# Control Strategy for an Active Suspension System

C. Alexandru, and P. Alexandru

Abstract—The paper presents the virtual model of the active suspension system used for improving the dynamic behavior of a motor vehicle. The study is focused on the design of the control system, the purpose being to minimize the effect of the road disturbances (which are considered as perturbations for the control system). The analysis is performed for a quarter-car model, which corresponds to the suspension system of the front wheel, by using the DFC (Design for Control) software solution EASY5 (Engineering Analysis Systems) of MSC Software. The controller, which is a PID-based device, is designed through a parametric optimization with the Matrix Algebra Tool (MAT), considering the gain factors as design variables, while the design objective is to minimize the overshoot of the indicial response.

Keywords—Active suspension, Controller, Dynamics, Vehicle

#### I. INTRODUCTION

THE suspension mechanism determines the position of the ■ wheels relative to the car body, and it overtakes the contact forces between wheels/tires and road. Equipping vehicles with advanced suspension systems, able to make a barrier of vibration and noise, became a necessity, more so as the speed of the vehicles on bumpy roads is not limited by the propulsion system performance, but rather the quality of the suspension system. Evaluation of the suspension quality in terms of comfort involves taking into account a variety of functional situations for the vehicle and a number of criteria by which to define and measure the quality parameters of the suspension system. Generally, the quality of comfort is difficult to be quantified, being also a psychological concept. The stability parameters can be more easily determined and measured, the evaluation being performed on the basis of generally accepted criteria, even if these criteria can sometimes become contradictory. The recent progresses in microelectronics, sensor technology and actuating systems favor the design of intelligent suspension systems, which provides improved comfort and stability relative to the traditional (passive) suspensions. An onboard computer detects body movement from sensors located throughout the vehicle and, using data calculated by opportune control techniques, controls the action of the suspension [1]. The dynamic behavior of a suspension system can be changed by modifying the characteristics of the springs and dampers, or by changing the properties of the compliant joints. The passive suspension system has inherent

limitations as a consequence of the choice of elastic & damping characteristics to ensure an acceptable behavior for the entire working frequency range. As is known, a highdamping system has an acceptable behavior around the resonant frequency, but poorly at values far away from this frequency, while a low damping system behaves inversely. The need to obtain a compromise between these conflicting requirements justify the research of the intelligent systems, where the elastic and damping characteristics can be controlled in closed-loops using external power sources and actuators controlled in feedback.In the case of passive suspension, the system characteristics remain constant, and the behavior is affected only by the physical quantities that directly affect the response. In addition, the behavior of the intelligent suspension systems depends on the physical quantities that do not directly affect behavior. A physical quantity which directly affects the response of the suspension system is the compression - extension speed of the damper, while the roll or pitch angular velocity of the car body can be considered as a measure that has no direct effect on the suspension system function. The intelligence of the suspension system is determined by a controller that takes data from the vehicle dynamics and transmits signals to the suspension system (feedback control). The response of the passive suspension (fig. 1,a) at a given excitation (e.g. road bump) is only affected by the excitation and system parameters that have a direct action on the suspension system. Instead, the intelligent suspension (fig. 1,b) is affected by indirect parameters, such as acceleration of the roll, pitch or vertical oscillations. The way to implement intelligence in a suspension system is to use variable damping (fig. 2,a), or to create a counter-force/counter-movement system (fig. 2,b).

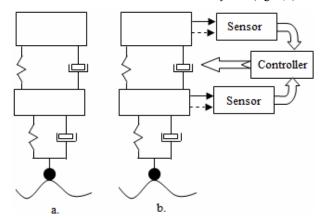


Fig. 1 Passive (a) and intelligent (b) suspension systems

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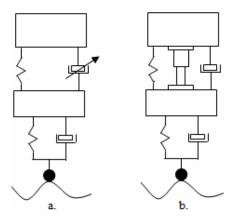


Fig. 2 Variable damping (a) and counter-force (b) systems

Active suspensions can be generally divided into two main classes: pure active suspensions and semi-active suspensions. Active suspensions use separate actuators which can exert an independent force on the suspension to improve the riding characteristics. Semi-active systems can only change the damping coefficient of the shock absorber, and do not add external energy to the system [2]-[4]. Obviously, an active suspension system provides superior dynamic performance compared to passive system, but it involves a more complex and expensive system, and consumes far more energy. When designing an active suspension, two important issues must be considered: the possible failure of the external energy source, and the transfer of a large quantity of mechanical energy in a structure that has the potential to destabilize the controlled system. However, the active suspension systems significantly improve car comfort, handling performance and driving safety, realizing an improved compromise among different vibrations modes of the vehicle (bounce, roll, pitch) [5]–[7].

In this paper, the design and simulation of an active suspension system is approached. The analysis is performed for a quarter-car model, which corresponds to the suspension system of the front wheel. The study is focused on the design & optimization of the control system, the purpose being to minimize the effect of the road disturbances (which are considered as perturbations for the control system). The mechatronic system was designed by using the DFC (Design for Control) software solution EASY5 of MSC Software. An important issue is related to the controller design, which is performed using a parametric optimization with the Matrix Algebra Tool (MAT).

## II. DESIGNING THE CONTROL SYSTEM

In the relative motion to car body, the front and rear wheels of the motor vehicles are usually guided by linkage mechanisms, which contain binary links and/or kinematics chains. A passive suspension contains an energy dissipating element, which is the damper, and an energy-storing element, which is the spring. In addition, the suspension includes other elements with elastic and damping properties, such as the bumpers, rebound elements, and anti-roll bar.

In this paper, the analysis is performed for a 2-DOF quarter car model, which is one of the most addressed models in the literature [2], [4], [8]–[10], being useful for preliminary design, before the development of the full-vehicle model. For the model in study, which corresponds to the suspension system of a front wheel (fig. 3), the guidance is assured by a four-bar mechanism. This uses two control arms to hold the wheel carrier and control its movements. The lower and upper wishbones are connected to the car body through bushings. Spherical joints constrain the wheel carrier to the control arms. Tie rod attaches to the steering part and to the wheel carrier through spherical joints. Revolute joint connect the wheel carrier to the wheel mount part. The spring is disposed between the lower strut of the damper and the car body. The car body equilibrium is assured with a translational joint to ground along the vertical axis, in the median plane of vehicle.



Fig. 3 The suspension system with four-bar guiding mechanism

By placing a force actuator, in parallel to passive suspension, an active suspension system is obtained (see fig. 2,b). The active suspension uses sensors to measure the accelerations of sprung mass and unsprung mass, the analog signals from the sensors being transmitted to the controller, which communicates with the force actuator. The simplified model of the quarter-car active suspension is shown in figure 4, with the following notations: m<sub>2</sub> - the sprung mass (the quarter car body);  $m_1$  - the unsprung mass (the wheel assembly, including the guiding mechanism); k2, c2 - the stiffness and damping coefficients of the passive spring and shock absorber; k<sub>1</sub>, c<sub>1</sub> - the stiffness and damping properties of the tire;  $z_2$  - the car body travel;  $z_1$  - the wheel travel; z - the road disturbance; u - the force generated by the actuator. For this paper, we have considered the following values of the parameters:  $m_1$ =60 kg,  $m_2$ =350 kg,  $k_1$ =255000 N/m,  $c_1$ =2500 Ns/m,  $k_2$ =45000 N/m,  $c_2$ =3500 Ns/m; these correspond to the suspension system of a domestic off-road vehicle.

The differential dynamic equations for the active suspension system shown in figure 4 have been developed considering the Newton-Euler formalism, as follows:

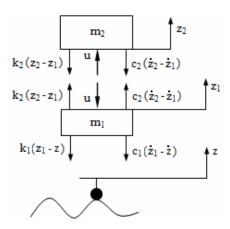


Fig. 4 The simplified quarter-car model of the active suspension

$$m_2 \ddot{z}_2 = -c_2 (\dot{z}_2 - \dot{z}_1) - k_2 (z_2 - z_1) + u,$$
  

$$m_1 \ddot{z}_1 = c_2 (\dot{z}_2 - \dot{z}_1) + k_2 (z_2 - z_1) - u + c_1 (\dot{z} - \dot{z}_1) + k_1 (z - z_1).$$
(1)

Equations (1) can be rewritten in matrix form using the Laplace transform:

$$\begin{bmatrix} m_2 s^2 + c_2 s + k_2 & -(c_2 s + k_2) \\ -(c_3 s + k_2) & m_3 s^2 + (c_1 + c_2) s + (k_1 + k_2) \end{bmatrix} \cdot \begin{bmatrix} Z_2 \\ Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ c_1 s + k_1 \end{bmatrix} \cdot Z + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \cdot U. \quad (2)$$

This is a MIMO (Multi-Input Multi-Output) system (fig. 5,a), having as inputs the road disturbance (z) and the force control signal (u), while the outputs are the wheel travel ( $z_1$ ) and the vertical displacement of the sprung mass ( $z_2$ ). We have selected as performance index for the dynamic response the relative displacement  $z_2$ - $z_1$  (fig. 5,b). Because this is a linear system, we can use the superposition principle: the output  $z_2$ - $z_1$  is the combined effect of the input signals z and u (fig. 5,c). In these terms, the transfer functions  $G_Z(s)$  and  $G_U(s)$  are obtained from (2):

$$G_{U}(s) = \frac{Z_{2} - Z_{1}}{U} = \frac{(m_{1} + m_{2})s^{2} + c_{1}s + k_{1}}{\Delta},$$

$$G_{Z}(s) = \frac{Z_{2} - Z_{1}}{Z} = \frac{-m_{2}c_{1}s^{3} - m_{2}k_{1}s^{2}}{\Delta},$$

$$\Delta = (m_{2}s^{2} + c_{2}s + k_{2}) \cdot (m_{1}s^{2} + (c_{1} + c_{2})s + (k_{1} + k_{2})) - (c_{2}s + k_{2})^{2}.$$

$$\frac{Z}{u} = Suspension \begin{cases} Z_{1} & Z \\ Suspension \end{cases}$$

$$Suspension \begin{cases} Z_{2} - Z_{1} \\ Suspension \end{cases}$$

$$Suspension \begin{cases} Z_{2} - Z_{1} \\ Suspension \end{cases}$$

$$Suspension \end{cases}$$

Fig. 5 The modeling of the MIMO system

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The response of the passive suspension (considering u=0 in figure 5), for a step input signal with the amplitude 0.1 m, is shown in figure 6, the simulation being performed by using the DFC (Design for Control) environment EASY5 of MSC Software. There are the following performance indexes of the passive suspension:  $\sigma \cong 30\%$  (overshoot),  $t_s \cong 0.9s$  (settling time). The car body oscillates on a period less than 1 second, the maximum elongation (relative to the wheel axis) being around 0.03 m. For a comfortable suspension, the settling time should be less than 2 seconds, while the maximum acceptable value for the overshoot is 5% [11]. In these terms, the control system of the active suspension aims to decrease the overshoot, without adversely affecting the settling time.

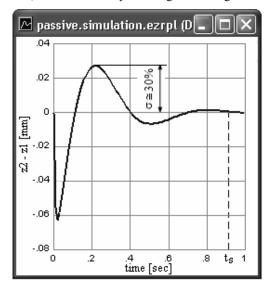


Fig. 6 The indicial response of the passive suspension

The control strategy is developed considering the road disturbance (i.e. the input signal z) as a perturbation to be eliminated by the control system, which generates the force signal u (fig. 7,a). Using the diagrams' algebra, the block scheme of the control system is transformed in one shown in figure 7,b. The transfer function of the controlled system is  $G_U$ , the perturbation being filtered by the transfer function  $G_F = G_Z/G_U$ . The inconvenient of the transfer function  $G_F(s)$  is that the denominator degree is less than the numerator degree, therefore the system is undetermined. The solution is to transform the step signal into a pulse signal, in this way the denominator having the same degree as the numerator. The amplitude and duration of the pulse signal have been chosen in accordance with the step signal value (0.1 m).

Considering the numerical values of the suspension's parameters, the transfer function  $Z(s) \cdot G_F(s)$  is obtained:

$$Z(s) \cdot G_F(s) = \frac{1}{s} \cdot \frac{-112.9s^3 - 7258s^2}{0.03467 s^2 + 0.02721 s + 17.48} =$$

$$= 1 \cdot \frac{-112.9s^2 - 7258 s}{0.03467 s^2 + 0.02721 s + 17.48}.$$
(4)

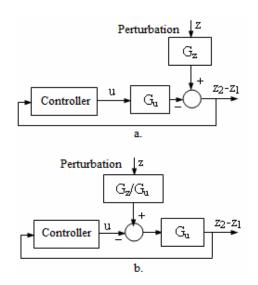


Fig. 7 The block scheme of the control system

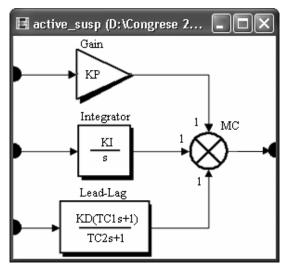


Fig. 8 The block model of the controller

For controlling the active suspension, we have designed a PID-based regulator, whose model in EASY5 is shown in figure 8. Lead-Lag block models a continuous first order transfer function, using the time constant format. The numerator time constant (TC1) is used as a lag time constant to calculate an approximate derivative from the signal, while the denominator time constant (TC2) is used to help prevent an implicit loop. The time constant represents the time it takes the system's response to reach 1-1/e  $\approx$  63,2% of its final value, where "e" is the mathematical constant (e= 2,71828.....). In these terms the block diagram of the control system, which was developed using EASY5, is shown in figure 9, in accordance with the control scheme in figure 7,b.

The tuning of the controller, in order to establish the values of the gain factors ( $K_P$  - proportional gain,  $K_I$  - integral gain,  $K_D$  - derivative gain), was made by performing a parametric optimization with the Matrix Algebra Tool (MAT). This is an interactive tool which is specifically designed to be used in

conjunction with EASY5 to perform such tasks as control system design, model data preparation, and post processing of simulation results. For beginning, the control system was exported as MAT EMX function. In this way, MAT and EASY5 will be used together to design the controller. The design variables used in the optimization study are the gain factors of the controller, while the design objective is to minimize the indicial response of the suspension (i.e. the relative displacement between the sprung and unsprung masses,  $z_2$ - $z_1$ ), mainly the overshoot of the response.

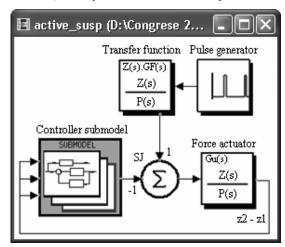


Fig. 9 The control system diagram of the active suspension

### III. OPTIMIZING THE CONTROLLER

The optimization study is made in two stages: performing parametric studies, and optimizing the controller. The parametric studies represent sets of simulations that help to adjust a parameter to measure its effect on the performance of the controller, by sweeping the variable through a range of values and then simulating the behavior of the various designs in order to understand the sensitivity of the overall system to these design variations. As result, the parametric studies allow to identify the main design variables, with great influence on the design objective.

The parametric studies have been successively performed for each design variable, in the variation field "1,...,106", keeping the rest of the variables fixed at their nominal value "0". For performing the parametric studies, a function script (\*.ezemf) has been developed in MAT. The script contains the instructions used to setup and run multiple simulations while varying a design variable, as follows: defining the plot and simulation commands (the instructions "setup plot" and "setup sim"), defining the loop used to change the gain values ("for"), defining the design variables to be used in the simulation ("sprintf"), loading the gain value, setting the plot parameters and the simulation parameters ("simulate"), loading the EASY5 plot data into MAT ("load plot"), and setting the plot data ("label data", "sprint", "plot", drawplot"). In these terms, the results of the parametric studies for the gain factors of the controller are shown in figures 10-12.

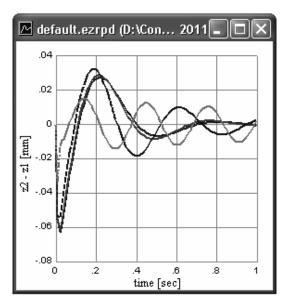


Fig. 10 The parametric study for the proportional gain (K<sub>P</sub>)

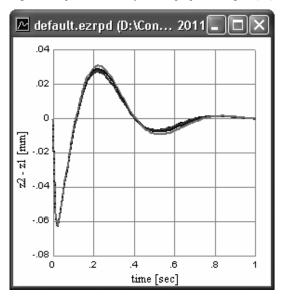


Fig. 11 The parametric study for the integral gain (K<sub>I</sub>)

From these results we can conclude that the main variables, with great influence on the design objective, are the proportional and derivative gains of the controller, while the integral gain is a secondary variable. Afterwards, the optimization study is performed considering the main design variables ( $K_P$  and  $K_D$ ). For performing the optimization, the "minimize\_v" function has been used. This function has the following syntax,  $[x,f] = minimize\_v(funcname, x0, H0, tol, delx)$ , where: funcname - name of function used to setup minimization, x0 - initial guess for minimizer, t0 - initial guess for Hessian, tol - relative tolerance for t0, delx - relative step size for computing gradients by differencing, t0 - minimizer of the function, or last search position, t0 - value of the function at t0.

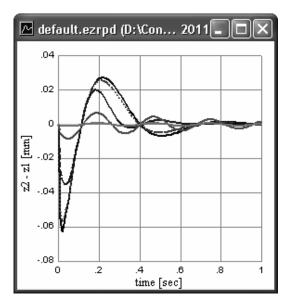


Fig. 12 The parametric study for the derivative gain (K<sub>D</sub>)

The next step is to create a MAT function that will perform the optimization we require. This function will take the proportional and derivative gains of the controller model as inputs and return the root mean square of the indicial response of the active suspension. The script file that defines the minimize function contains MAT instructions for setting the values of the gains ("sprintf"), setting the initial time and conditions ("initial time", "initial conditions"), setting the integration time ("Tmax"), performing the integration ("calc\_xic", "simulate"), getting the output value ("get\_value"), and calculating the objective function to minimize ("mean", "sqrt").

The optimization is performed by calling the "minimize\_v" function with the minimizer function as the first argument. MAT will repeatedly call the minimizer function as it performs the minimization procedure. The minimize function will set the proportional and derivative gains appropriately, perform a simulation, and retrieve the output value. The function returns the error in the simulations, defined as the difference between the simulation and desired value. The final values of the design variables result in a simulation that meets the design requirements, as follows:  $K_P\cong 1.6e5$ ,  $K_D\cong 2e5$ ; therefore, there is necessary, and sufficient, a PD device to control the active suspension system.

With these values, the indicial response of the active suspension is shown in figure 13. The overshoot of the output signal is very small ( $\sigma \approx 1.5\%$ ), while the settling time remains around 1 second. These results are in the recommended field for a comfortable suspension, and this demonstrates the viability of the control strategy.

The final issue is to verify the frequency response of the active suspension system, which is the measure of the system's output spectrum in response to the input signal. The frequency response is characterized by the magnitude of the system's response and the phase, versus frequency.

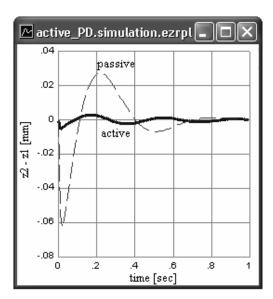


Fig. 13 The indicial response of the active suspension

We have obtained the frequency response by plotting the magnitude and phase measurements through the Bode plot, which is a graph of the transfer function of the system versus frequency, plotted with a logarithmic-frequency axis. In accordance with the plot shown in figure 14, the system acts as a filter, the maximum value of the resonance amplitude being -18 dB; consequently, the system filters out very well.

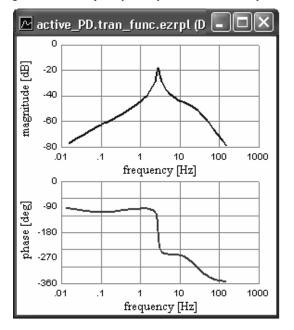


Fig. 14 The frequency response of the active suspension system

### IV. FINAL REMARKS

The future researches in the field will be focused on the research of more complex suspension models, such as half-car and full-car models, and for other control strategies & controller types (e.g. fuzzy logic). At the same time, we intend

to develop and simulate the virtual prototype of the active suspension system in the concurrent engineering concept. This is a control loop composed by the multi-body (MBS) mechanical model connected with the dynamic model of the actuator and with the controller model. The idea is to replace the transfer function  $G_Z$  from the basic control scheme shown in figure 7.a with the MBS model of the of the suspension mechanism (see figure 3), which is developed by using the virtual prototyping environment ADAMS. The mechanical model and the control system will be tested together (from one database), and in this way the risk of the control law being poorly matched to the real suspension can be minimized.

Finally, the virtual prototype will be refined by modeling the mechanical structure with finite elements, for identifying the eigenshapes and eigenfrequencies. Integrating the finite element model in the multi-body system analysis, we can quickly build a parametric flexible body representation of a component, analyze the system, make changes to the flexible body and evaluate the effect of the changes. This will help us to take quick decisions on any design changes without going through expensive hardware prototype building and testing.

#### ACKNOWLEDGMENT

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