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Decision Making using Maximization of Negret

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***Abstract*—**We analyze the problem of decision making under ignorance with regrets. Recently, Yager has developed a new method for decision making where instead of using regrets he uses another type of transformation called negrets. Basically, the negret is considered as the dual of the regret. We study this problem in detail and we suggest the use of geometric aggregation operators in this method. For doing this, we develop a different method for constructing the negret matrix where all the values are positive. The main result obtained is that now the model is able to deal with negative numbers because of the transformation done in the negret matrix. We further extent these results to another model developed also by Yager about mixing valuations and negrets. Unfortunately, in this case we are not able to deal with negative numbers because the valuations can be either positive or negative.

***Keywords*—**Decision Making, Aggregation operators, Negret, OWA operator, OWG operator.

I. Introduction

IN the literature, we find a wide range of aggregation operators for fusing the information such as the ordered weighted averaging (OWA) operator and the ordered weighted geometric (OWG) operator. The OWA operator was introduced by Yager [1] and it provides a parameterized family of aggregation operators that includes the maximum, the minimum and the average, among others. The OWG operator is a geometric version of the OWA operator introduced in [2] and it also provides a parameterized family of aggregation operators. For further reading on the OWA or the OWG operator, see for example [3] – [24].

In [25], [26], Savage introduced the concept of decision making with minimization of regret. It consists in a decision process where the payoffs are transformed in regret values that express the regret against the optimal choice for each state of nature. Recently, Yager [20] has suggested a different method for dealing with regrets. He develops a process that uses the dual of the regret. He refers to these values as the negret against the optimal choice. Then, by using the OWA operator, this method provides a parameterized family of negret aggregation operators. Moreover, this method can also be mixed with the usual valuation methods because in both cases the optimal choice is the one with the highest value.

In this paper, we suggest a new method for decision making under ignorance with negrets. We propose the use of geometric aggregation operators in decision making with maximization of negret. For doing this, we will develop a new procedure for constructing the negret matrix where we will transform all the negret values in positive numbers. Then, we will be able to use the OWG operator because it can only deal with positive numbers. Furthermore, we will apply this new approach in Yagers model [20] about mixing valuation and regret methods. Unfortunately, in this case, we are not able to deal with negative numbers when using the OWG operator because the usual valuations can be either positive or negative. It is also interesting to note that other transformations could be developed in the negret matrix. Among them, one possible construction could be the construction used in the Analytic Hierarchy Process (AHP) [27]. The problem found in this particular construction is that it cannot deal with negative numbers when using geometric aggregation operators because the results become inconsistent. Therefore, in this paper we prefer to focus on a method that is able to deal with negative numbers.

In order to do so, the remainder of the paper is organized as follows. In Section II, we briefly comment some basic aggregation operators to be used throughout the paper. In Section III, we analyze the decision making problem with maximization of negret. In Section IV, we study a more general model about mixing valuation and regret methods. Finally, in Section V, we give an illustrative example where we can see the different results obtained by using the new approaches suggested in the paper.

II. Preliminaries

A. OWA Operator

The OWA operator was introduced in [1] and it provides a parameterized family of aggregation operators which have been used in a wide range of applications [9] – [22]. In the following, we provide a definition of the OWA operator as introduced by Yager [1].

Definition 1: An OWA operator of dimension n is a mapping $OWA{:}R^n \rightarrow R$ that has an associated weighting vector W of dimension n having the properties:

1) $w_j \in [0, 1]$

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2) $\sum_{j=1}^{n} w_j = 1$

and such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \quad (1)$$

where b_j is the jth largest of the a_i.

From a generalized perspective of the reordering step, we have to distinguish between the Descending OWA (DOWA) operator and the Ascending OWA (AOWA) operator [14]. The OWA operator is a mean or averaging operator. This is a reflection of the fact that the operator is commutative, monotone, bounded and idempotent. It can also be demonstrated that the OWA operator has as special cases the maximum, the minimum and the average criteria among others [1], [9], [11], [15] – [19], [21].

B. Geometric Mean

The geometric mean is a traditional aggregation operator which has been used for different applications such as in [28], [29], for ratio-scale judgements. It is defined as follows:

Definition 2: A geometric mean operator of dimension n is a mapping GM: $R^{+n} \rightarrow R^{+}$, defined as:

$$GM(a_1, a_2, \ldots, a_n) = \prod_{i=1}^{n} (a_i)^{\frac{1}{n}} \quad (2)$$

where R^+ is the set of positive real numbers. The geometric mean is commutative, monotonic, bounded and idempotent. Note that it is also possible to consider a situation where the weights of the arguments have different degrees of importance. Then, we are using the weighted geometric mean (WGM).

C. OWG Operator

The OWG operator was introduced in [2] and it provides a family of aggregation operators similar to the OWA operator. It uses in the same aggregation the OWA operator and the geometric mean. In the following, we provide a definition of the OWG operator as introduced by [13].

Definition 3: An OWG operator of dimension n is a mapping OWG:$R^{+n} \rightarrow R^{+}$ that has an associated weighting vector W of dimension n having the properties:

1) $w_j \in [0, 1]$
2) $\sum_{j=1}^{n} w_j = 1$

and such that:

$$OWG(a_1, a_2, \ldots, a_n) = \prod_{j=1}^{n} b_j^{w_j} \quad (3)$$

where b_j is the jth largest of the a_i, and R^+ is the set of positive real numbers.

From a generalized perspective of the reordering step in the OWG operator, we have to distinguish between the Descending OWG (DOWG) operators and the Ascending OWG (AOWG) operators [13]. The weights of these operators are related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the DOWG (or OWG) operator and w^*_{n+1-j} the jth weight of the AOWG operator. Note that it accomplishes similar properties than the OWA operator [2] – [10]. For example, in this operator it is also found the maximum and the minimum as particular cases. Other families found in this aggregation are the geometric mean, the weighted geometric mean, the Hurwicz geometric criteria, etc.

III. Decision Making Using Maximization of Minimal Negret

A. Introduction

The use of maximization of minimal regret in decision making was introduced by Yager [20]. This model is similar to the minimization of regret process. The difference is that the negret process considers first the payoff c_{ij} while the regret process considers first the maximal payoff C_j for each state of nature. That is, the regret is calculated as: $C_j - c_{ij}$; while the negret as: $c_{ij} - C_j$. With this information, we can summarize the basic steps when taking decisions with the negret method as follows.

Assume we have a decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$ with states of nature $\{S_1, \ldots, S_n\}$. c_{ij} is the payoff to the decision maker if he selects alternative A_i and the state of nature is S_j. The matrix E whose components are the e_{ij}, is the negret matrix. The objective of the problem is to select the alternative which best satisfies the payoff to the decision maker. In order to do this, the following steps should be taken:

Step 1: Calculate the payoff matrix.
Step 2: Calculate $C_j = \text{Max}\{c_{ij}\}$ for each S_j.
Step 3: Calculate $e_{ij} = c_{ij} - C_j$; for each pair A_i and S_j.
Step 4: Calculate $E_i = OWA(e_{i1}, \ldots, e_{in})$ using (1), for each A_i.
Step 5: Select A_{i*} such that $E_{i*} = \text{Max}\{E_i\}$.

As we can see, once we calculate the negret matrix, we aggregate the information obtained with the OWA operator. This method suggested by Yager is a general one that includes among others the pessimistic, the optimistic and the average criteria. These particular situations are obtained by using a different manifestation in the weighting vector of Step 4. Then:

1) When $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$; we are using an optimistic aggregation operator.
2) When $w_n = 1$ and $w_j = 0$, for all $j \neq n$; we are using a pessimistic criteria.
3) When $w_j = 1/\text{n}$, for all j; we are aggregating the negret matrix with the average criteria.

Note that we will refer to this decision process as the Max-OWA-Negret procedure. Also note that other families of Max-OWA-Negret operators could be used in the aggregation of the negret matrix such as the step-OWA, the window-OWA, the olympic-OWA, the OWA median, the centered-OWA, the S-OWA, the maximal entropy OWA, etc.

B. Using the OWG Operator

The use of the OWG operator in decision making with maximization of negret is an alternative when taking decisions with negret methods. It consists in using the OWG operator in the aggregation step of the negret matrix. When using geometric operators, we need to modify the negret matrix because it cannot deal with negative numbers. This problem has also been considered for the regret matrix [10]. Then, the transformation we suggest is to sum the minimum argument in absolute numbers plus the maximum argument and plus one: $c_{ij} - C_j + |\text{ Min}\{c_{ij}\} + C_j | + 1$. With this construction in the negret matrix, we are able to aggregate with geometric aggregation operators because now, all the arguments are positive. The decision process will be the same as for the case with OWA operators with the differences commented above. We can summarize the procedure as follows:

Assume we have a decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$ with states of nature $\{S_1, \ldots, S_n\}$. c_{ij} is the payoff to the decision maker if he selects alternative A_i and the state of nature is S_j. The matrix E whose components are the e_{ij}, is the negret matrix. The objective of the problem is to select the alternative which best satisfies the payoff to the decision maker. Note that we refer to this process as the Max-OWG-Negret. In order to do this, we should follow the following steps:

Step 1: Calculate the payoff matrix.

Step 2: Calculate $C_j = \text{Max}\{c_{ij}\}$ for each S_j.

Step 3: Calculate $e_{ij} = c_{ij} - C_j + |\text{ Min}\{c_{ij}\}| + |C_j| + 1$; for each pair A_i and S_j.

Step 4: Calculate $E_i = OWG(e_{i1}, \ldots, e_{in})$ using (3), for each A_i.

Step 5: Select A_{i*} such that $E_{i*} = \text{Max}\{E_i\}$.

As we can see, the main difference in this decision procedure is that now we use geometric aggregation operators. Therefore, we need to develop a different negret matrix in order to obtain positive numbers because the OWG operator cannot aggregate negative numbers.

From a generalized perspective of the reordering step, we have to distinguish between the descending Max-OWG-Negret operator and the ascending Max-OWG-Negret operator. Note that they can be used in situations where the highest value is the best result and in situations where the lowest value is the best result. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the Max-DOWG-Negret and w^*_{n-j+1} the jth weight of the Max-AOWG-Negret operator. As we can see, the main difference is that in the Max-AOWG-Negret operator, the elements e_j (j= 1, 2, …, n) are ordered in an increasing way: $e_1 \leq e_2 \leq \ldots \leq e_n$ while in the Max-DOWG-Negret (or Max-OWG-Negret) they are ordered in a decreasing way.

Another interesting issue to consider is the properties of this generalized Max-OWG-Negret method:

1) Commutativity: any permutation of the arguments has the same evaluation.
2) Monotonicity: If $e_i \geq d_i$ for all $i \Rightarrow OWG(e_1,\ldots, e_n) \geq OWG(d_1,\ldots, d_n)$.
3) Boundedness: $\text{Min}\{e_i\} \leq OWG(e_1,\ldots, e_n) \leq \text{Max}\{e_i\}$.
4) Idempotency: If $e_i = e$, for all $i \Rightarrow OWG(e_1,\ldots, e_n) = e$.

As we can see, the generalized Max-OWG-Negret method accomplishes the same properties as the original OWG operator.

In this case, it is also included as particular cases the maximum and the minimum. The maximum is obtained when $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$; and the minimum when $w_n = 1$ and $w_j = 0$, for all $j \neq n$. The geometric mean is also a special type of aggregation operator found in this model. It appears when $w_j = 1/\text{n}$, for all j.

Other families of OWG operators could be used such as the S-OWG operator, the olympic-OWG, the E-Z OWG weights, the OWG median, the centered-OWG operator, etc. For example, if $w_1 = w_n = 0$, and for all others $w_{j*} = 1/(n-2)$, we are using the Max-olympic-OWG-Negret which has the same methodology than the OWA version [18]. Note that if $n = 3$ or $n = 4$, the Max-olympic-OWG-Negret is transformed in the Max-median-OWG-Negret and if $m = n - 2$ and $k = 2$, the Max-window-OWG-Negret is transformed in the Max-olympic-OWG-Negret.

Another interesting family is the Max-S-OWG-Negret operator which is based on [15], [17]. It can be subdivided in three classes, the "orlike", the "andlike" and the generalized Max-S-OWG-Negret. The "orlike" Max-olympic-OWG-Negret operator is found when $w_1 = (1/n)(1 - \alpha) + \alpha$, and $w_j = (1/n)(1 - \alpha)$ for $j = 2$ to n with $\alpha \in [0, 1]$. Note that if $\alpha = 0$, we get the Max-GM-Negret and if $\alpha = 1$, we get the maximum. The "andlike" Max-S-OWG-Negret operator is found when $w_n = (1/n)(1 - \beta) + \beta$ and $w_j = (1/n)(1 - \beta)$ for $j = 1$ to $n - 1$ with $\beta \in [0, 1]$. Note that in this class, if $\beta = 0$ we get the Max-GM-Negret and if $\beta = 1$, we get the minimum. Finally, the generalized Max-S-OWG-Negret operator is obtained when $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, the generalized Max-S-OWG-Negret becomes the "andlike" Max-S-OWG-Negret and if $\beta =$

0, it becomes the "orlike" Max-S-OWG-Negret operator. Also note that if $\alpha + \beta = 1$, the generalized Max-S-OWG-Negret operator becomes the Max-Hurwicz-OWG-Negret criteria.

We note that the median and the weighted median can also be used as Max-OWG-Negret operators. For the Max-median-OWG-Negret, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others. For the weighted Max-median-OWG-Negret, we select the argument b_k that has the kth largest argument such that the sum of the weights from 1 to k is equal or higher than 0.5 and the sum of the weights from 1 to $k - 1$ is less than 0.5.

A further family of aggregation operator that could be used is the Max-centered-OWG-Negret operator. Note that this type of aggregation operator is based on the OWA version developed recently by Yager [21]. We can define a Max-centered-OWG-Negret operator as a centered aggregation operator if it is symmetric, strongly decaying and inclusive. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \le (n+1)/2$ then $w_i < w_j$ and when $i > j \ge (n+1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \le w_j$ instead of $w_i < w_j$. We shall refer to this as softly decaying Max-centered-OWG-Negret operator. Note that the Max-GM-Negret is an example of this particular case. Another particular situation of the Max-centered-OWG-Negret operator appears if we remove the third condition. We shall refer to it as a non-inclusive Max-centered-OWG-Negret operator. For this situation, we find the Max-median-OWG-Negret as a particular case.

IV. Using Valuation and Negret Methods in the Same Decision Process

A. Introduction

A more general formulation for decision making was introduced by Yager in [20] where he suggested a combination between valuation and negret methods in the same decision model. This process is summarized as follows.

Assume we have a decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$ with states of nature $\{S_1, \ldots, S_n\}$. c_{ij} is the payoff to the decision maker if he selects alternative A_i and the state of nature is S_j. Let $C_j = \text{Max}\{c_{ij}\}$ for each S_j. Then:

Step 1: Let $m_{ij} = c_{ij} - \alpha C_j$ where $\alpha \in [0, 1]$.
Step 2: For each alternative A_i, find $M_i = OWA(m_{i1}, \ldots, m_{in})$.
Step 3: Select the alternative A_q such that $M_q = \text{Max}_i [M_i]$.

This process can be denoted as Max-OWA/α-Val/Neg method. As we can see, if $\alpha = 0$, we get the usual Max-OWA-Val method and if $\alpha = 1$, we get the Max-OWA-Negret method. Note that m_{ij} can be also formulated as $m_{ij} = \alpha e_{ij} + (1 - \alpha) c_{ij}$. Also note that it is possible to consider a wide range of families of Max-OWA/α-Val/Neg such as the Max-step-OWA/α-Val/Neg, the Max-window-OWA/α-Val/Neg, the Max-centered-OWA/α-Val/Neg, the Max-SOWA/α-Val/Neg, etc.

In this process we could also study different properties such as commutativity, monotonicity, boundedness and idempotency. It is commutative because any permutation of the arguments has the same evaluation. That is, $OWA(m_1, m_2, \ldots, m_n) = OWA(p_1, p_2, \ldots, p_n)$, where $(p_1, \ldots, p_n)$ is any permutation of the arguments $(m_1, \ldots, m_n)$. It is monotonic because if $m_i \ge p_i$, for all m_i, then, $OWA(m_1, m_2, \ldots, m_n) \ge OWA(p_1, p_2, \ldots, p_n)$. It is bounded because the OWA aggregation is delimitated by the minimum and the maximum. That is, $\text{Min}\{m_i\} \le OWA(m_1, m_2, \ldots, m_n) \le \text{Max}\{m_i\}$. It is idempotent because if $m_i = m$, for all m_i, then, $OWA(m_1, m_2, \ldots, m_n) = m$.

B. Mixing Valuations and Negret Methods with the OWG Operator

Now we are going to further extend the previous method when using geometric aggregation operators. The process is very similar with the difference that now we use the OWG operator in the aggregation step. The process can be summarized as follows.

Assume we have a decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$ with states of nature $\{S_1, \ldots, S_n\}$. c_{ij} is the payoff to the decision maker if he selects alternative A_i and the state of nature is S_j. Let $C_j = \text{Max}\{c_{ij}\}$ for each S_j. Then:

Step 1: Let $m_{ij}^* = c_{ij} + \alpha[|\text{Min}\{c_{ij}\}| + |\text{Max}\{c_{ij}\}| - C_j + 1]$ where $\alpha \in [0, 1]$. Note that this result is equivalent to $m_{ij}^* = m_{ij} + \alpha[|\text{Min}\{c_{ij}\}| + |\text{Max}\{c_{ij}\}| + 1]$.
Step 2: For each alternative A_i, calculate $M_i^* = OWG(m_{i1}^*, \ldots, m_{in}^*)$.
Step 3: Select the alternative A_q such that $M_q^* = \text{Max}[M_i^*]$.

This process can be denoted as Max-OWG/α-Val/Neg method. From a generalized perspective of the reordering step, we have to distinguish between the descending Max-OWG/α-Val/Neg operator and the ascending Max-OWG/α-Val/Neg operator. Note that they can be used in situations where the highest value is the best result and in situations where the lowest value is the best result. But in a more efficient context, it is better to use one of them for one situation and the other one for the dual situation. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the Max-DOWG/α-Val/Neg and w^*_{n-j+1} the jth weight of the Max-AOWG/α-Val/Neg operator.

Note that different properties could be studied in this method. It is easy to see that this method is monotonic, commutative, idempotent and bounded. It is monotonic because if $m_i^* \ge p_i^*$, for all m_i^*, then, $OWG(m_1^*, m_2^*, \ldots, m_n^*) \ge OWG(p_1^*, p_2^*, \ldots, p_n^*)$. It is commutative because any permutation of the arguments has the same evaluation. That is, $OWG(m_1^*, m_2^*, \ldots, m_n^*) = OWG(p_1^*, p_2^*, \ldots, p_n^*)$, where $(p_1^*, \ldots, p_n^*)$ is any permutation of the arguments $(m_1^*, \ldots,$

m_n*). It is idempotent because if m_i* = m*, for all m_i*, then, $OWG(m_1$*, m_2*,…, m_n*) = m*. It is bounded because the OWG aggregation is delimitated by the minimum and the maximum. That is, Min{m_i*} ≤ $OWG(m_1$*, m_2*,…, m_n*) ≤ Max{m_i*}.

As we can see, if α = 0, we get the usual Max-OWG-Val method and if α = 1, we get the Max-OWG-Negret method. Note that it is possible to consider a wide range of families of Max-OWG/α-Val/Neg such as the Max-step-OWG/α-Val/Neg, the Max-window-OWG/α-Val/Neg, the Max-centered-OWG/α-Val/Neg, the Max-SOWG/α-Val/Neg, etc.

For example, if w_k = 1 and w_j = 0, for all $j \neq k$, we get the Max-step-OWG/α-Val/Neg method. The Max-GM/α-Val/Neg method is found when $w_j = 1/n$, for all $\tilde{a}_i$.

When $w_{j*} = 1/m$ for $k \leq j* \leq k + m - 1$ and $w_{j*} = 0$ for $j* > k + m$ and $j* < k$, we are using the Max-window-OWG/α-Val/Neg operator. Note that k and m must be positive integers such that $k + m - 1 \leq n$. Also note that if $m = k = 1$, the Max-window-OWG/α-Val/Neg is transformed in the maximum. If $m = 1$, $k = n$, the Max-window-OWG/α-Val/Neg becomes the minimum. And if $m = n$ and $k = 1$, the Max-window-OWG/α-Val/Neg is transformed in the geometric mean.

Another type of aggregation that could be used is the Max-EZ-OWG/α-Val/Neg weights. In this case, we should distinguish between two classes. In the first class, we assign $w_{j*} = (1/q)$ for $j* = 1$ to q and $w_{j*} = 0$ for $j* > q$, and in the second class, we assign $w_{j*} = 0$ for $j* = 1$ to $n - q$ and $w_{j*} = (1/q)$ for $j* = n - q + 1$ to n. If $q = 1$ for the first class, the Max-EZ-OWG/α-Val/Neg becomes the maximum. And if q = 1 for the second class, the Max-EZ-OWG/α-Val/Neg becomes the minimum. Note that the Max-EZ-OWG/α-Val/Neg is transformed in the Max-GM/α-Val/Neg if $q = n$. If $q = m$ and k = 1, the Max-EZ-OWG/α-Val/Neg becomes the Max-window-OWG/α-Val/Neg for the first class. And for the second class, it is found the Max-window-OWG/α-Val/Neg if $q = m$ and $k = n - q + 1$.

When $w_1 = w_n = 0$, and for all others $w_{j*} = 1/(n - 2)$, we are using the Max-olympic-OWG/α-Val/Neg. Note that if n = 3 or n = 4, the Max-olympic-OWG/α-Val/Neg is transformed in the Max-median-OWG/α-Val/Neg and if $m = n - 2$ and k = 2, the Max-window-OWG/α-Val/Neg is transformed in the Max-olympic-OWG/α-Val/Neg.

Note that the median and the weighted median can also be used as Max-OWG/α-Val/Neg operators. For the Max-median-OWG/α-Val/Neg, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others. For the weighted Max-median-OWG/α-Val/Neg, we select the argument b_k that has the kth largest argument such that the sum of the weights from 1 to k is equal or higher than 0.5 and the sum of the weights from 1 to $k - 1$ is less than 0.5.

A further type of aggregation operator that could be used is the Max-centered-OWG/α-Val/Neg. We can define a Max-centered-OWG/α-Val/Neg as a centered aggregation operator if it is symmetric, strongly decaying and inclusive. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n + 1)/2$ then $w_i < w_j$ and when $i > j \geq (n + 1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \leq w_j$ instead of $w_i < w_j$. We shall refer to this as softly decaying Max-centered-OWG/α-Val/Neg operator. Note that the Max-GM/α-Val/Neg is an example of this particular case of Max-centered-OWG/α-Val/Neg operator. Another particular situation of the Max-centered-OWG/α-Val/Neg operator appears if we remove the third condition. We shall refer to it as a non-inclusive Max-centered-OWG/α-Val/Neg operator. For this situation, we find the Max-median-OWG/α-Val/Neg as a particular case.

A further interesting family is the Max-S-OWG/α-Val/Neg operator. It can be subdivided in three classes, the "orlike", the "andlike" and the generalized Max-S-OWG/α-Val/Neg. The "orlike" Max-S-OWG/α-Val/Neg operator is found when $w_1 = (1/n)(1 - \alpha) + \alpha$, and $w_j = (1/n)(1 - \alpha)$ for j = 2 to n with $\alpha \in [0, 1]$. Note that if α = 0, we get the geometric mean and if α = 1, we get the maximum. The "andlike" Max-S-OWG/α-Val/Neg operator is found when $w_n = (1/n)(1 - \beta) + \beta$ and $w_j = (1/n)(1 - \beta)$ for j = 1 to $n - 1$ with $\beta \in [0, 1]$. Note that in this class, if β = 0 we get the geometric mean and if β = 1, we get the minimum. Finally, the generalized Max-window-OWG/α-Val/Neg operator is obtained when $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for j = 2 to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if α = 0, the generalized Max-S-OWG/α-Val/Neg operator becomes the "andlike" Max-S-OWG/α-Val/Neg operator and if β = 0, it becomes the "orlike" Max-S-OWG/α-Val/Neg operator. Also note that if $\alpha + \beta$ = 1, the generalized Max-S-OWG/α-Val/Neg operator becomes the Max-Hurwicz-OWG/α-Val/Neg criteria.

V. Illustrative Example

In the following, we are going to develop an example in order to understand numerically all the procedures commented above. We will develop a decision making problem under ignorance about selection of investments. We will develop different transformations in the initial payoff matrix such as the usual regret matrix, the geometric regret matrix, the arithmetic negret matrix, the geometric negret matrix, the arithmetic combination between valuations and negrets, and the geometric combination between valuations and negrets. Then, we will aggregate these matrixes with different types of aggregation operators. For the arithmetic matrixes, we will consider the average (AM), the weighted average (WA), the OWA operator and the AOWA operator, and for the geometric ones, the geometric mean (GM), the weighted geometric mean (WGM), the OWG and the AOWG operators.

We should note that in these methods the results obtained from the aggregations are relevant for selecting an alternative but not for considering the specific result obtained. In this example, we will assume the following weighting vector when necessary: W = (0.1, 0.1, 0.2, 0.3, 0.3). For the parameter α to be used in the combination between valuations and negrets, we will consider that $\alpha = 0.5$.

Step 1: Assume an investment company has five possible investments and they want to select the alternative that better adapts to his interests.

1) A_1 is a car company.
2) A_2 is a food company.
3) A_3 is a computer company.
4) A_4 is a chemical company.
5) A_5 is a TV company.

The possible results depending on the state of nature that happens in the future are shown in Table 1.

TABLE I
PAYOFF MATRIX

	S_1	S_2	S_3	S_4	S_5
A_1	30	60	80	40	40
A_2	40	20	90	30	70
A_3	30	50	70	60	60
A_4	80	80	20	20	40
A_5	20	10	30	80	90

Step 2: Calculate the transformed matrixes. For the regret matrix we will use $C_j = \text{Max}\{c_{ij}\}$ for each S_j and $r_{ij} = C_j - c_{ij}$; for each pair A_i and S_j, and for the geometric regret matrix we will consider $r_{ij} = C_j - c_{ij} + 1$. For the negret matrix we will use $e_{ij} = c_{ij} - C_j$; for each pair A_i and S_j, and for the geometric negret matrix we will consider $e_{ij} = c_{ij} - C_j + |\text{Min}\{c_{ij}\}| + |C_j| + 1$. For the combination between valuations and negrets we will use $m_{ij} = c_{ij} - \alpha C_j$ where $\alpha = 0.5$, and for the geometric version, $m_{ij}{*} = c_{ij} + \alpha[|\text{Min}\{c_{ij}\}| + |\text{Max}\{c_{ij}\}| - C_j + 1]$. The results are shown in Tables II – VII.

TABLE II
REGRET MATRIX

	S_1	S_2	S_3	S_4	S_5
A_1	50	20	10	40	50
A_2	40	60	0	50	20
A_3	50	30	20	20	30
A_4	0	0	70	60	50
A_5	60	70	60	0	0

TABLE III
REGRET MATRIX FOR THE GEOMETRIC OPERATORS

	S_1	S_2	S_3	S_4	S_5
A_1	51	21	11	41	51
A_2	41	61	1	51	21
A_3	51	31	21	21	31
A_4	1	1	71	61	51
A_5	61	71	61	1	1

TABLE IV
NEGRET MATRIX

	S_1	S_2	S_3	S_4	S_5
A_1	–50	–20	–10	–40	–50
A_2	–40	–60	0	–50	–20
A_3	–50	–30	–20	–20	–30
A_4	0	0	–70	–60	–50
A_5	–60	–70	–60	0	0

TABLE V
NEGRET MATRIX FOR THE GEOMETRIC OPERATORS

	S_1	S_2	S_3	S_4	S_5
A_1	51	81	61	61	51
A_2	61	41	51	51	81
A_3	51	71	81	81	71
A_4	101	101	41	41	51
A_5	41	31	101	101	101

TABLE VI
COMBINATION BETWEEN VALUATIONS AND NEGRETS

	S_1	S_2	S_3	S_4	S_5
A_1	–10	20	35	0	–5
A_2	0	–20	45	–10	25
A_3	–10	10	25	20	15
A_4	40	40	–25	–20	–5
A_5	–20	30	–15	40	45

TABLE VII
GEOMETRIC COMBINATION BETWEEN VALUATIONS AND NEGRETS

	S_1	S_2	S_3	S_4	S_5
A_1	40.5	70.5	70.5	50.5	45.5
A_2	50.5	30.5	70.5	40.5	75.5
A_3	40.5	60.5	75.5	70.5	65.5
A_4	90.5	90.5	30.5	30.5	45.5
A_5	30.5	20.5	65.5	90.5	95.5

Step 3: Aggregate the previous matrixes with their corresponding aggregation operators. We will aggregate the regret matrix (Table II), the negret matrix (Table IV) and the combination matrix between valuations and negrets (Table VI), with the AM, the WA, the OWA and the AOWA operator. The regret matrix for the geometric operators (Table III), the negret matrix for the geometric operators (Table V) and the geometric combination between valuations and negrets (Table VII) will be aggregated with the GM, the WGM, the OWG and the AOWG operator. The results obtained for each aggregation operator are shown in Tables VIII – XIII. Note that we are interested in establishing an ordering of the alternatives but not in the particular values obtained in the aggregation because each matrix has used a different construction process. Therefore, the values obtained in each matrix are completely different and cannot be compared with other matrixes.

TABLE VIII
AGGREGATED REGRET

	AM	*WA*	*OWA*	*AOWA*
A_1	34	36	27	41
A_2	34	31	25	43
A_3	30	27	26	34
A_4	36	47	23	49
A_5	38	25	25	51

TABLE IX
AGGREGATED REGRET FOR THE GEOMETRIC TRANSFORMATION

	GM	*WGM*	*OWG*	*AOWG*
A_1	30,08	32.16	23.61	38.32
A_2	19.3	17.73	11.70	31.81
A_3	29.3	26.81	25.79	33.29
A_4	11.71	26.18	5.07	27.07
A_5	12.14	5.25	5.25	28.05

TABLE X
AGGREGATED NEGRET

	AM	*WA*	*OWA*	*AOWA*
A_1	–34	–36	–27	–41
A_2	–34	–31	–25	–43
A_3	–30	–27	–26	–34
A_4	–36	–47	–23	–49
A_5	–38	–25	–25	–51

TABLE XI
AGGREGATED NEGRET FOR THE GEOMETRIC TRANSFORMATION

	GM	*WGM*	*OWG*	*AOWG*
A_1	60.09	58.41	56.36	64.08
A_2	55.5	58.36	50.93	60.49
A_3	70.05	73.36	66.00	74.34
A_4	61.42	52.42	51.29	73.56
A_5	66.59	82.01	54.07	82.01

TABLE XII
AGGREGATED RESULTS OF THE COMBINATION BETWEEN VALUATIONS AND NEGRETS

	AM	*WA*	*OWA*	*AOWA*
A_1	8	6.5	1	15
A_2	8	11.5	–2	18
A_3	12	15.5	7.5	16.5
A_4	6	–4.5	–6.5	18.5
A_5	16	23.5	4	28

TABLE XIII
AGGREGATED RESULTS OF THE GEOMETRIC COMBINATION BETWEEN VALUATIONS AND NEGRETS

	GM	*WGM*	*OWG*	*AOWG*
A_1	54.07	52.91	48.97	59.71
A_2	50.61	54.20	43.73	58.56
A_3	61.13	65.13	56.57	66.07
A_4	51.04	42.74	41.06	63.45
A_5	51.26	66.65	39.42	66.65

Step 4: Select the optimal investment for each method. As we can see, we will select A_3 for the Min-AM-Regret, the Min-AOWA-Regret, the Max-AM-Negret, the Max-AOWA-Negret, the Max-OWA/α-Val/Neg, the Max-GM-Negret, the Max-OWG-Negret, the Max-GM/α-Val/Neg, and for the Max-OWG/α-Val/Neg. A_4 will be selected if we use the Min-OWA-Regret, the Max-OWA-Negret, the Min-GM-Regret, the Min-OWG-Regret and the Min-AOWG-Regret. Finally, we will select A_5 if we use the Min-WA-Regret, the Max-WA-Negret, the Max-AM/α-Val/Neg, the Max-WA/α-Val/Neg, the Max-AOWA/α-Val/Neg, the Min-WGM-Regret, the Max-WGM-Negret, the Max-AOWG-Negret, the Max-WGM/α-Val/Neg and the Max-AOWG/α-Val/Neg.

Another possibility is to establish an ordering of the investments. The results are shown in Table XIV. Note that $\succ$ means *preferred to*.

TABLE XIV
ORDERING OF THE INVESTMENTS

Min-AM-Regret	$A_3 \succ A_1 = A_2 \succ A_4 \succ A_5$
Min-WA-Regret	$A_5 \succ A_3 \succ A_2 \succ A_1 \succ A_4$
Min-OWA-Regret	$A_4 \succ A_2 = A_5 \succ A_3 \succ A_1$
Min-AOWA-Regret	$A_3 \succ A_1 \succ A_2 \succ A_4 \succ A_5$
Max-AM-Negret	$A_3 \succ A_1 = A_2 \succ A_4 \succ A_5$
Max-WA-Negret	$A_5 \succ A_3 \succ A_2 \succ A_1 \succ A_4$
Max-OWA-Negret	$A_4 \succ A_2 = A_5 \succ A_3 \succ A_1$
Max-AOWA-Negret	$A_3 \succ A_1 \succ A_2 \succ A_4 \succ A_5$
Max-AM/α-Valuation/Negret	$A_5 \succ A_3 \succ A_1 = A_2 \succ A_4$
Max-WA/α-Valuation/Negret	$A_5 \succ A_3 \succ A_2 \succ A_1 \succ A_4$
Max-OWA/α-Valuation/Negret	$A_3 \succ A_5 \succ A_1 \succ A_2 \succ A_4$
Max-AOWA/α-Valuation/Negret	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
Min-GM-Regret	$A_4 \succ A_5 \succ A_2 \succ A_3 \succ A_1$
Min-WGM-Regret	$A_5 \succ A_2 \succ A_4 \succ A_3 \succ A_1$
Min-OWG-Regret	$A_4 \succ A_5 \succ A_2 \succ A_1 \succ A_3$
Min-AOWG-Regret	$A_4 \succ A_5 \succ A_2 \succ A_3 \succ A_1$
Max-GM-Negret	$A_3 \succ A_5 \succ A_4 \succ A_1 \succ A_2$
Max-WGM-Negret	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
Max-OWG-Negret	$A_3 \succ A_1 \succ A_5 \succ A_4 \succ A_2$
Max-AOWG-Negret	$A_5 \succ A_3 \succ A_4 \succ A_1 \succ A_2$
Max-GM/α-Valuation/Negret	$A_3 \succ A_1 \succ A_5 \succ A_4 \succ A_2$
Max-WGM/α-Valuation/Negret	$A_5 \succ A_3 \succ A_2 \succ A_1 \succ A_4$
Max-OWG/α-Valuation/Negret	$A_3 \succ A_1 \succ A_2 \succ A_4 \succ A_5$
Max-AOWG/α-Valuation/Negret	$A_5 \succ A_3 \succ A_4 \succ A_1 \succ A_2$

As we can see, depending on the decision process used, the ordering of the investments will be different. Note that each decision maker will select a different process depending on its own characteristics and interests.

VI. Conclusion

In this paper we have developed a new approach for decision making under ignorance. We have introduced the use of geometric aggregation operators in decision making with maximization of negret. We have seen that the negret matrix cannot be constructed in the same way as with the arithmetic version because the OWG operator cannot aggregate negative numbers. Therefore, a new scheme has been suggested for constructing the negret matrix. With this new method, we have been able to transform the negative numbers of the initial negret matrix in positive numbers that can be used with the OWG operator. From a general point of view, this method is very practical in the sense that it permits to deal with negative numbers when using the OWG operator because of the transformation done in the negret matrix.

Furthermore, we have extended Yager's method about mixing valuation and negret methods in the same decision process for the case when using geometric aggregation operators. We have seen that this method permits to mix the payoffs with the regrets. Unfortunately, this method is not able to deal with negative numbers because the valuation results can be either positive or negative.

Finally, an illustrative example has been given about the use of the new approaches suggested in the paper. We have focused in an investment selection problem where we have seen the different results obtained depending on the method used.

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