

Delay-Dependent Stability Analysis for Neutral Type Neural Networks with Uncertain Parameters and Time-Varying Delay

Qingqing Wang, Shouming Zhong

Abstract—In this paper, delay-dependent stability analysis for neutral type neural networks with uncertain parameters and time-varying delay is studied. By constructing new Lyapunov-Krasovskii functional and dividing the delay interval into multiple segments, a novel sufficient condition is established to guarantee the globally asymptotically stability of the considered system. Finally, a numerical example is provided to illustrate the usefulness of the proposed main results.

Keywords—Neutral type neural networks, Time-varying delay, Stability, Linear matrix inequality(LMI).

I. INTRODUCTION

NEUTRAL type neural networks have been extensively studied in recent years due to its wide application in contemporary society of science and technology such as image processing, automatic control, pattern recognition, and so on [1,2]. Among various behaviors, the stability has proven to be the most important one that has received considerable research attention, see, for example, [3-20] and references cited therein. On the other hand, systems with uncertain parameters have been attracting increasing research attention [20-22]. In [21], the problem of robust stability criteria for recurrent neural networks with time-varying delays are investigated based on linear matrix inequality (LMI) approach. In addition, the authors in [22] discuss the problem of robust stability for Hopfield neural networks of neutral-type via constructing a new Lyapunov-Krasovskii functional. At recent times, the authors in [23-26] have provided a less conservative stability condition for delayed systems by using delay partitioning approach. The advantage of this approach is to get more tighter upper bound of the terms calculated by time-derivative of Lyapunov functional.

Motivated by the above discussion, in this paper, the stability analysis for neutral-type neural networks with uncertain parameters and time-varying delay is considered. Some novel delay-dependent stability criteria based on LMI for neutral-type neural networks will be proposed by partitioning the delay interval into multiple segments, and constructing new Lyapunov-Krasovskii functional. The obtained criteria are less conservatism which can be easily

Qingqing Wang and Shouming Zhong are with the School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China.

Shouming Zhong is with Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China.

(E-mail address: wangqqchenbc@163.com).

checked by using the MATLAB LMI Toolbox. Finally, in order to show the feasibility of the proposed criteria in this paper, a numerical example is considered.

II. PROBLEM STATEMENT

Consider the following neutral-type neural networks with time varying delays described by

$$\dot{y}(t) = -Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t))) + E\dot{y}(t-d) + I_0 \quad (1)$$

where $y(t) = [y_1(t), y_2(t), \dots, y_n(t)]^T \in R^n$ is the neuron state vector, $g(y(t)) = [g_1(y_1(t)), g_2(y_2(t)), \dots, g_n(y_n(t))]^T$ denotes the neuron activation function, and $I_0 = [I_1, I_2, \dots, I_n]^T \in R^n$ is a constant input vector, $A = \text{diag}\{a_i\} \in R^n$ is a positive diagonal matrix, $B = (b_{ij})_{n \times n} \in R^n$ is the connection weight matrix, $C = (c_{ij})_{n \times n} \in R^n$, and $E = (e_{ij})_{n \times n} \in R^n$ are the delayed connection weight matrices, d is the constant neutral time delay.

The following assumptions are adopted throughout the paper. **Assumption 1:** The delay $\tau(t)$ is time-varying continuous function and satisfies:

$$0 \leq \tau(t) \leq \tau, \dot{\tau}(t) \leq \mu \leq 1 \quad (2)$$

Assumption 2: Each neuron activation function $g_i(\cdot)$, $i = 1, 2, \dots, n$, in (1) satisfies the following condition:

$$l_i^- \leq \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \leq l_i^+, \forall \alpha, \beta \in R, \alpha \neq \beta \quad (3)$$

where l_i^-, l_i^+ , $i = 1, 2, \dots, n$ are constants.

Based on Assumption 1-2, it can be easily proven that there exists one equilibrium point for (1) by Brouwer's fixed-point theorem. Assuming that $y^* = [y_1^*, y_2^*, \dots, y_n^*]^T$ is the equilibrium point of (1) and using the transformation $x(\cdot) = y(\cdot) - y^*$, system (1) can be converted to the following system:

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf(x(t-\tau(t))) + E\dot{x}(t-d) \quad (4)$$

where

$$x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T, f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t))]^T,$$

$$f_i(x_i(\cdot)) = g_i(y_i(\cdot) + y_i^*) - g_i(y_i^*), i = 1, 2, \dots, n.$$

From Eq.(3), $f_i(\cdot)$ satisfies the following condition:

$$l_i^- \leq \frac{f_i(\alpha)}{\alpha} \leq l_i^+, \forall \alpha \neq 0, i = 1, 2, \dots, n. \quad (5)$$

Due to the disturbance frequent occurs in many applications,so by translating A, B, C and E to function $A(t), B(t), C(t)$ and $E(t)$ respectively, we have

$$\dot{x}(t) = -A(t)x(t) + B(t)f(x(t)) + C(t)f(x(t - \tau(t))) + E(t)\dot{x}(t - d) \quad (6)$$

Assumption 3: Assumption that $A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), C(t) = C + \Delta C(t),$ and $E(t) = E + \Delta E(t)$ $\Delta A(t), \Delta B(t), \Delta C(t), \Delta E(t)$ are unknown constant matrices representing time-varying parametric uncertainties, and are of linear fractional forms:

$$[\Delta A(t), \Delta B(t), \Delta C(t), \Delta E(t)] = DF(t)[\tilde{E}_a, \tilde{E}_b, \tilde{E}_c, \tilde{E}_e] \quad (7)$$

with

$$F^T(t)F(t) \leq I \quad (8)$$

where $D, \tilde{E}_a, \tilde{E}_b, \tilde{E}_c, \tilde{E}_e$ are known constant matrices of appropriate dimensions.

Lemma 1 [27]. For any constant positive-definite matrix $M \in R^{n \times n}$ and $h_1 \leq h_2$, the following inequalities hold:

$$(h_2 - h_1) \int_{h_1}^{h_2} \dot{x}^T(s)M\dot{x}(s)ds \geq \left(\int_{h_1}^{h_2} \dot{x}(s)ds \right)^T M \left(\int_{h_1}^{h_2} \dot{x}(s)ds \right) \quad (9)$$

Lemma 2 (Schur complement [28]). For any constant matrix $H_1, H_2, H_3,$ where $H_1 = H_1^T$ and $H_2 = H_2^T > 0.$ Then $H_1 + H_3^T H_2^{-1} H_3 < 0$ if and only if $\begin{bmatrix} H_1 & H_3^T \\ H_3 & -H_2 \end{bmatrix} < 0$ or $\begin{bmatrix} -H_2 & H_3 \\ H_3^T & H_1 \end{bmatrix} < 0.$

Lemma 3 [29]. Given symmetric matrices Ω and $D_1, D_2,$ of appropriate dimensions, $\Omega + D_1 F(t) D_2 + D_2^T F^T(t) D_1^T < 0$ for all $F(t)$ satisfying $F^T(t)F(t) \leq I,$ if and only if there exists some $\varepsilon > 0$ such that $\Omega + \varepsilon^{-1} D_1 D_1^T + \varepsilon D_2^T D_2 < 0.$

III. MAIN RESULTS

In this section, a less conservative delay-dependent stability criterion is obtained on the condition of $\Delta A(t) = \Delta B(t) = \Delta C(t) = \Delta E(t) = 0$ in system (6). For representation convenience, the following notations are introduced:

$$\begin{aligned} L_1 &= \text{diag}\{l_1^-, l_2^-, \dots, l_n^-\} \\ L_2 &= \text{diag}\{l_1^+, l_2^+, \dots, l_n^+\} \\ L_3 &= \text{diag}\{l_1^- l_1^+, l_2^- l_2^+, \dots, l_n^- l_n^+\} \\ L_4 &= \text{diag}\left\{\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, \dots, \frac{l_n^- + l_n^+}{2}\right\} \end{aligned}$$

Theorem 1 Given that the Assumption 1-2 hold, the system (6) globally asymptotically stable if there exist symmetric positive definite matrices $P, Q_1, Q_2, Q_3, Q_4, R_1, R_2, R_3,$ $\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}$ positive diagonal matrices $W = \text{diag}\{w_i\}, \Lambda = \text{diag}\{\delta_i\},$

K_1, K_2 with appropriate dimensions, such that the following LMIs holds:

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & \Omega_{13} & R_2 & R_3 & \Omega_{16} & \Omega_{17} & \Omega_{18} & -A^T R \\ * & \Omega_{22} & 0 & 0 & 0 & 0 & K_2 L_4 & 0 & 0 \\ * & * & \Omega_{33} & -G_{12} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & B^T R \\ * & * & * & * & * & * & \Omega_{77} & 0 & C^T R \\ * & * & * & * & * & * & * & -Q_4 & E^T R \\ * & * & * & * & * & * & * & * & -R \end{bmatrix} < 0 \quad (10)$$

where

$$\begin{aligned} \Omega_{11} &= -PA - AP - 2(L_2 \Lambda - L_1 W)A + Q_1 + Q_3 + G_{11} \\ &\quad - R_1 - R_2 - R_3 - K_1 L_3 \end{aligned}$$

$$\Omega_{13} = G_{12} + R_1$$

$$\Omega_{16} = PB - A(W - \Lambda) + (L_2 \Lambda - L_1 W)B + K_1 L_4$$

$$\Omega_{17} = PC + (L_2 \Lambda - L_1 W)C$$

$$\Omega_{18} = PE + (L_2 \Lambda - L_1 W)E$$

$$\Omega_{22} = -(1 - \mu)Q_1 - K_2 L_3$$

$$\Omega_{33} = G_{22} - G_{11} - R_1$$

$$\Omega_{44} = -G_{22} - R_2, \Omega_{55} = -Q_3 - R_3$$

$$\Omega_{66} = 2(W - \Lambda)B + Q_2 - K_1, \Omega_{67} = (W - \Lambda)C$$

$$\Omega_{68} = (W - \Lambda)E, \Omega_{77} = -(1 - \mu)Q_2 - K_2,$$

$$R = \frac{\tau^2}{9} R_1 + \frac{4\tau^2}{9} R_2 + dR_3 + Q_4$$

Proof: Construct a new class of Lyapunov functional candidate as follow:

$$V(x_t) = \sum_{i=1}^4 V_i(x_t)$$

with

$$\begin{aligned} V_1(x_t) &= x^T(t)Px(t) + 2 \sum_{i=1}^n w_i \int_0^{x_i(t)} (f_i(s) - l_i^- s) ds \\ &\quad + 2 \sum_{i=1}^n \delta_i \int_0^{x_i(t)} (l_i^+ s - f_i(s)) ds \end{aligned}$$

$$\begin{aligned} V_2(x_t) &= \int_{t-\tau(t)}^t x^T(s)Q_1 x(s)ds + \int_{t-\tau(t)}^t f^T(x(s))Q_2 f(x(s))ds \\ &\quad + \int_{t-d}^t x^T(s)Q_3 x(s)ds + \int_{t-d}^t \dot{x}^T(s)Q_4 \dot{x}(s)ds \end{aligned}$$

$$V_3(x_t) = \int_{t-\frac{\tau}{3}}^t \begin{bmatrix} x(s) \\ x(s-\frac{\tau}{3}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-\frac{\tau}{3}) \end{bmatrix} ds$$

$$V_4(x_t) = \frac{\tau}{3} \int_{-\frac{\tau}{3}}^0 \int_{t+\theta}^t \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta + d \int_{-d}^0 \int_{t+\theta}^t \dot{x}^T(s) R_3 \dot{x}(s) ds d\theta + \frac{2\tau}{3} \int_{-\frac{2\tau}{3}}^0 \int_{t+\theta}^t \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta$$

Then, taking the time derivative of $V(x_t)$ with respect to t along the system (6) yield

$$\dot{V}(x_t) = \sum_{i=1}^4 \dot{V}_i(x_t)$$

where

$$\dot{V}_1(x_t) = 2x^T(t)P\dot{x}(t) + 2(f(x(t)) - L_1x(t))^T W\dot{x}(t) + (L_2x(t) - f(x(t)))^T \Lambda\dot{x}(t) \tag{11}$$

$$\dot{V}_2(x_t) = x^T(t)(Q_1 + Q_3)x(t) - x^T(t-d)Q_3x(t-d) - (1-\mu)x^T(t-\tau(t))Q_1x(t-\tau(t)) + f^T(x(t))Q_2f(x(t)) - (1-\mu)f^T(x(t-\tau(t)))Q_2f(x(t-\tau(t))) + \dot{x}^T(t)Q_4\dot{x}(t) - \dot{x}^T(t-d)Q_4\dot{x}(t-d)$$

$$\dot{V}_3(x_t) = \begin{bmatrix} x(t) \\ x(t-\frac{\tau}{3}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\frac{\tau}{3}) \end{bmatrix} - \begin{bmatrix} x(t-\frac{\tau}{3}) \\ x(t-\frac{2\tau}{3}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} x(t-\frac{\tau}{3}) \\ x(t-\frac{2\tau}{3}) \end{bmatrix} \tag{13}$$

$$\dot{V}_4(x_t) = \dot{x}^T(t)R\dot{x}(t) - \frac{\tau}{3} \int_{t-\frac{\tau}{3}}^t \dot{x}^T(s)R_1\dot{x}(s)ds - \frac{2\tau}{3} \int_{t-\frac{2\tau}{3}}^t \dot{x}^T(s)R_2\dot{x}(s)ds - d \int_{t-d}^t \dot{x}^T(s)R_3\dot{x}(s)ds \tag{14}$$

Using Lemma 1, we can obtain that

$$\begin{aligned} & -\frac{\tau}{3} \int_{t-\frac{\tau}{3}}^t \dot{x}^T(s)R_1\dot{x}(s)ds \\ & \leq -[x(t) - x(t-\frac{\tau}{3})]^T R_1 [x(t) - x(t-\frac{\tau}{3})] \\ & -\frac{2\tau}{3} \int_{t-\frac{2\tau}{3}}^t \dot{x}^T(s)R_2\dot{x}(s)ds \\ & \leq -[x(t) - x(t-\frac{2\tau}{3})]^T R_2 [x(t) - x(t-\frac{2\tau}{3})] \\ & -d \int_{t-d}^t \dot{x}^T(s)R_3\dot{x}(s)ds \\ & \leq -[x(t) - x(t-d)]^T R_3 [x(t) - x(t-d)] \end{aligned} \tag{15}$$

For positive diagonal matrices $K_i, i = 1, 2$, we can get from (5) that

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -K_1L_3 & K_1L_4 \\ * & -K_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \geq 0 \tag{18}$$

$$\begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} -K_2L_3 & K_2L_4 \\ * & -K_2 \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \geq 0 \tag{19}$$

From (11)-(19), we can obtain that:

$$\dot{V}(x_t) \leq \xi^T(t)\Pi\xi(t) \tag{20}$$

where

$$\xi^T(t) = [x^T(t), x^T(t-\tau(t)), x^T(t-\frac{\tau}{3}), x^T(t-\frac{2\tau}{3}), x^T(t-d), f^T(x(t)), f^T(x(t-\tau(t))), \dot{x}^T(t-d)]$$

$$\Pi = \begin{bmatrix} \pi_{11} & 0 & \pi_{13} & R_2 & R_3 & \pi_{16} & \pi_{17} & \pi_{18} \\ * & \pi_{22} & 0 & 0 & 0 & 0 & K_2L_4 & 0 \\ * & * & \pi_{33} & -G_{12} & 0 & 0 & 0 & 0 \\ * & * & * & \pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \pi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \pi_{66} & \pi_{67} & \pi_{68} \\ * & * & * & * & * & * & \pi_{77} & 0 \\ * & * & * & * & * & * & * & \pi_{88} \end{bmatrix}$$

$$\pi_{11} = -PA - AP - 2(L_2\Lambda - L_1W)A + Q_1 + Q_3 + G_{11} - R_1 - R_2 - R_3 - K_1L_3 + A^T RA$$

$$\pi_{13} = G_{12} + R_1$$

$$\pi_{16} = PB - A(W - \Lambda) + (L_2\Lambda - L_1W)B + K_1L_4 - A^T RB$$

$$\pi_{17} = PC + (L_2\Lambda - L_1W)C - A^T RC$$

$$\pi_{18} = PE + (L_2\Lambda - L_1W)E - A^T RE$$

$$\pi_{22} = -(1-\mu)Q_1 - K_2L_3$$

$$\pi_{33} = G_{22} - G_{11} - R_1$$

$$\pi_{44} = -G_{22} - R_2, \pi_{55} = -Q_3 - R_3$$

$$\pi_{66} = 2(W - \Lambda)B + Q_2 - K_1 + B^T RB$$

$$\pi_{67} = (W - \Lambda)C + B^T RC$$

$$\pi_{68} = (W - \Lambda)E + B^T RE$$

$$\pi_{77} = -(1-\mu)Q_2 - K_2 + C^T RE,$$

$$\pi_{88} = -Q_4 + E^T RE$$

Using Lemma 2, we can obtain that $\dot{V}(x_t) < 0$ on the condition of (10), therefore, the system (6) is asymptotically stable. This completes the proof. ■

Remark 1 Firstly, in this paper, dividing the delay interval $[0, \tau]$ into three different ones $[0, \frac{\tau}{3}]$, $[\frac{\tau}{3}, \frac{2\tau}{3}]$, $[\frac{2\tau}{3}, \tau]$; Secondly, constructing new Lyapunov functional which contains some new integral terms. It has potential to yield less conservative results.

Based on Theorem 1, we have the following result for uncertain neutral-type neural networks with time-varying delay.

Theorem 2 Given that the Assumption 1-3 hold, the system (6) globally asymptotically stable if there exist symmetric positive definite matrices $P, Q_1, Q_2, Q_3, Q_4, \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}$, R_1, R_2, R_3 , positive diagonal matrices $W = \text{diag}\{w_i\}$, $K_1, K_2, \Lambda = \text{diag}\{\delta_i\}$ with appropriate dimensions, and a scalar $\varepsilon > 0$ such that the following LMIs holds:

$$\begin{bmatrix} \Omega & \Psi & \varepsilon \Phi^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \quad (21)$$

where

$$\Psi = [PD + (L_2\Lambda - L_1W)D, 0_{n \times 4n}, (W - \Lambda)D, 0_{n \times 2n}, D^T R]$$

$$\Phi = [-\tilde{E}_a, 0_{n \times 4n}, \tilde{E}_b, \tilde{E}_c, \tilde{E}_e, 0_n]$$

Proof: Replacing A, B, C, E in (10) with $A + DF(t)\tilde{E}_a, B + DF(t)\tilde{E}_b, C + DF(t)\tilde{E}_c, E + DF(t)\tilde{E}_e$, respectively, (10) is equivalent to the following condition:

$$\Omega + \Psi F(t)\Phi + \Phi^T F^T(t)\Psi^T < 0 \quad (22)$$

According to Lemma 3, one can obtain (22) is equivalent to the following inequalities on the condition of $F^T(t)F(t) \leq I$:

$$\Omega + \varepsilon^{-1}\Psi\Psi^T + \varepsilon\Phi^T\Phi < 0 \quad (23)$$

Using Lemma 2, we know that (23) is equivalent to (21). This completes the proof. ■

Remark 2 In this paper, Theorem 1 and Theorem 2 require the upper bound μ of the time-varying delay $\tau(t)$ to be known. However, in many cases μ is unknown, considering this situation, we can set $Q_1 = Q_2 = 0$ in $V(x_t)$, and employ the similar methods in Theorem 1 and Theorem 2, we can derive the delay-derivative-independent and delay-dependent stability criteria.

IV. EXAMPLE

Example 1 Consider delayed neutral-type neural networks (6) with the following parameters:

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.7 \end{bmatrix}, B = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, C = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix}$$

$$E = \begin{bmatrix} 1.2135 & 0 \\ -0.3412 & 0.2257 \end{bmatrix}, \tilde{E}_a = \tilde{E}_b = \tilde{E}_c = \tilde{E}_e = I$$

The neuron activation functions are assumed to satisfy Assumption 2 with $L_1 = \text{diag}\{0, 0\}$, $L_2 = \text{diag}\{0.3, 0.8\}$.

TABLE I
ADMISSIBLE UPPER BOUND τ FOR DIFFERENT d AND μ .

Method	Theorem 1
$d = 0.1, \mu = 0.2$	3.4055
$d = 0.3, \mu = 0.4$	4.0657
$d = 0.5, \mu = 0.6$	4.3121
$d = 0.7, \mu = 0.8$	4.5602

In this example, by applying Theorem 1 and solving the LMI (10) using MATLAB LMI Control Toolbox, we can obtain the maximum allowable upper bounds of delay for various of d and μ . From Table I, it can be seen that our results show significant improvements and less conservative.

V. CONCLUSION

In this paper, the problem of stability analysis for delayed neutral-type neural networks with uncertain parameters has been investigated. By choosing new Lyapunov-Krasovskii functional, dividing the delay interval into multiple segments, and combining linear matrix inequalities (LMI) techniques, two new sufficient criteria ensuring the global stability asymptotic stability of delayed neutral-type neural networks is obtained. Finally, one example is given to show the effectiveness of our obtained criteria.

ACKNOWLEDGMENT

The authors would like to thank the editors and the reviewers for their valuable suggestions and comments which have led to a much improved paper. This work was supported by the National Basic Research Program of China (2010CB32501).

REFERENCES

- [1] T.L. Liao, F.C. Wang, Global stability for cellular neural networks with time delay, IEEE Trans, Neural Networks 11(6)(2000)1481-1484.
- [2] S.Arik, Global asymptotic stability of a larger class of neural networks with constant time delay, Phys. Lett. A 311(2002)504-511.
- [3] L.Cheng, Z.G.Hou, M.Tan, A neutral-type delayed projection neural network for solving nonlinear variational inequalities, IEEE Trans. Circuits Syst.H: Express Briefs 55(2008)06-810.
- [4] Ju.H.Park, O.M.Kwon, Synchronization of neural networks of neutral-type with stochastic perturbation, Mod.Phys.Lett. B 23(2009)1743-1751.
- [5] Ju.H.Park, O.M.Kwon, S.M.Lee, LMI optimization approach on stability for delayed neural networks of neutral-type, Appl.Math.Comput. 196(2008)236-244.
- [6] J.D.Chen, New stability criteria for a class of neutral systems with discrete and distributed time-delays: an LMI approach, Appl. Math. Comput,150(2004)719-736.
- [7] Tian J K,Zhong S M.Improved delay-dependent stability criterion for neural networks with time-varying delay.Applied Mathematics and Computation 2011;217:10278-88.
- [8] P.L.Liu,Improved delay-dependent robust stability criteria for recurrent neural networks with time-varying delays,ISA Transactions,52 (2013)30-35.
- [9] X. Liu, Y. Wang, Delay-dependent exponential stability for neural networks with time-varying delays, Phys. Lett. A 373(2009) 4066-4072.
- [10] P.G. Park, J.W. Ko, C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, Automatica 47(2011) 235-238.
- [11] X. Liu, C. Dang, Stability analysis of positive switched linear systems with delays, IEEE Trans. Autom. Control 56(2011) 1684-1690.
- [12] O.M. Kwon, J.H. Park, Delay-dependent stability for uncertain cellular neural networks with discrete and distributed time-varying delays,J.Franklin Inst. 345(2008) 766-778.

- [13] Z.G. Wu, J.H. Park, H.Y. Su, J. Chu, New results on exponential passivity of neural networks with time-varying delays, *Nonlinear Anal. Real World Appl.* 13(2012) 1593-1599.
- [14] S.M. Lee, O.M. Kwon, J.H. Park, A novel delay-dependent criterion for delayed neural networks of neutral type, *Phys. Lett. A* 374(2010) 1843-1848.
- [15] J.H. Park, O.M. Kwon, Synchronization of neural networks of neutral type with stochastic perturbation, *Mod. Phys. Lett. B* 23(2009) 1743-1751.
- [16] Y.K. Li, X.L. Fan, Existence and globally exponential stability of almost periodic solution for Cohen-Grossberg BAM neural networks with variable coefficients, *Applied Mathematical Modelling* 33(2009)2114C2120.
- [17] C. Lien, L. Chung, Global asymptotic stability for cellular neural networks with discrete and distributed time-varying delays, *Chaos Solitons Fract* 34(2007) 1213-1219.
- [18] Z.G. Wu, J.H. Park, H. Su, J. Chu, Dissipativity analysis for singular systems with time-varying delays, *Appl. Math. Comput.* 218(2011) 4605-4613.
- [19] T. Li, L. Guo, C. Sun, C. Lin, Further result on delay-dependent stability criteria of neural networks with time-varying delays, *IEEE Trans. Neural Networks* 19(2008) 726-730.
- [20] Liu P.L. Robust exponential stability for uncertain time-varying delay systems with delay dependence. *Journal of The Franklin Institute* 2009;346(10):958-968.
- [21] P.L. Liu, Improve delay-dependent robust stability criteria for recurrent neural networks with time-varying delays. *ISA Transactions* 52(2013)30-35.
- [22] W. Qian, T. Li, S. Cong, S. M. Fei, Improved stability analysis on delayed neural networks with linear fractional uncertainties. *Applied Mathematics and Computation* 217(2010)3596-3606.
- [23] C. Lin, Q. G. Wang, T. H. Lee, A less conservative robust stability test for linear uncertain time-delay systems, *IEEE Trans. Automat. Control* 51(2006)87-91.
- [24] K. Gu, V. L. Kharitonov, J. Chen, *Stability of Time-Delay System*, Birkhauser, Boston, 2003.
- [25] Liu P.L. Robust exponential stability for uncertain time-varying delay systems with delay dependence. *Journal of The Franklin Institute* 2009;346(10):958-968.
- [26] Z. Wang, H. Shu, H. R. Liu, Robust stability analysis of generalized neural networks with discrete and distributed time delays, *Chaos Soliton. Fract.* 30(2006)886-896.
- [27] R. Song, Z. Wang, Neural networks with discrete and discrete and distributed time-varying delays: A general stability analysis, *Chaos Soliton. Fract.* 37(2008)1538-1547.
- [28] S. Boyd, L. E. Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.
- [29] Boyd S, Ghaoui LE, Feron E, Balakrishnan V. *Linear matrix inequalities in system and control theory*. Society and Applied Mathematics PA: Philadelphia 1994.

Qingqing Wang was born in Anhui Province, China, in 1989. She received the B.S. degree from Anqing University in 2012. She is currently pursuing the M.S. degree from University of Electronic Science and Technology of China. Her research interests include neural networks, switch and delay dynamic systems.

Shouming Zhong was born in 1955 in Sichuan, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yatsen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo, Waterloo, Canada. He is currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations, neural networks, biomathematics and robust control. He has authored more than 80 papers in reputed journals such as the *International Journal of Systems Science*, *Applied Mathematics and Computation*, *Chaos, Solitons and Fractals*, *Dynamics of Continuous, Discrete and Impulsive Systems*, *Acta Automatica Sinica*, *Journal of Control Theory and Applications*, *Acta Electronica Sinica*, *Control and Decision*, and *Journal of Engineering Mathematics*.