed identification of experiment variation which leads to know changes in each factor adjustment
  - Comparison of effects on a factor, with basic variations in order to find the factor's validity probability
  - Not only gaining the prediction of average result value in a successful performance, but also to find a rang for the results
- Furthermore, analyzing the variation sources and percentage of each from total variation could be taken into account as an ANOVA function.

As it was mentioned, there are different techniques of ANOVA in software and for instance Excel spread sheet – containing techniques: ANOVA Single-Factor, ANOVA Two-Factor with Replication, and ANOVA Two-Factor without Replication – would be explained.

**A. ANOVA Single-Factor**

This scheme contribute to the appliance of one way analyses in which the average of an ideal variable will be compared to two or more independent samples (this technique won't contain post hoc experiences).

**B. ANOVA Two-Factor with Replication**

This topic covers simple repetitive measurement in ANOVA. A factor represents independents samples while another factor describes repetitive measurement. This technique compares average of an ideal variable from independent samples, average of repetitive factor, and experiments for an effect between these two factors.

**C. ANOVA Two-Factor without Replication**

This method implements two ways ANOVA that compare average of an ideal from independent samples which are clustered based on two factors.

To do the R&R index calculations by ANOVA technique several software may use that as the most relevant Minitab could be named. Regarding to the high calculative capabilities of Excel spread sheet, some notes to this topic are discussed. As it was mentioned before, in order to do the ANOVA calculations in Excel, three ways are offered: ANOVA Single-Factor, ANOVA Two-Factor with Replication, and ANOVA Two-Factor without Replication. For solving R&R problems two methods of ANOVA Two-Factor with Replication and ANOVA Two-Factor without Replication simply can be used which usually presents by ANOVA Two Way. Two questions asked in this regards are:

1. If there is any meaningful effect between those two factors?
2. Would the measurement result change, if any changes in each of the factors occur?

In Fig. 4, two schema of using ANOVA in Excel are shown as an example [5].

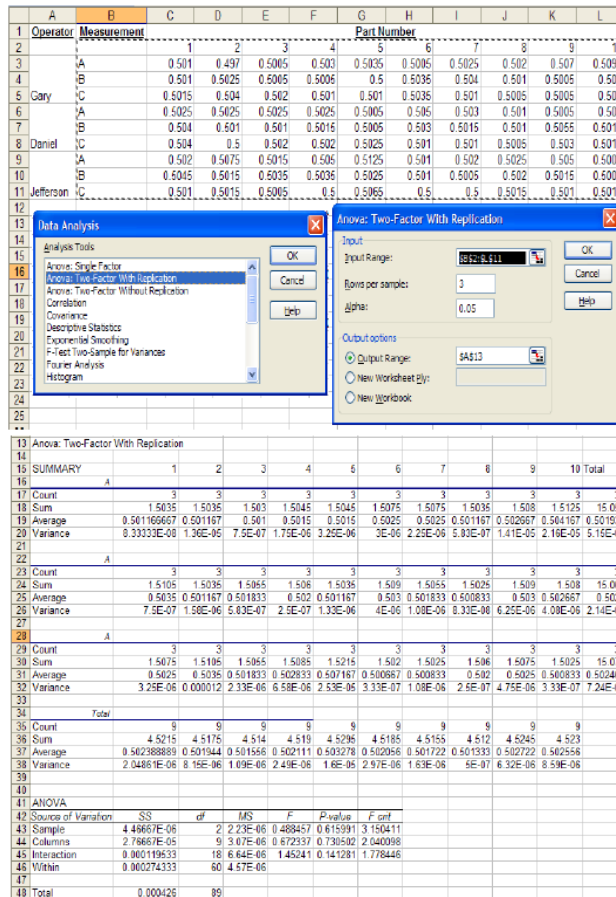


Fig. 4 ANOVA in Excel [4]

To summarize this title, it could be said that ANOVA is a useful and powerful tool for determining if differences are statistically significant. It can also be used to establish cause and affect relationships with a specific degree of certainty. Excel has three ANOVA functions that can be used for basic analysis of variance and especially profitable result in measurement system's errors.

**V. CONCLUSION**

The efficiency of a measurement system is with exact relation to the accuracy of measurement instruments. Usual measurement tools such as calipers and micrometers are the ones that could cause the most anxiety with their wrong usage. The reason is that equipment and processes measurements have to be controlled appropriately to ensure gaining the proper data. Thus, using measurement system analysis (MSA) and its evaluation would be a great support for assurance of activities accuracy. So that measurement analysis could be named as a tracing way for the important reasons of process variability.

To emphasis on the Measurement System Analysis (MSA) necessity, a reference to the statement of Dr. Walter A. Shewhart will be proposed in here [7]:

“An element can affect the measurement process

accidentally. So, it could be possible that measured values contain unknown elements. Therefore, when it seems that what has been measured is stable, these process measurements are certainly under the effect of unknown factors.”

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