International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:8, No:5, 2014

Damage Localization of Deterministic-Stochastic Systems

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***Abstract*—**A scheme integrated with deterministic–stochastic subspace system identification and the method of damage localization vector is proposed in this study for damage detection of structures based on seismic response data. A series of shaking table tests using a five-storey steel frame has been conducted in National Center for Research on Earthquake Engineering (NCREE), Taiwan. Damage condition is simulated by reducing the cross-sectional area of some of the columns at the bottom. Both single and combinations of multiple damage conditions at various locations have been considered. In the system identification analysis, either full or partial observation conditions have been taken into account. It has been shown that the damaged stories can be identified from global responses of the structure to earthquakes if sufficiently observed. In addition to detecting damage(s) with respect to the intact structure, identification of new or extended damages of the as-damaged (ill-conditioned) counterpart has also been studied. The proposed scheme proves to be effective.

***Keywords*—**Damage locating vectors, deterministic-stochastic subspace system, shaking table tests, system identification.

I. Introduction

OVER the last decade, structural health monitoring (SHM) has attracted a great deal of attention in civil engineering. Development of promising SHM systems to efficiently assess the integrity of critical buildings, industrial manufactories and infrastructures right after the strike of an earthquake is in urgent demand. A SHM system is pragmatic only if integrated with reliable measures in response monitoring, system identification and damage detection. System identification schemes that make a direct use of the recorded data acquired from limited locations to reconstruct the full (or equivalent) state-space system parameters representative of the physical structures are desired. Among the damage detection techniques based on variation of the physical parameters as the structure deteriorates, the flexibility-based approaches have shown to be very promising and computationally efficient. Pandey and Biswas [1] demonstrated that the damage locations of a wide-flange steel beam could be identified by interrogating the change of the flexibility matrix. The method of *Damage Localization Vector (DLV)* proposed by Bernal is of great potential in advancing structural health monitoring into practical application [2], [3]. This flexibility-based approach is capable of identifying multiple damages in the structure via a truncated modal basis without a predetermined analytical model. Structural members with nearly zero stress under the loading of the *DLV* are considered potentially damaged. The *DLV* are determined by performing *Singular Value Decomposition* of the flexibility differential before and after the damage state of the structure. The flexibility matrix is obtained from system identification analysis of the structure via natural or artificial dynamic testing.

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To facilitate implementation of the *DLV* method, system identification technique that identifies the system matrix and output (observation) matrix of a state-space model is considered. The method of *System Realization using Information Matrix* (*SRIM*) proposed by Juang [4] is based on a deterministic state-space system. The equivalent system matrix is identified from the covariance matrix of the input and output signals. This simple and elegant approach works well for systems in response to transient excitation if the noise level is negligible. The performance degrades, however, as the noise becomes pronounced [5], [6]. On the contrary, the *Stochastic Subspace Identification* (*SSI*)method based on a stochastic model ([7],[8]) without knowing the input is less sensitive to noise and works well if the excitation is Gaussian white noise. This scheme is not sufficient, however, for systems under transient excitations such as earthquakes [5], [6]. Therefore, a mixed deterministic-stochastic model is considered more robust to transient systems with non-negligible noise contamination. The stochastic realization theory initiated by Akaike [9] and Faure [10] is non-iterative and convergence-guaranteed. The covariance matrix is first constructed from block Hankle matrix of shifted process sequences. The state-space model is in turn realized from the observability and/or controllability matrix via *Singular Value Decomposition* of the covariance matrix. The theory of covariance-driven subspace method has been unified by Van Overschee and De Moor [8] for deterministic, stochastic and combined systems by defining the estimated state sequences as the projection of input-output data. The projected state sequences turn out to be the outputs of non-steady state Kalman filter banks. A numerically stable and efficient algorithm has been devised by Overschee and De Moor [11] to solve for the system matrix. The algorithm is adopted in this study.

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II. Theoretical Backgrounds

A. Deterministic-Stochastic Subspace System Identification

A deterministic-stochastic linear time-invariant system is represented in a discrete-time state-space model as:

$$\mathbf{z}_{k+1} = \mathbf{A}\mathbf{z}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \tag{1a}$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{z}_k + \mathbf{D}\mathbf{u}_k + \mathbf{v}_k \tag{1b}$$

where $\mathbf{z}_k \in R^{2n\times 1}$ and $\mathbf{y}_k \in R^{m\times 1}$ are respectively the state and output vectors, and $\mathbf{u}_k \in R^{r\times 1}$ is the input vector at time instant k. $\mathbf{A} \in R^{2n\times 2n}$ is the system matrix, $\mathbf{B} \in R^{2n\times r}$ is the input influence matrix, $\mathbf{C} \in R^{m\times 2n}$ is the observation matrix and $\mathbf{D} \in R^{m\times r}$ is the direct transmission matrix. $\mathbf{w}_k \in R^{2n\times 1}$ and $\mathbf{v}_k \in R^{m\times 1}$ are the un-measurable vector signals assumed to be zero-mean, stationary white noise vector sequences. By recursive substitution into the state-space equations of consecutive shifted processes, it leads to [6]

$$\mathbf{Y}_{0|i-1} = \mathbf{\Gamma}_i\mathbf{Z}_0^d + \mathbf{H}_i\mathbf{U}_{0|i-1} + \mathbf{Y}_{0|i-1}^s \tag{2a}$$

$$\mathbf{Y}_{i|2i-1} = \mathbf{\Gamma}_i\mathbf{Z}_i^d + \mathbf{H}_i\mathbf{U}_{i|2i-1} + \mathbf{Y}_{i|2i-1}^s \tag{2b}$$

$$\mathbf{Z}_i^d = \mathbf{A}_i\mathbf{Z}_0^d + \mathbf{\Lambda}_i\mathbf{U}_{0|i-1} \tag{2c}$$

where

$$\mathbf{Y}_{0|i-1} = \begin{bmatrix} \mathbf{y}_0 & \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_{j-1} \\ \mathbf{y}_1 & \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_j \\ \mathbf{y}_2 & \mathbf{y}_3 & \mathbf{y}_4 & \cdots & \mathbf{y}_{j+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{i-1} & \mathbf{y}_i & \mathbf{y}_{i+1} & \cdots & \mathbf{y}_{i+j-2} \end{bmatrix} \in R^{mi\times j}$$ is the output block Hankle matrix of the past;

$$\mathbf{Y}_{0|i-1}^s = \begin{bmatrix} \mathbf{y}_0^s & \mathbf{y}_1^s & \mathbf{y}_2^s & \cdots & \mathbf{y}_{j-1}^s \\ \mathbf{y}_1^s & \mathbf{y}_2^s & \mathbf{y}_3^s & \cdots & \mathbf{y}_j^s \\ \mathbf{y}_2^s & \mathbf{y}_3^s & \mathbf{y}_4^s & \cdots & \mathbf{y}_{j+1}^s \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{i-1}^s & \mathbf{y}_i^s & \mathbf{y}_{i+1}^s & \cdots & \mathbf{y}_{i+j-2}^s \end{bmatrix} \in R^{mi\times j}$$ is the stochastic output block Hankle matrix of the past;

$$\mathbf{U}_{0|i-1} = \begin{bmatrix} \mathbf{u}_0 & \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{j-1} \\ \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \mathbf{u}_j \\ \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{u}_4 & \cdots & \mathbf{u}_{j+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{u}_{i-1} & \mathbf{u}_i & \mathbf{u}_{i+1} & \cdots & \mathbf{u}_{i+j-2} \end{bmatrix} \in R^{ri\times j}$$ is the input block Hankle matrix of the past;

$\mathbf{Z}_i^d = \begin{bmatrix} \mathbf{z}_i^d & \mathbf{z}_{i+1}^d & \mathbf{z}_{i+2}^d & \cdots & \mathbf{z}_{i+j-1}^d \end{bmatrix} \in R^{2n\times j}$ is the deterministic state matrix;

$$\mathbf{\Gamma}_i = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{i-1} \end{bmatrix} \in R^{mi\times 2n}$$ is the observability matrix;

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{D} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CB} & \mathbf{D} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CAB} & \mathbf{CB} & \mathbf{D} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{CA}^{i-2}\mathbf{B} & \mathbf{CA}^{i-3}\mathbf{B} & \mathbf{CA}^{i-4}\mathbf{B} & \cdots & \mathbf{D} \end{bmatrix} \in R^{mi\times ri}$$ is the triangular Toeplits matrix;

$\mathbf{\Lambda}_i = \begin{bmatrix} \mathbf{A}^{i-1}\mathbf{B} & \mathbf{A}^{i-2}\mathbf{B} & \cdots & \mathbf{AB} & \mathbf{B} \end{bmatrix} \in R^{2n\times ri}$ is the controllability matrix, and

$\mathbf{A}_i = \begin{pmatrix} \mathbf{A}^T & (\mathbf{A}^T)^2 & (\mathbf{A}^T)^3 & \dots & (\mathbf{A}^T)^i \end{pmatrix}^T \in R^{2ni\times 2n}$

Equation (2c) can be easily extended from (2b) for the "future" of the shifted sequences.

The stochastic covariance equations of the subspace can be defined as

$$\mathbf{P}^s = \mathrm{E}\left[\mathbf{z}_k^s\ (\mathbf{z}_k^s)^T\right] = \mathbf{A}\mathbf{P}^s\mathbf{A}^T + \mathbf{R}_{\mathbf{ww}}(0) \tag{3a}$$

$$\mathbf{G} = \mathrm{E}\left[\mathbf{z}_{k+1}^s\ (\mathbf{y}_k^s)^T\right] = \mathbf{A}\mathrm{P}^s\mathbf{C}^T + \mathbf{R}_{\mathbf{wv}}(1) \tag{3b}$$

$$\boldsymbol{\lambda}_0 = \mathrm{E}\left[\mathbf{y}_k^s\ (\mathbf{y}_k^s)^T\right] = \mathbf{C}\mathbf{P}^s\mathbf{C}^T + \mathbf{R}_{\mathbf{vv}}(0) \tag{3c}$$

The stochastic Toeplitz matrices are defined as

$$\mathbf{\Lambda}_{0|i-1}^s = \frac{1}{N}\mathbf{Y}_{0|i-1}\mathbf{Y}_{0|i-1}^T = \begin{bmatrix} \boldsymbol{\lambda}_0 & \boldsymbol{\lambda}_{-1} & \boldsymbol{\lambda}_{-2} & \dots & \boldsymbol{\lambda}_{1-i} \\ \boldsymbol{\lambda}_1 & \boldsymbol{\lambda}_0 & \boldsymbol{\lambda}_{-1} & \dots & \boldsymbol{\lambda}_{2-i} \\ \boldsymbol{\lambda}_2 & \boldsymbol{\lambda}_1 & \boldsymbol{\lambda}_0 & \dots & \boldsymbol{\lambda}_{3-i} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{\lambda}_{i-1} & \boldsymbol{\lambda}_{i-2} & \boldsymbol{\lambda}_{i-3} & \dots & \boldsymbol{\lambda}_0 \end{bmatrix} \in R^{mi\times mi} \tag{4a}$$

$$\mathbf{\Lambda}_{i|2i-1}^s = \frac{1}{N}\mathbf{Y}_{i|2i-1}\mathbf{Y}_{0|i-1}^T = \begin{bmatrix} \boldsymbol{\lambda}_i & \boldsymbol{\lambda}_{i-1} & \boldsymbol{\lambda}_{i-2} & \dots & \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_{i+1} & \boldsymbol{\lambda}_i & \boldsymbol{\lambda}_{i-1} & \dots & \boldsymbol{\lambda}_2 \\ \boldsymbol{\lambda}_{i+2} & \boldsymbol{\lambda}_{i+1} & \boldsymbol{\lambda}_i & \dots & \boldsymbol{\lambda}_3 \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{\lambda}_{2i-1} & \boldsymbol{\lambda}_{2i-2} & \boldsymbol{\lambda}_{2i-3} & \dots & \boldsymbol{\lambda}_i \end{bmatrix} = \mathbf{\Gamma}_i\mathbf{\Lambda}_i^s \in R^{mi\times mi} \tag{4b}$$

where $\boldsymbol{\lambda}_i = \mathrm{E}\left[\mathbf{y}_{k+i}^s\ (\mathbf{y}_k^s)^T\right] = \begin{cases} \mathbf{CA}^{i-1}\mathbf{G} & i > 0 \\ \boldsymbol{\lambda}_0 & i = 0 \\ \mathbf{G}^T(\mathbf{A}^T)^{-i-1}\mathbf{C}^T & i < 0 \end{cases}$ and

$\mathbf{\Lambda}_i^s = \begin{bmatrix} \mathbf{A}^{i-1}\mathbf{G} & \mathbf{A}^{i-2}\mathbf{G} & \mathbf{A}^{i-3}\mathbf{G} & \dots & \mathbf{G} \end{bmatrix} \in R^{2n\times mi}$ is the stochastic controllability matrix.

Projecting the future output state matrix $\mathbf{Y}_{i|2i-1}$ onto the input matrix $\mathbf{U}_{0|2i-1}$ and the past output state matrix $\mathbf{Y}_{0|i-1}$, the output-input block Hankel matrices are constructed as

$$\overline{\mathbf{Y}}_i = \mathbf{Y}_{i|2i-1} \Big/ \begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i-1} \end{pmatrix} = \mathbf{Y}_{i|2i-1}\begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i-1} \end{pmatrix}^T \left(\begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i-1} \end{pmatrix}\begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i-1} \end{pmatrix}^T\right)^* \begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i-1} \end{pmatrix} \tag{5a}$$

and

$$\overline{\mathbf{Y}}_{i+1} = \mathbf{Y}_{i+1|2i-1} \Big/ \begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i} \end{pmatrix} = \mathbf{Y}_{i+1|2i-1} \begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i} \end{pmatrix}^{T} \left(\begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i} \end{pmatrix} \begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i} \end{pmatrix}^{T} \right)^{*} \begin{pmatrix} \mathbf{U}_{0|2i-1} \\ \mathbf{Y}_{0|i} \end{pmatrix} \quad (5b)$$

where $\bullet^{*}$ is the pseudo-inverse of $\bullet$.

To further simplify (5a) and (5b), the deterministic subspace and stochastic subspace are utilized to define what follows,

$$\lim_{j\to\infty} \frac{1}{j} \begin{pmatrix} \mathbf{U}_{0|i-1} \\ \hline \mathbf{U}_{i|2i-1} \\ \hline \mathbf{Z}_0^d \end{pmatrix} \left(\mathbf{U}_{0|i-1}^{T} \mid \mathbf{U}_{i|2i-1}^{T} \mid (\mathbf{Z}_0^d)^T \right) = \left[\begin{array}{cc|c} \mathbf{R}_{11} & \mathbf{R}_{12} & \mathbf{S}_1^T \\ \mathbf{R}_{12}^T & \mathbf{R}_{22} & \mathbf{S}_2^T \\ \hline \mathbf{S}_1 & \mathbf{S}_2 & P^d \end{array} \right] \quad (6a)$$

$$= \left[\begin{array}{c|c} \mathbf{R} & \mathbf{S}^T \\ \hline \mathbf{S} & \mathbf{P}^d \end{array} \right] \in R^{(2ri+2n)\times(2ri+2n)}$$

and

$$\lim_{j\to\infty} \frac{1}{j} \begin{pmatrix} \mathbf{Y}_{0|i-1}^{s} \\ \hline \mathbf{Y}_{i|2i-1}^{s} \end{pmatrix} \left((\mathbf{Y}_{0|i-1}^{s})^{T} \mid (\mathbf{Y}_{i|2i-1}^{s})^{T} \right) = \left(\begin{array}{c|c} \mathbf{\Lambda}_{0|i-1}^{s} & (\mathbf{\Lambda}_{i|2i-1}^{s})^{T} \\ \hline \mathbf{\Lambda}_{i|2i-1}^{s} & \mathbf{\Lambda}_{0|i-1}^{s} \end{array} \right) \quad (6b)$$

Assuming that the deterministic input $\mathbf{u}_k$ and the deterministic state $\mathbf{z}_k^d$ are independent of the stochastic output $\mathbf{y}_k^s$, then (5a) and (5b) can be simplified as

$$\overline{\mathbf{Y}}_i = \mathbf{\Gamma}_i \hat{\mathbf{Z}}_i + \mathbf{H}_i \mathbf{U}_{i|2i-1.} \quad (7a)$$

and

$$\overline{\mathbf{Y}}_{i+1} = \mathbf{\Gamma}_{i-1} \hat{\mathbf{Z}}_{i+1} + \mathbf{H}_{i-1} \mathbf{U}_{i+1|2i-1.} \quad (7b)$$

where

$$\hat{\mathbf{Z}}_i = \left(\mathbf{A}^i - \mathbf{Q}_i \mathbf{\Gamma}_i \mid \mathbf{\Lambda}_i - \mathbf{Q}_i \mathbf{H}_i \mid \mathbf{Q}_i \right) \begin{pmatrix} \mathbf{S}\mathbf{R}^{-1}\mathbf{U}_{0|2i-1} \\ \hline \mathbf{U}_{0|i-1} \\ \hline \mathbf{Y}_{0|i-1} \end{pmatrix}$$

$$\hat{\mathbf{Z}}_{i+1} = \left(\mathbf{A}^{i+1} - \mathbf{Q}_{i+1} \mathbf{\Gamma}_{i+1} \mid \mathbf{\Lambda}_{i+1} - \mathbf{Q}_{i+1} \mathbf{H}_{i+1} \mid \mathbf{Q}_{i+1} \right) \begin{pmatrix} \mathbf{S}\mathbf{R}^{-1}\mathbf{U}_{0|2i-1} \\ \hline \mathbf{U}_{0|i} \\ \hline \mathbf{Y}_{0|i} \end{pmatrix}$$

$$\mathbf{Q}_i = \mathbf{x}_i \mathbf{\psi}_i^{-1}$$

in which

$$\mathbf{x}_i = \mathbf{A}^i \left(\mathbf{P}^d - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T \right) \mathbf{\Gamma}_i^T + \mathbf{\Lambda}_i^s$$
$$\mathbf{\psi}_i = \mathbf{\Gamma}_i \left(\mathbf{P}^d - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T \right) \mathbf{\Gamma}_i^T + \mathbf{\Lambda}_{0|i-1}^s$$

Equations (5a) and (5b) can be rearranged as

$$\hat{\mathbf{Z}}_i = \mathbf{\Gamma}_i^{*} \left(\overline{\mathbf{Y}}_i - \mathbf{H}_i \mathbf{U}_{i|2i-1} \right) \quad (8a)$$

and

$$\hat{\mathbf{Z}}_{i+1} = \mathbf{\Gamma}_{i-1}^{*} \left(\overline{\mathbf{Y}}_{i+1} - \mathbf{H}_{i-1} \mathbf{U}_{i+1|2i-1} \right) \quad (8b)$$

where $\mathbf{\Gamma}_i^{*}$ and $\mathbf{\Gamma}_{i-1}^{*}$ are respectively the pseudo-inverse of $\mathbf{\Gamma}_i$ and $\mathbf{\Gamma}_{i-1}$.

The non-stationary Kalman state vector $\hat{\mathbf{z}}_k$ can be defined as

$$\hat{\mathbf{z}}_k = \mathbf{A}\hat{\mathbf{z}}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{K}_{k-1}\left(\mathbf{y}_{k-1} - \mathbf{C}\hat{\mathbf{z}}_{k-1} - \mathbf{D}\mathbf{u}_{k-1} \right) \quad (9)$$

where

$$\mathbf{K}_{k-1} = \left(\mathbf{A}\mathbf{P}_{k-1}\mathbf{C}^T + \mathbf{G} \right) \left(\mathbf{\lambda}_0 + \mathbf{C}\mathbf{P}_{k-1}\mathbf{C}^T \right)^{-1} \quad (10)$$

and

$$\mathbf{P}_k = \mathbf{A}\mathbf{P}_{k-1}\mathbf{A}^T - \mathbf{K}_{k-1}(\mathbf{A}\mathbf{P}_{k-1}\mathbf{C}^T + \mathbf{G})^T \quad (11)$$

By assembling j consecutive Kalman states together, (9) can be expanded as

$$\begin{aligned} \hat{\mathbf{Z}}_{i+1} &= \mathbf{A}\hat{\mathbf{Z}}_i + \mathbf{B}\mathbf{U}_{i|i} + \mathbf{K}_i\left(\mathbf{Y}_{i|i} - \mathbf{C}\hat{\mathbf{Z}}_i - \mathbf{D}\mathbf{U}_{i|i} \right) \\ &= \mathbf{A}\hat{\mathbf{Z}}_i + \mathbf{B}\mathbf{U}_{i|i} + \mathbf{W}_{i|i} \end{aligned} \quad (12)$$

where

$$\mathbf{W}_{i|i} = \left(\mathbf{w}_i \quad \mathbf{w}_{i+1} \quad \mathbf{w}_{i+2} \quad \cdots \quad \mathbf{w}_{i+j-1} \right) \quad (13a)$$

$$\begin{aligned} \mathbf{Y}_{i|i} &= \mathbf{C}\hat{\mathbf{Z}}_i + \mathbf{D}\mathbf{U}_{i|i} + \left(\mathbf{Y}_{i|i} - \mathbf{C}\hat{\mathbf{Z}}_i - \mathbf{D}\mathbf{U}_{i|i} \right) \\ &= \mathbf{C}\hat{\mathbf{Z}}_i + \mathbf{D}\mathbf{U}_{i|i} + \mathbf{V}_{i|i} \end{aligned} \quad (13b)$$

and

$$\mathbf{V}_{i|i} = \left(\mathbf{v}_i \quad \mathbf{v}_{i+1} \quad \mathbf{v}_{i+2} \quad \cdots \quad \mathbf{v}_{i+j-1} \right) \quad (13c)$$

Combining (12) and (13b), one gets

$$\begin{pmatrix} \hat{\mathbf{Z}}_{i+1} \\ \mathbf{Y}_{i|i} \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{C} \end{pmatrix} \hat{Z}_i + \begin{pmatrix} \mathbf{B} \\ \mathbf{D} \end{pmatrix} \mathbf{U}_{i|i} + \begin{pmatrix} \mathbf{W}_{i|i} \\ \mathbf{V}_{i|i} \end{pmatrix} \quad (14)$$

Substituting (8a) and (8b) for $\hat{\mathbf{Z}}_i$ and $\hat{\mathbf{Z}}_{i+1}$ into (14), it leads to

$$\begin{pmatrix} \mathbf{\Gamma}_{i-1}^{*} \overline{\mathbf{Y}}_{i+1} \\ \mathbf{Y}_{i|i} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{K}_{12} \\ \mathbf{C} & \mathbf{K}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{\Gamma}_i^{*} \overline{\mathbf{Y}}_i \\ \mathbf{U}_{i|2i-1} \end{pmatrix} + \begin{pmatrix} \mathbf{W}_{i|i} \\ \mathbf{V}_{i|i} \end{pmatrix} \quad (15)$$

where

$$\mathbf{K}_{12} = \left(\mathbf{B} - \mathbf{A}\mathbf{\Gamma}_i^{*} \begin{pmatrix} \mathbf{D} \\ \mathbf{\Gamma}_{i-1}\mathbf{B} \end{pmatrix} \middle| \; \mathbf{\Gamma}_{i-1}^{*}\mathbf{H}_{i-1} - \mathbf{A}\mathbf{\Gamma}_i^{*} \begin{pmatrix} \mathbf{0} \\ \mathbf{H}_{i-1} \end{pmatrix} \right)$$

and

$$\mathbf{K}_{22} = \left(\mathbf{D} - \mathbf{C}\mathbf{\Gamma}_i^{*} \begin{pmatrix} \mathbf{D} \\ \mathbf{\Gamma}_{i-1}\mathbf{B} \end{pmatrix} \middle| \; -\mathbf{C}\mathbf{\Gamma}_i^{*} \begin{pmatrix} \mathbf{0} \\ \mathbf{H}_{i-1} \end{pmatrix} \right)$$

By forcing the noise terms in (14) to be zero, the coefficient matrices may be resolved as

$$\begin{pmatrix} \mathbf{A} & \mathbf{K}_{12} \\ \mathbf{C} & \mathbf{K}_{22} \end{pmatrix} = \begin{pmatrix} \mathbf{\Gamma}_{i-1}^{*} \overline{\mathbf{Y}}_{i+1} \\ \mathbf{Y}_{i|i} \end{pmatrix} \begin{pmatrix} \mathbf{\Gamma}_i^{*} \overline{\mathbf{Y}}_i \\ \mathbf{U}_{i|2i-1} \end{pmatrix}^{*} \quad (16)$$

where $\begin{pmatrix} \mathbf{\Gamma}_i^* \overline{\mathbf{Y}}_i \\ \mathbf{U}_{i|2i-1} \end{pmatrix}^*$ is the pseudo-inverse of $\begin{pmatrix} \mathbf{\Gamma}_i^* \overline{\mathbf{Y}}_i \\ \mathbf{U}_{i|2i-1} \end{pmatrix}$. A numerically stable and efficient algorithm referred to as the N4SID devised by Overschee and De Moor [11] is adopted in this study to solve for the system matrix.

B. Damage Localization Vector

Bernal [2] proposed that the structure subjected to the damage locating vectors $\mathbf{L}$ would undergo the same deformation before and after the damaged state. This statement immediately leads to

$$\mathbf{D}_F \mathbf{L} = \mathbf{0} \tag{17}$$

where $\mathbf{D}_F$ is the flexibility differential of the structure before and after damaged. When $rank(\mathbf{D}_F) < n$ (n is the degree of freedom of the structure), the basis corresponds to the null space of $\mathbf{D}_F$ is the damage locating vectors $\mathbf{L}$, which can be derived from singular value decomposition of the flexibility differentia. Members with nearly zero stress under the loadings of DLVs are considered potentially damaged.

The flexibility matrix of the structure can be expressed with the system matrices of the continuous-time state-space representation as

$$\mathbf{F} = -\mathbf{C}_0 \mathbf{A}_c^{-1} \mathbf{H}^{-1} \mathbf{C}_0{}^T \tilde{\mathbf{D}} = \mathbf{Q}\tilde{\mathbf{D}} \tag{18}$$

where $\mathbf{A}_c = \frac{\ln(\mathbf{A})}{\Delta t} \in R^{2n \times 2n}$ is the continuous-time system matrix; $\mathbf{C}_0 = [\mathbf{I} \quad \mathbf{0}] \in R^{n \times 2n}$; $\mathbf{H} = \begin{bmatrix} \mathbf{C}_0 \\ \mathbf{C}_0 \mathbf{A}_c \end{bmatrix} \in R^{2n \times 2n}$; $\tilde{\mathbf{D}} = \mathbf{C}_0 \mathbf{A}_c \mathbf{B}_c = -\mathbf{M}^{-1}$ ($\mathbf{M}$ being the mass matrix of the system). With (18), the flexibility differential $\mathbf{D}_F$ can be expressed as

$$\mathbf{D}_F = \Delta \mathbf{Q} \tilde{\mathbf{D}}^i + \mathbf{Q}^d \Delta \tilde{\mathbf{D}} = \Delta \mathbf{Q} \tilde{\mathbf{D}}^i \tag{19}$$

where $\Delta \mathbf{Q} = \mathbf{Q}^d - \mathbf{Q}^i$ and $\Delta \tilde{\mathbf{D}} = \tilde{\mathbf{D}}^d - \tilde{\mathbf{D}}^i = \mathbf{0}$ since the mass matrix is unchanged. By taking the singular value decomposition of $\Delta \mathbf{Q}$, the eigen-vectors $\mathbf{V}_0^{\Delta \mathbf{Q}}$ correspond to the singular eigen-values is the damage locating vector $\mathbf{L} \in R^{n \times q}$.
The normalized weighted stress index WSI_j is defined as

$$\mathrm{WSI}_j = \sum_{i=1}^{q} nsi_{j,\, i} \tag{20}$$

where $nsi_{j,\, i}$ is the normalized stress index of the *j*-th member or d.o.f. subjected to the *i*-th DLV. Member *j* (or storey *j*) is considered seriously damaged when the normalized weighted stress index $\mathrm{nWSI}_j \leq 0.1$ in which $\mathrm{nWSI}_j = WSI_j / WSI_{j,\max}$, whereas it is considered moderately damaged as $0.1 < n\mathrm{WSI}_j \leq 0.2$.

III. Experimental Setup

This series of shaking table tests has been carried out in NCREE, Taiwan using a benchmark model, as illustrated in Fig. 1. It is a five-storey steel frame with 4.5 ton weight at each floor. Accelerometers have been implemented at the mass center of each floor and the base to monitor the dynamic responses serving as the basis for system identification.

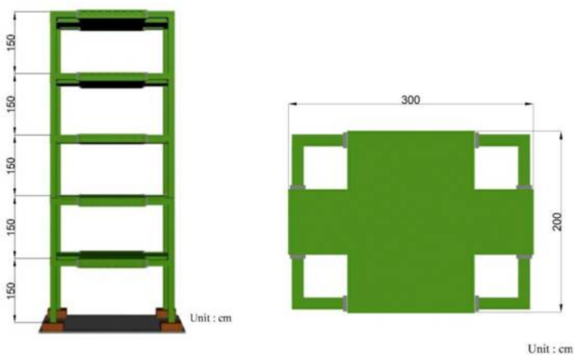

Fig. 1 Front and top views of the benchmark model

Damage of the structure is simulated by cutting out a small portion of the flange near the bottom of the column(s), as shown in Fig. 2. In order to sufficiently examine the damage at various extents, the damages were progressively enforced on one side of the frame from the 1st storey to the 3rd storey. It is meant to represent a moderate damage condition as a single column being damaged and a serious damage condition as two columns being damaged in the same storey. Totally six damage conditions have been considered in the tests. The 1940 El Centro earthquake has been adopted as the input with the PGA scaled to 0.1g.

Fig. 2 Flange partially cut-out at the bottom end of column

IV. Test Results

The test results are analyzed under considerations of full observation (utilizing acceleration responses of all floors), partial observation (ignoring some floor accelerations) and ill-conditioned condition where the reference structure has been wounded in the previous tests.

A. Full Observation

The six damaged conditions simulated in the tests are designated as:

M1: Single column damaged at the first floor, representing a *m*oderate damage condition of the first storey;

S1: Two columns damaged at the first storey, representing a *s*erious damage condition of the first storey;

S1M2: Two columns damaged at the first storey and single column damaged at the second storey;

S12: Two columns damaged at both the first and second stories;

S12M3: Two columns damaged at both the first and second stories and single column damaged at the third storey;

S123: Two columns damaged from storey 1 to 3.

The assessment results of various damage conditions based on full observation data of the structure are summarized in Table I where the shaded area corresponds to those being screened out as potentially damaged stories. It is evident that, under a full observation condition, the damaged location(s) are successfully identified, regardless of single or multiple damage conditions.

TABLE I
SUMMARY OF DAMAGE ASSESSMENT W/ FULL OBSERVATION

	nWSI$_j$#					
Case	M1	S1	S1M2	S12	S12M3	S123
1F	0.09	0.05	0.04	0.07	0.01	0.04
2F	1.00	0.44	0.12	0.06	0.01	0.01
3F	0.97	0.90	1.00	0.81	0.11	0.04
4F	0.72	1.00	0.89	0.90	0.51	0.83
5F	0.86	0.56	0.45	1.00	1.00	1.00
Performance	Good	Good	Good	Good	Good	Good

#nWSI$_j$ ≦0.1 indicates serious damage; 0.1<nWSI$_j$ ≦0.2 indicates moderate damage

B. Partial Observation

As a further step to exam if the scheme is also valid as the vibration data is not fully available for all floors, the system identification analysis utilizing only part of the information is considered. Designation of the cases for partial observation is first the damage condition followed with the observed floors after a "/". Take M1/135 for example, it stands for a moderate damage condition at the first storey and only the 1st, 3rd and 5th floors are observed.

TABLEII
SUMMARY OF DAMAGE ASSESSMENT W/ PARTIAL OBSERVATION

	WSI$_j$					
Case	M1/135	S1/135	M1/124	S1/124	S1M/124	S1M2/125
1F	0.14	0.11	0.17	0.16	0.41	0.41
2F	-	-	1.00	1.00	1.00	1.00
3F	0.74	0.72	-	-	-	-
4F	-	-	0.41	0.43	0.33	-
5F	1.00	1.00	-	-	-	0.38
PMC*	Good	Fair	Good	Fair	Fail	Fail

	WSI$_j$					
Case	S1M2/1235	S12/124	S12/125	S12/1235	S12M3/1235	S123/1235
1F	0.02	0.96	1.00	0.01	0.09	0.14
2F	0.24	1.00	0.27	0.20	0.70	0.15
3F	1.00	-	-	1.00	1.00	0.48
4F	-	0.07	-	-	-	-
5F	0.66	-	0.81	0.78	0.63	1.00
PMC	Poor	Fail	Fail	Fair	Fail	Poor

***Good** indicates the damaged location(s) being identified without miss-judgment.
Fair indicates the damaged storey being identified but the extent might be underestimated.
Poor indicates failing to identify one of the damaged stories.
Fail indicates failing to identify more than one of the damaged stories.

The assessment results of various damage conditions based on partial observation data of the structure are summarized in Table II where the shaded area corresponds to those being screened out as potentially damaged stories. The scheme with partial observation remains effective for single damage conditions, as in the cases of M1/135, S1/135, M1/124 and S1/124. The scheme fails, however, to locate the damaged stories in multiple damage conditions except for case S12/1235 where the first two stories are seriously damaged and 4 out of 5 stories are observed.

C. Ill-Conditioned Structures

If the structural health monitoring system is introduced after the target building has been previously damaged, it is of interest to verify if the scheme is able to identify new or extended damage(s) of an ill-conditioned structure damaged earlier after another earthquake event. The system identification analysis will be based on full observation data as it provides more reliable structural information for damage assessment. Designation of the cases under study for ill-conditioned structures is first the current damage condition followed with the one serving as the basis for comparison after a "/". Take S1/M1 for example, it stands for a serious damage condition at the first storey of the current state versus the one with its first storey moderately damaged earlier.

The assessment results of various damage conditions in respect to an ill-conditioned structure damaged earlier are summarized in Table III. The performance index (PMC) is similarly defined as in Table II. In all the cases considered, the scheme proves to be sufficient in identifying new or extended damage(s) without exception, under a full observation condition.

TABLE III
DAMAGE ASSESSMENT OF ILL-CONDITIONED STRUCTURES

	WSI_j				
Case	S1/M1	S1M2/S1	S12/S1M2	S12M3/S12	S123/S12M3
1F	0.16	0.10	0.26	0.24	0.32
2F	0.46	0.01	0.03	0.46	0.55
3F	1.00	0.29	0.36	0.16	0.17
4F	0.58	0.85	0.73	1.00	1.00
5F	0.90	1.00	1.00	0.62	0.72
PMC	Good	Good	Good	Good	Good
	WSI_j				
Case	S12/M1	S12M3/M1	S123/M1	S12/S1	S123/S1
1F	0.01	0.01	0.02	0.48	1.00
2F	0.02	0.02	0.01	0.07	0.03
3F	0.13	0.05	0.10	0.35	0.08
4F	0.72	0.21	0.67	0.43	0.34
5F	1.00	1.00	1.00	1.00	0.69
PMC	Good	Good	Good	Good	Good

V. CONCLUSION

Experimental verification of damage detection of structures using seismic response data (accelerations) has been carried out under realistic earthquake simulations via shaking table. The method of damage locating vector is adopted utilizing system parameters identified by the N4SID algorithm developed for deterministic-stochastic subspace models. The algorithm is applicable for both full and partial observation conditions. Based on the test results, the conclusion is drawn as the following,

A. Damage localization utilizing seismic response data, in particular the floor accelerations, proves feasible.
B. Under a full observation condition where all floors are observed, the damaged location(s) can be successfully identified, regardless of single or multiple damage conditions.
C. Under a partial observation condition where 3 out of 5 floors are observed, only structures with single damage can be identified if the damaged storey is co-located with one of the observed floor. The scheme fails, however, to locate the damaged stories in multiple damage conditions in general.
D. The scheme proves to be sufficient in identifying new or extended damage(s) without exception under a full observation condition.

ACKNOWLEDGMENT

This work is supported by the National Science Council of Republic of China under contract ***NSC 102-2221-E-009-086***.

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