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Cost-Optimized SSB Transmitter with High Frequency Stability and Selectivity

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Abstract**—**Single side band modulation is a widespread technique in communication with significant impact on communication technologies such as DSL modems and ATSC TV. Its widespread utilization is due to its bandwidth and power saving characteristics. In this paper, we present a new scheme for SSB signal generation which is cost efficient and enjoys superior characteristics in terms of frequency stability, selectivity, and robustness to noise. In the process, we develop novel Hilbert transform properties.

Keywords**—**Crystal filter, frequency drift, frequency mixing, Hilbert transform, phasing, selectivity, single side band AM.

I. Introduction

SINGLE side band modulation is an improvement over standard amplitude modulation (AM). The refinement of the technique lies in its power efficiency and reduced bandwidth. While standard AM produces a modulated signal that has twice the bandwidth of the information-bearing baseband signal, SSB reduces this bandwidth and associated power by half at the cost of device complexity, increased cost, and sensitivity to frequency drift [1].

In retrospect, SSB appears to be simple, trivial, and as old as telephony itself. Historically, SSB was pioneered by telephone companies in the 1930s as the core modulation for frequency division multiplexing (FDM) for voice transmission over long distance channels. With its bandwidth saving property, SSB allows many voice signals (double the number in standard AM) to be transmitted via a single channel. This is usually done by separating the channel carriers by 4 KHz, thus offering a speech bandwidth of nominally 0.3 – 3.3 KHz (one extra KHz of bandwidth is reserved as guardband).

Nowadays, SSB is employed by many advanced communication technologies as a key component of quadrature amplitude modulation (QAM) and FDM. Most commonly, suppressed carrier SSB modulation is implemented in DSL modems and in ATSC (a digital TV standard developed by the American Television Systems Committee that uses MPEG-2 codec for transport).

The rest of this paper is organized as follows. In section II, we present a mathematical background of the Hilbert transform since it is a foundation block of the SSB modulator, highlighting a number of properties that we have independently developed. In section III, we present an overview of traditional schemes used in SSB signal generation and then present our novel scheme. In section IV, we conclude with practical considerations when implementing the different SSB schemes.

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II. Background on the Hilbert Transform

A. Definition

The Hilbert transform of a signal $m(t)$ is defined as [2]

$$\mathrm{H}\,m(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{m(\tau)}{\tau - t}d\tau\,, \tag{1}$$

or, simply stated, it is the convolution of $m(t)$ with $(\pi t)^{-1}$. Strictly speaking, the Hilbert transform is mathematically defined to be the limit of integrals taken over bounded intervals with regions around t deleted (in simpler terms, the integral is taken in the principal value sense), specifically,

$$\mathrm{H}\,m(t) = \frac{1}{\pi}\lim_{\varepsilon\to 0,\mathrm{L}\to\infty}\left(\int_{-\mathrm{L}}^{t-\varepsilon}\frac{m(\tau)}{\tau - t}d\tau + \int_{t+\varepsilon}^{\mathrm{L}}\frac{m(\tau)}{\tau - t}d\tau\right). \tag{2}$$

The definition of (2) resolves the apparent singularity at 0 for a suitable signal $m(t)$. The Hilbert transform of a square wave is illustrated in Fig. 1.

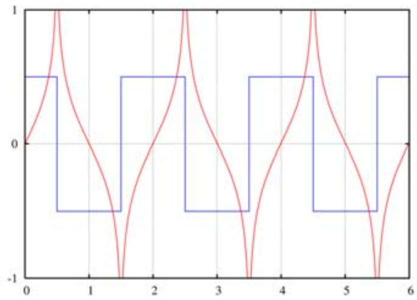

Fig. 1 The Hilbert transform of a square wave

B. Properties

The Hilbert transform enjoys a number of elegant properties.

1) Bounded operation: The Hilbert transform is a *bounded*

operator on $L^2(\boldsymbol{R})$:

$$\|\mathrm{H}m(t)\| \le \|m(t)\|, m(t) \in L^2(\boldsymbol{R}) . \tag{3}$$

2) Linearity and superposition:

$$\mathrm{H}\left(\sum_{i=1}^{N} \alpha_i x_i(t)\right) = \sum_{i=1}^{N} \alpha_i \mathrm{H} x_i(t) . \tag{4}$$

3) The Hilbert transform of a Hilbert transform is the negative of the original signal:

$$\mathrm{H}^2 m(t) = -m(t) , \tag{5}$$

or, more generally,

$$\begin{aligned} \mathrm{H}^n m &= -m \ (n = 2, 6, 10, \cdots); \mathrm{H}m (n = 1, 5, 8, \cdots); \\ &- \mathrm{H}m (n = 3, 7, 11, \cdots); m (n = 4, 8, 12, \cdots) \end{aligned} \tag{6}$$

This results from the fact that two successive 90° phase shifts causes a total 180° phase shift or a sign reversal.

4) Orthogonality:

$$\int_{-\infty}^{\infty} m(t) \mathrm{H}m(t) dt = 0, m(t) \in \boldsymbol{R} . \tag{7}$$

This property results from the fact that the product of in-phase and quadrature-phase components integrates to 0, noting that a cosine wave is Hilbert transformed into a sinewave and vice versa (except for sign change).

5) Differentiation:

$$\mathrm{H}m'(t) = (\mathrm{H}m)'(t) , \tag{8}$$

that is, the Hilbert transform of the derivative is the derivative of the Hilbert transform.

6) Convolution: $\mathrm{H}(f * g) = \mathrm{H}f * g = f * \mathrm{H}g$, or

$$\begin{aligned} &\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\int_{-\infty}^{\infty} f(\tau - \zeta) g(\zeta) d\zeta}{t - \tau} d\tau = \\ &\int_{-\infty}^{\infty} \mathrm{H}f(t - \tau) g(\tau) d\tau = \int_{-\infty}^{\infty} f(t - \tau) \mathrm{H}g(\tau) d\tau \end{aligned} . \tag{9}$$

This means that the Hilbert transform of the convolution of two signals is the convolution of one signal with the Hilbert transform of the other signal.

7) Energy conservation:

$$\int_{-\infty}^{\infty} |m(t)|^2 dt = \int_{-\infty}^{\infty} |\mathrm{H}m(t)|^2 dt . \tag{10}$$

This results from the fact that the Hilbert transform causes a phase-shift in the signal without changing the amplitude of the spectrum, and therefore the energy of the signal is preserved.

8) Even/odd symmetry/antisymmetry:

$$\begin{aligned} m(t) = m(-t) &\Rightarrow \mathrm{H}m(t) = -\mathrm{H}m(-t), \\ m(t) = -m(-t) &\Rightarrow \mathrm{H}m(t) = \mathrm{H}m(-t) \end{aligned} , \tag{11}$$

that is, the Hilbert transform of an even function is an odd function and the Hilbert transform of an odd function is an even function.

9) Baseband and passband mixing:
If $m(t)$ is a low pass signal (baseband) and $s(t)$ is a bandpass signal with non overlapping spectra, then

$$\mathrm{H}\{m(t)s(t)\} = m(t)\mathrm{H}s(t) . \tag{12}$$

10) Scaling:

$$\mathrm{H}\{m(at)\} = \mathrm{sgn}(a)\mathrm{H}m(at) . \tag{13}$$

11) Time shifting:

$$\mathrm{H}\{m(t - t_0)\} = \mathrm{H}m(t - t_0) . \tag{14}$$

We note that properties (8), (9), (13), and (14) have been independently developed by the author of this paper, and to the best of our knowledge, treatment of these properties is not available elsewhere in the literature. On another note related to notation, the Hilbert transform $\mathrm{H}m(t)$ symbolized by a Euclidean H is also denoted by $m(t)$.

C. Table of Transformation

The Hilbert transform of a few basic functions is provided in Table I. We note that the Hilbert transforms of the functions Rect(t), sinc(t), e^{-t^2}, $\left(t^2+1\right)^{-1}$, $\left(\pi(t-t_0)\right)^{-1}$, $\delta'(t)$, and of the Rep(.) and Comb(.) operators have been independently derived by the author of this paper. In Table I, Rep(.) and Comb(.) are respectively the replica and comb operators, and the function erfi(t) is the imaginary error function, also expressed in terms of the real error function as –$j$erf(jt). It is worth mentioning that the Hilbert transform is an *intra*-temporal transform. In contrast, the Fourier transform is *inter*-domain (time-to-frequency domains).

TABLE I
HILBERT TRANSFORM OF SOME BASIC FUNCTIONS AND OPERATORS

$z(t)$	$\mathrm{H}z(t)$ or $\hat{z}(t)$
K	0
$\delta(t-t_0)$	$\frac{1}{\pi(t-t_0)}$
$\delta'(t-t_0)$	$-\frac{1}{\pi(t-t_0)^2}$
$m(t)\cos(2\pi f_c t+\theta)$	$m(t)\sin(2\pi f_c t+\theta)$
$m(t)\sin(2\pi f_c t+\theta)$	$-m(t)\cos(2\pi f_c t+\theta)$
$e^{j2\pi f_0 t}$	$-je^{j2\pi f_0 t}$
$\mathrm{Rect}(t)$	$\frac{1}{\pi}\ln\left\lvert\frac{t+\frac{1}{2}}{t-\frac{1}{2}}\right\rvert$
$\mathrm{sinc}(t)$	$\frac{1-\cos(\pi t)}{\pi t}$
$\frac{1}{t^2+1}$	$\frac{t}{t^2+1}$
$\frac{1}{\pi(t-t_0)}$	$-\delta(t-t_0)$
$\underset{T}{\mathrm{Rep}}\left[m(t)\right]$	$\underset{T}{\mathrm{Rep}}\left[\mathrm{H}m(t)\right]$
$\underset{T}{\mathrm{Comb}}\left[m(t)\right]$	$\frac{1}{\pi}\sum_{n=-\infty}^{\infty}\frac{m(nT)}{t-nT}$
e^{-t^2}	$e^{-t^2}\mathrm{erfi}(t)$
$\mathrm{Re}(m(t))$	$\mathrm{Im}(m(t))$
$\mathrm{Im}(m(t))$	$-\mathrm{Re}(m(t))$

III. SSB SYSTEM DESIGN

A. Conventional SSB Signal Generators

The *basic method* of generating an SSB-AM signal is to remove one of the sidebands of a double side band (DSB) generated signal using a bandpass filter (BPF), leaving only either the upper sideband (USB) or lower sideband (though less conventional).

An alternate method is to suppress the unwanted sideband via phase shift filters or Hilbert transformation. The device that generates SSB signals under this scheme is called a *quadrature modulator* or *phase discriminator*. This method is illustrated by the block diagram of Fig. 2, where the modulating signal $m(t)$ is multiplied by (or mixed with) the sinusoid carrier $\cos(wt)$ (or $\cos(2\pi f_c t)$), generated by an oscillator. The signal $m(t)$ is also Hilbert transformed or passed through a quadrature phase shift filter (QPSF) and the output $\hat{m}(t)=\mathrm{H}m(t)$ is multiplied by the quadrature carrier $\sin(wt)$. When the two product signals are subtracted (upper product minus lower product), the result is a USB-SSB signal. On the other hand, if the two product signals are added, an LSB-SSB signal is obtained [3-5].

B. Proposed SSB Signal Generator

The functional block diagram of our proposed SSB modulator is shown in Fig. 3. We refer to this method as

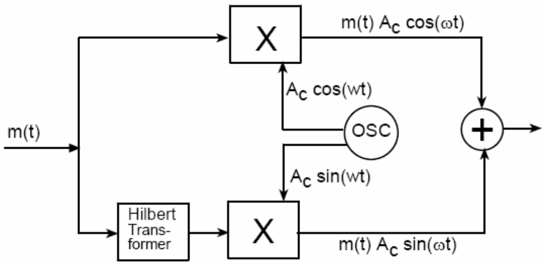

Fig. 2 SSB signal generation using the phasing method

hybrid phasing-bandpass filtering. In this system, two versions of the original modulating signal $m(t)$ are generated which are mutually 90^o out of phase. The signal m(t) is added to the quadrature carrier $\sin(2\pi f_c t)$ generated by an oscillator, whereas the Hilbert transformed signal $\hat{m}(t)$ is added to the carrier $\cos(2\pi f_c t+\pi)$ produced by passing the quadrature carrier through a quadrature phase shift filter (QPSF). By multiplying the resulting aggregated signals and then passing the product signal through a bandpass filter with cutoffs at (2/3)f_c and (4/3)f_c, an upper sideband signal results.

Key to the proper operation of this scheme is to select a real-valued baseband signal $m(t)\in\Re$ with bandwidth $B_m \leq f_c/3$ and the observation that the spectrum of the product signal $m(t)m(t)$ is zero in the Nyquist domain $\lvert f\rvert \geq 2B_m$.

We now study the practical implication of these methods.

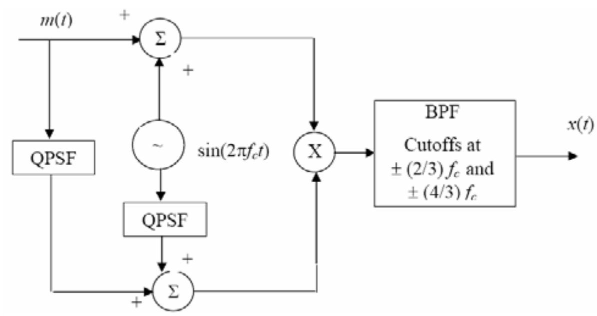

Fig. 3 Novel hybrid phasing-bandpass filtering SSB scheme

IV. PRACTICAL CONSIDERATIONS

A. Conventional Basic BPF Method

Although simple and low cost, the basis method is hampered by the use of a BPF with a sharp cutoff at the carrier frequency f_c. This is an *ideal* formulation that requires very high precision which cannot be realized in practice. Besides, sharp filters are most sensitive to temperature, humidity change, and physical damage, rendering them unattractive for communication systems.

Even if an exact sharp BPF cutoff is achieved, the transmitted SSB signal would still be very sensitive to frequency deviation (as little as this might be). Since the carrier frequency is exactly at the edge of the BPF, a small frequency shift outside the BPF bandwidth will inhibit the receiver from recovering the carrier. Frequency shifts are usually caused by Doppler shift or oscillator drift and can cause frequency shift to an adjacent channel, thus causing interference. Oscillator drift is an arbitrary offset of an oscillator's frequency from its nominal frequency, caused by changes in temperature which can alter the piezoelectric effect in a quartz filter (used in BPF design).

B. Conventional Phasing Method

The phasing method relies heavily on the use of Hilbert transformers (HT) or quadrature phase shift filter (QPSF). The drawback of HT is that the impulse response $1/(\pi t)$ is a non-causal filter and therefore cannot be implemented as is. The filter also has infinite support which can cause problems in some applications. Another problem is the DC component (or zero frequency), but this can be avoided by ensuring that the signal does not contain a DC component. Despite all these obstacles, Hilbert transformers can be digitally implemented (approximately), as we will discuss in the next section.

Another drawback of the phasing method is the use of two multiplying devices (mixers). Such devices (along with dividers and square-root devices) suffer from poor frequency stability and poor selectivity (i.e., high sensitivity to small difference). Since the VCO output is multiplied by the modulating message, the output of the VCO must be a smooth noise-free voltage signal. Any noise on this signal causes frequency drift and slow response time.

Noting that a quadrature filter

$$q(t) = \left(\delta(t) + \frac{j}{\pi t}\right) * m(t) \tag{15}$$

is the analytical signal of the real-valued signal $m(t)$, the SSB-LSB signal can be conceived as a quadrature filter at certain time instants. Set $\theta(t) = 2\pi f_c t$ and consider $\theta(t) = (n+1/4)\pi, n = 0, \pm1, \pm2, \cdots$, or $t_n = (n+1/4)/2f_c$, we then obtain

$$\begin{aligned} x_{SSB}^{(LSB)}(t_n) &= m(t_n)\cos(\theta(t_n)) + jm(t_n)\sin(\theta(t_n)) \\ &= \pm\frac{\sqrt{2}}{2}\left(m(t_n) + jm(t_n)\right) = Gq(t_n). \end{aligned} \tag{16}$$

Thus the SSB-LSB signal at the time instants $t = t_n = (n+1/4)/2f_c$ is a quadrature filter (except for a trivial constant gain $G = \pm\sqrt{2}/2$), that is, $x_{SSB}^{(LSB)}(t_n) = Gq(t_n)$. The practical implications of this is that a quadrature filter is difficult to approximate as a filter which is either causal or of finite support, or both.

C. Novel Hybrid Phasing-BPF Method

The functional block diagram of our proposed SSB scheme (Fig. 3) utilizes 2 op-amp summers and 1 mixer. By comparison, the classical phasing SSB generator utilizes 2 mixers and 1 op-amp summer (Fig. 2). Addition is a simpler operation than multiplication and is easier to implement electronically (multiplication problems have been already discussed in the previous section). The advantage of this system comes, however, at the cost of the added cost of a BPF. Such BPF does not face the problems of the conventional basic BPF method because the carrier frequency is not at the edge of the filter (cutoffs at 2/3 f_c and 4/3 f_c), making it *robust* to possible erroneous frequency shifts. In addition, such a BPF can be efficiently implemented using a crystal filter (a special form of a quartz filter) with exceptionally high frequency *stability* and high *selectivity*. The crystal filter provides a very precisely defined center frequency and very steep bandpass characteristics with a very high Q-factor (quality factor).

The drawback of using Hilbert transformers can be easily overcome if the HT is digitally implemented via approximate discrete Fourier transform (DFT) as shown in Fig. 4.

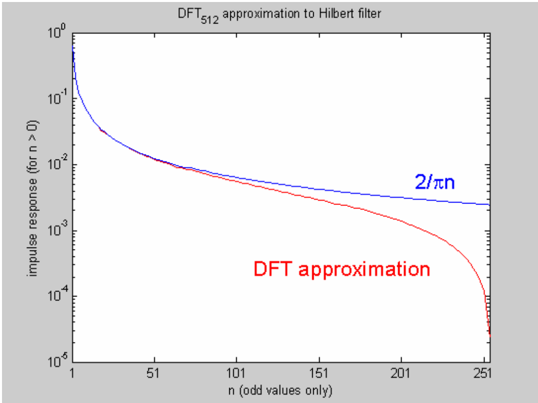

Fig. 4 Discrete Hilbert transform
(ordinate axis logarithmically scaled)

Another obstacle that might be faced by our proposed method is amplitude compression by the transmitter. This amplitude can be easily restored by an expander at the receiver if ACSB (amplitude compandored sideband) is employed (a narrowband modulation using a single sideband with an added pilot tone).

REFERENCES

[1] L. W. Couch II, *Digital and Analog Communication Systems*, Fifth Edition, Prentice Hall, Upper Saddle River, NJ, 1997.
[2] S. Haykin, *Communication Systems*, John Wiley & Sons, 2000.
[3] B. Sklar, *Digital Communications Fundamentals and Applications*, Prentice Hall Editions, 2002.
[4] B. P. Lathi, *Modern Digital and Analog Communication Systems*, Oxford University Press, 2003.
[5] J. G. Proakis and M. Salehi, *Communication Systems Engineering*, Prentice Hall, 1st edition, 1994.