

Construction and Analysis of Samurai Sudoku

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Abstract—Samurai Sudoku consists of five Sudoku square designs each having nine treatments in each row (column or sub-block) only once such the five Sudoku designs overlaps. Two or more Samurai designs can be joint together to give an extended Samurai design. In addition, two Samurai designs, each containing five Sudoku square designs, are mutually orthogonal (Graeco). If we superimpose two Samurai designs and obtained a pair of Latin and Greek letters in each row (column or sub-block) of the five Sudoku designs only once, then we have Graeco Samurai design. In this paper, simple method of constructing Samurai designs and mutually orthogonal Samurai design are proposed. In addition, linear models and methods of data analysis for the designs are proposed.

Keywords—Samurai design, Graeco samurai design, sudoku design, row or column swap.

I. INTRODUCTION

SUDOKU square design consists of treatments that are arranged in a square array such that each row, column or sub-square of the design contains each of the treatments only once [1]. The standard Sudoku square of order 9 entails, a 9×9 array of numbers 1 through 9, such that every row, every column and every 3×3 sub-block contains each number exactly once. Sudoku squares of any other order $k = pq$ are also similarly defined as an $k \times k$ array of numbers 1 through k such that every row, every column and every $p \times q$ internal block contains each number exactly once, see [2] and [3]. The basic properties of $k \times k$ Sudoku squares with construction procedure are discussed by [1]. In [1] and [4], Sudoku Square is made into a new design and was applied to field experiments. This design can make a layout of k treatments with k replications and control the three-way soil-environmental variation was discussed by [5]. A part from the construction procedure of Sudoku squares also presented the mathematical model and statistical method of analyzing data from a Sudoku square design [1]. They also compared the Latin square design with the Sudoku square design and stated that Sudoku adds a term of box effect in the source of variation, and that the soil-environment variation can be better controlled when the Sudoku square is used in a field experiment. It should show a smaller error, and more precise tests for treatment means. Particularly, when a field experiment is conducted on the area with lumpy soil variation, the Sudoku square design should be recommended [1]. In another approach, [6] discussed the construction of a class of Sudoku designs of order m^2 . They presented four different models for analyzing the data obtained from such design. They also gave an illustration of analysis and application these designs in various fields of agriculture. A paper presented by

[7] discussed the orthogonal Sudoku square. Two Sudoku squares of order m^2 are said to be mutually orthogonal Sudoku squares. If we superimpose the two Sudoku squares then one may get each pair of numbers only once as discussed in [7]-[9].

Construction of Sudoku and mutually orthogonal Sudoku squares has been considered by [2], [6]-[10]. These researchers used integers at different modules in the construction of Sudoku squares. For example, [6] outlined the following steps for construction of Sudoku designs of order m^2 sequentially:

1. Write the m^2 numbers from 1 to m^2 in a matrix form sequentially starting from row 1 to row m .
2. Write the m columns obtained in step 1, one by one to get a column of order m^2 .
3. Column 2 can be obtained from column 1 by adding 1 to each of its elements and reduce to mod m^2 if it exceeds the value m^2 . Proceed in the similar way to complete all the columns.

In construction of mutually orthogonal Sudoku squares, [9] considered $m^2 (= 2n + 1)$ to be the number of symbols coded by $(1, 2, 3, \dots, m^2)$ and is denoted by the set $S = (1, 2, 3, \dots, m^2)$. A pair of orthogonal Latin squares is obtained by generating the two initial rows mod(m^2), where m^2 and m are odd numbers, see [9].

One of the problems of these methods is that Latin letters cannot be used directly to represent treatments, and instead codes or numbers 1 to m^2 are used. If treatments are in Latin letter, addition of 1 to each elements (treatments) is impossible, since Latin letters are not real numbers or integers. In addition, only odd order Sudoku design can be constructed with this procedure, while if $k = p \times q$, where $p < q$, then this procedure cannot be used to construct Sudoku designs.

The application of Kronecker product (direct product) to two Latin squares $A_{n \times n}$ and $B_{m \times m}$ to obtain a Latin square $A \otimes B$ of order $n \times m$ is considered by [13]–[17]. Instead of using Kronecker product to Latin squares, [16] and [17] work with two Sudoku Latin squares of order ab and cd and then apply the row and column permutation to obtain a mutually orthogonal Sudoku Latin square of order $abcd$. The problem of this method is that some mutually orthogonal Sudoku Latin squares cannot be constructed, especially of order 4 and 9 (since there is no Sudoku square of order 2 or 3)

A new class of permutation matrices based on a tensor product of permutation matrices of reverse cyclic stride was proposed by [18]. A permutation matrix is a square matrix with one and only one-unit element in each row and column. In this method, permutation matrix is obtained by cyclic shifting of columns. The problems of this procedure are that matrix must have one and only one-unit element in each row

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and column, and rearrangement of elements of a given vector after multiplication by a permutation matrix. In addition, Latin letters cannot be used directly in this method.

Sudoku based space filling designs were studied by [11]. Reference [12] discussed joining several Sudoku squares to form what is called Samurai design. A Samurai design consists of five overlapping Sudoku grids, for which several entries are provided, and the remaining entries must be filled subject to each row, column and three-by-three sub-square containing the integers 1 to 9 precisely once [12].

This paper proposed a simple method of constructing of Sudoku design and Samurai designs by using the cyclic permutations of rows (or columns) of an initial sub-block of k treatments. Methods of data analysis from Samurai and orthogonal (Gaeco) Samurai designs are also proposed.

II. CONSTRUCTION

Let D be a block matrix and D_{11} a sub-block of D . Suppose that D contains n Latin letters. To construct a Sudoku design is to simply fix D_{11} in the first row and first column of D . Then to obtain D_{12} in the first row and second column of D , perform row swap of D_{11} in cyclic order and so on until each Latin letter occurs once in each row of the first row-block of D . Similarly, to obtain D_{21} in the second row and first column, perform column swap of D_{11} in cyclic order, and then obtain D_{22} in the second row and second column of D by performing row swap of D_{21} in cyclic order and so on until each Latin letter appears once in each row of the second row-block of D . Repeat this procedure until each Latin letter appears once in each column of the column-blocks of D . Then, the block matrix D obtained using this procedure is a Sudoku design.

Alternatively, Let A be an $n \times m$ matrix and B be a $p \times q$ matrix. The direct (or Kronecker) product of A and B (written $A \otimes B$) is defined as the $np \times mq$ matrix [19].

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1m}B \\ \vdots & \cdots & \vdots \\ a_{n1}B & \cdots & a_{nm}B \end{pmatrix}$$

Let A be an $n \times m$ matrix of ones. Suppose that B_i^r and B_j^c are sub-block matrices having the same elements with B , where $i, j = 1, 2, \dots, m$. Then, the from Kronecker product of the matrix of ones (A) and B , we can form a block matrix whose elements are sub-block matrices formed by row or column swap in a cyclic order. That is,

$$A \otimes B = \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \cdots & \vdots \\ 1 & \cdots & 1 \end{pmatrix} \otimes B = \begin{pmatrix} B & B_1^r & B_2^r & \cdots & B_{m-1}^r \\ B^{c_1} & B_1^{c_1r} & B_2^{c_1r} & \cdots & B_{m-1}^{c_1r} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ B^{c_{m-1}} & B_1^{c_{m-1}r} & B_2^{c_{m-1}r} & \cdots & B_{m-1}^{c_{m-1}r} \end{pmatrix}$$

Note that to construct a Sudoku square design using Kronecker product, A and B must be $n \times n$ matrices or $n = q$

and $m = p$ (i.e., $A_{q \times p}$ and $B_{p \times q}$) in the case where B is not a square matrix. To construct a mutually orthogonal Sudoku Square designs using this method, B must contain both the Latin and Greek letters and then perform the Kronecker product (modified) of A and B . Alternatively, if the elements of A are the Greek letters, and the elements of B are the Latin letters, then their Kronecker product gives all the elements of the Graeco Sudoku square, which after some arrangements can form mutually orthogonal Sudoku square or non-orthogonal square. An advantage of this method is that Latin letters, Greek letters, or integers can be used in the construction of Sudoku square designs. For example, if $k = 9$, $p = q = 3$, we have $k = 3^2$ and;

$$B = \begin{pmatrix} C & D & E \\ F & G & H \\ I & J & K \end{pmatrix}_{3 \times 3}$$

Then;

$$A \otimes B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \otimes B = \begin{pmatrix} C & D & E & F & G & H & I & J & K \\ F & G & H & I & J & K & C & D & E \\ I & J & K & C & D & E & F & G & H \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G & H & F & J & K & I & D & E & C \\ J & K & I & D & E & C & G & H & F \\ D & E & C & G & H & F & J & K & I \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ K & I & J & E & C & D & H & F & G \\ E & C & D & H & F & G & K & I & J \\ H & F & G & K & I & J & E & C & D \end{pmatrix}$$

To construct a samurai design, the row-blocks of D are swapped as in Fig. 1. An example of Samurai formed by D_{11} with 3×3 letters is presented in Fig. 2. Similarly, performing row swap of row-blocks of D and joining the grids with Fig. 1 will result in an extended Samurai design presented in Fig. 3. However, to construct Graeco Samurai design D_{11} must contains both Latin and Greek letter of which row (or column) swap of Greek letter be in the reverse cyclic order.

D_{11}	D_{12}	D_{13}		D_{21}	D_{22}	D_{23}
D_{21}	D_{22}	D_{23}		D_{33}	D_{32}	D_{31}
D_{31}	D_{32}	D_{33}	D_{32}	D_{11}	D_{12}	D_{13}
		D_{13}	D_{12}	D_{21}		
D_{21}	D_{22}	D_{23}	D_{22}	D_{31}	D_{32}	D_{33}
D_{31}	D_{32}	D_{33}		D_{11}	D_{12}	D_{13}
D_{11}	D_{12}	D_{13}		D_{21}	D_{22}	D_{23}

Fig. 1 Swap of row-blocks

Similarly, construction of other forms of Samurai designs can be made by using the above procedure. For example, if D_{11} is a 4×2 initial sub-block, then we have the Samurai in Fig. 2.

D ₁₁	D ₁₂	D ₁₃	D ₁₄			D ₁₃	D ₁₄	D ₁₁	D ₁₂
D ₂₁	D ₂₂	D ₂₃	D ₂₄	D ₂₁	D ₂₂	D ₂₃	D ₂₄	D ₂₁	D ₂₂
D ₁₂	D ₁₃	D ₁₄	D ₁₁	D ₁₂	D ₁₃	D ₁₄	D ₁₁	D ₁₂	D ₁₃
D ₂₁	D ₂₂	D ₂₃	D ₂₄			D ₂₄	D ₂₁	D ₂₂	D ₂₃

Fig. 2 Samurai for Sudoku with 4×2 sub-blocks

Example 1. Suppose that:

$$D_{11} = \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix}$$

Swapping row or columns of D_{11} gives the following Samurai design presented in Fig. 2. The resulted Samurai of Fig. 3 can be easily extended to give an extended Samurai design presented in Fig. 4.

A B C	D E F	G H I			B C A	E F D	H I G
D E F	G H I	A B C			E F D	H I G	B C A
G H I	A B C	D E F			H I G	B C A	E F D
B C A	E F D	H I G			C A B	F D E	I G H
E F D	H I G	B C A			F D E	I G H	C A B
H I G	B C A	E F D			I G H	C A B	F D E
C A B	F D E	I G H	F D E	A B C	D E F	G H I	
F D E	I G H	C A B	I G H	D E F	G H I	A B C	
I G H	C A B	F D E	C A B	G H I	A B C	D E F	
		G H I	D E F	B C A			
		A B C	G H I	E F D			
		D E F	A B C	H I G			
B C A	E F D	H I G	E F D	C A B	F D E	I G H	
E F D	H I G	B C A	H I G	F D E	I G H	C A B	
H I G	B C A	E F D	B C A	I G H	C A B	F D E	
C A B	F D E	I G H		A B C	D E F	G H I	
F D E	I G H	C A B		G H I	A B C	D E F	
I G H	C A B	F D E		B C A	E F D	H I G	
A B C	D E F	G H I		E F D	H I G	B C A	
D E F	G H I	A B C		H I G	B C A	E F D	
G H I	A B C	D E F					

Fig. 3 Samurai design

The following Extended Samurai Design is also formed with the above initial sub-block D_{11} :

A B C	D E F	G H I			B C A	E F D	H I G
D E F	G H I	A B C			E F D	H I G	B C A
G H I	A B C	D E F			H I G	B C A	E F D
B C A	E F D	H I G			C A B	F D E	I G H
E F D	H I G	B C A			F D E	I G H	C A B
H I G	B C A	E F D			I G H	C A B	F D E
C A B	F D E	I G H	F D E	A B C	D E F	G H I	
F D E	I G H	C A B	I G H	D E F	G H I	A B C	
I G H	C A B	F D E	C A B	G H I	A B C	D E F	
		G H I	D E F	B C A			
		A B C	G H I	E F D			
		D E F	A B C	H I G			
B C A	E F D	H I G	E F D	C A B	F D E	I G H	
E F D	H I G	B C A	H I G	F D E	I G H	C A B	
H I G	B C A	E F D	B C A	I G H	C A B	F D E	
C A B	F D E	I G H		A B C	D E F	G H I	
F D E	I G H	C A B		G H I	A B C	D E F	
I G H	C A B	F D E		B C A	E F D	H I G	
A B C	D E F	G H I		E F D	H I G	B C A	
D E F	G H I	A B C		H I G	B C A	E F D	
G H I	A B C	D E F					

Fig. 4 Extended Samurai Sudoku Design

Example 2. Suppose that:

$$D_{11} = \begin{pmatrix} A\alpha & D\theta & G\varepsilon \\ B\beta & E\phi & H\sigma \\ C\gamma & F\mu & I\lambda \end{pmatrix}$$

Performing row (or column) swapping in cyclic order for D_{11} as described above gives the Graeco Samurai design presented in Fig. 5.

Similarly, other forms of Samurai and Graeco Samurai designs can be constructed using this procedure. For example,

if $D_{11} = \begin{pmatrix} H & L \\ I & M \\ J & N \\ K & O \end{pmatrix}$, we have Samurai design presented in Fig. 6.

Similarly, we have Graeco Samurai design presented in Fig. 7 if

$$D_{11} = \begin{pmatrix} A\alpha & E\phi \\ B\beta & F\mu \\ C\gamma & G\varepsilon \\ D\theta & H\lambda \end{pmatrix}$$

III. ANALYSIS

Suppose that a Samurai design consists of g Sudoku square designs each of order k such that c sub-squares of the Sudoku designs overlapped. Then, the number of sub-squares of the samurai designs $r = g \times k - cs$, where s is the number of jointly Samurai designs, i.e., $s = 1$ for single Samurai design, $s = 2$ for two joint Samurai designs etc. The linear model proposed for Samurai square designs is as:

$$y_{(ij)lmx} = \mu + \theta_x + \alpha_i + \beta_{jx} + \delta_{lx} + \gamma_{mx} + \varepsilon_{(ij)lmx}$$

$$\left\{ \begin{array}{l} i = 1, 2, \dots, k \\ j = 1, 2, \dots, w \\ l = 1, 2, \dots, k \\ m = 1, 2, \dots, k \\ x = 1, 2, \dots, g \end{array} \right.$$

where $y_{(ij)lm}$ is an observed value of the plot in the l th row and m th column, subjected to the i th treatment, j th box of the x th Sudoku design; μ is the grand mean, $\alpha_i, \beta_j, \delta_l, \gamma_m, \theta_x$, are the main effects of the i th treatment, j th box, l th row, m th column, x th Sudoku design, respectively, ε_{ijm} is the random error.

Aα Dθ Gε	Bγ Eμ Hλ	Cβ Fφ Iσ				Bγ Eμ Hλ	Cβ Fφ Iσ	Aα Dθ Gε
Bβ Eφ Hσ	Cα Fθ Iε	Aγ Dμ Gλ				Cα Fθ Iε	Aγ Dμ Gλ	Bβ Eφ Hσ
Cγ Fμ Iλ	Aβ Dφ Gσ	Bα Eθ Hε				Aβ Dφ Gσ	Bα Eθ Hε	Cγ Fμ Iλ
Dε Gα Aφ	Eλ Hγ Bμ	Fσ Iβ Cφ				Eλ Hγ Bμ	Fσ Iβ Cφ	Dε Gα Aφ
Eσ Hβ Bφ	Fε Iα Cθ	Dλ Gγ Aμ				Fε Iα Cθ	Dλ Gγ Aμ	Eσ Hβ Bφ
Fλ Iγ Cμ	Dσ Gβ Aφ	Eε Hα Bθ				Dσ Gβ Aφ	Eε Hα Bθ	Fλ Iγ Cμ
Gθ Aε Dα	Hμ Bλ Eγ	Iφ Cσ Fβ	Gθ Aε Dα	Hμ Bλ Eγ	Iφ Cσ Fβ	Gθ Aε Dα	Hμ Bλ Eγ	Iφ Cσ Fβ
Hφ Bσ Eβ	Iθ Cε Fα	Gμ Aλ Dγ	Hφ Bσ Eβ	Iθ Cε Fα	Gμ Aλ Dγ	Hφ Bσ Eβ	Iθ Cε Fα	Gμ Aλ Dγ
Iμ Cλ Fγ	Gφ Aσ Dβ	Hθ Bε Eα	Iμ Cλ Fγ	Gφ Aσ Dβ	Hθ Bε Eα	Iμ Cλ Fγ	Gφ Aσ Dβ	Hθ Bε Eα
		Fσ Iβ Cφ	Dε Gα Aθ	Bγ Eμ Hλ				
		Dλ Gγ Aμ	Eσ Hβ Bφ	Cα Fθ Iε				
		Eε Hα Bθ	Fλ Iγ Cμ	Aβ Dφ Gσ				
Cβ Fφ Iσ	Aα Dθ Gε	Bγ Eμ Hλ	Cβ Fφ Iσ	Dε Gα Aθ	Eλ Hγ Bμ	Fσ Iβ Cφ		
Aγ Dμ Gλ	Bβ Eφ Hσ	Cα Fθ Iε	Aγ Dμ Gλ	Eσ Hβ Bφ	Fε Iα Cθ	Dλ Gγ Aμ		
Bα Eθ Hε	Cγ Fμ Iλ	Aβ Dφ Gσ	Bα Eθ Hε	Fλ Iγ Cμ	Dσ Gβ Aφ	Eε Hα Bθ		
Fσ Iβ Cφ	Dε Gα Aθ	Eλ Hγ Bμ						
Dλ Gγ Aμ	Eσ Hβ Bφ	Fε Iα Cθ						
Eε Hα Bθ	Fλ Iγ Cμ	Dσ Gβ Aφ						
Iφ Cσ Fβ	Gθ Aε Dα	Hμ Bλ Eγ						
Gμ Aλ Dγ	Hφ Bσ Eβ	Iθ Cε Fα						
Hθ Bε Eα	Iμ Cλ Fγ	Gφ Aσ Dβ						

Fig. 5 Graeco Samurai design

H L I M J N K O				J N K O H L I M
I M J N K O H L				K O H L I M J N
J N K O H L I M				H L I M J N K O
K O H L I M J N				I M J N K O H L
L H M I N J O K	L H M I N J O K	L H M I N J O K	L H M I N J O K	L H M I N J O K
M I N J O K L H	M I N J O K L H	M I N J O K L H	M I N J O K L H	M I N J O K L H
N J O K L H M I	N J O K L H M I	N J O K L H M I	N J O K L H M I	N J O K L H M I
O K L H M I N J	O K L H M I N J	O K L H M I N J	O K L H M I N J	O K L H M I N J
I M L N K O H L	I M J N K O H L	I M J N K O H L	I M J N K O H L	I M J N K O H L
J N K O H L I M	J N K O H L I M	J N K O H L I M	J N K O H L I M	J N K O H L I M
K O H L I M J N	K O H L I M J N	K O H L I M J N	K O H L I M J N	K O H L I M J N
H L I M J N K O	H L I M J N K O	H L I M J N K O	H L I M J N K O	H L I M J N K O
L H M I N J O K				O K L H M I N J
M I N J O K L H				L H M I N J O K
N J O K L H M I				M I N J O K L H
O K L H M I N J				N J O K L H M I

Fig. 6 Samurai design of 8x8 Sudoku design

Aα Eφ	Bθ Fλ	Cε Gγ	Dμ Hβ	Cγ Gε	Dβ Hμ	Aφ Eα	Bλ Fθ
Bβ Fμ	Cα Gφ	Dλ Hθ	Aε Eγ	Dθ Hλ	Aγ Eε	Bμ Fβ	Cφ Gα
Cγ Gε	Dβ Hμ	Aφ Eα	Bλ Fθ	Aα Eφ	Bθ Fλ	Cε Gγ	Dμ Hβ
Dθ Hλ	Aγ Eε	Bμ Fβ	Cφ Gα	Bβ Fμ	Cα Gφ	Dλ Hθ	Aε Eγ
Eλ Aθ	Fε Bγ	Gβ Cμ	Hα Dφ	Gμ Cβ	Hφ Dα	Eθ Aλ	Fγ Bε
Fφ Bα	Gλ Cθ	Hγ Dε	Eβ Aμ	Fα Bφ	Gλ Cθ	Hε Dγ	Eμ Aβ
Gμ Cβ	Hφ Dα	Eθ Aλ	Fγ Bε	Gβ Cμ	Hφ Dα	Eλ Aθ	Fε Bγ
Hε Dγ	Eμ Aβ	Fα Bφ	Gθ Cλ	Hγ Dε	Eμ Aβ	Fφ Bα	Gλ Cθ
Bθ Fλ	Cγ Gε	Dμ Hβ	Aφ Eα	Bλ Fθ	Cγ Gε	Dβ Hμ	Aα Eφ
Cα Gφ	Dθ Hλ	Aε Eγ	Bμ Fβ	Cφ Gα	Dθ Hλ	Aγ Eε	Bμ Fβ
Dβ Hμ	Aα Eφ	Bλ Fθ	Cε Gγ	Dμ Hβ	Aα Eφ	Bθ Fλ	Cγ Gε
Aγ Eε	Bβ Fμ	Cφ Gα	Dλ Hθ	Aε Eγ	Bβ Fμ	Cα Gφ	Dθ Hλ
Fφ Bα	Gλ Cθ	Hγ Dε	Eβ Aμ	Hφ Dα	Eλ Aθ	Fγ Bε	Gβ Cμ
Gμ Cβ	Hφ Dα	Eθ Aλ	Fγ Bε	Eμ Aβ	Fφ Bα	Gθ Cλ	Hγ Dε
Hε Dγ	Eμ Aβ	Fα Bφ	Gθ Cλ	Fε Bγ	Gμ Cβ	Hα Dφ	Eθ Aλ
Eλ Aθ	Fε Bγ	Gβ Cμ	Hα Dφ	Gλ Cθ	Hε Dγ	Eβ Aμ	Fα Bφ

Fig. 7 Graeco Samurai with 8x8 Sudoku

TABLE I
ANOVA OUTLINE FOR SAMURAI DESIGNS OF DATA FROM G SUDOKU
SQUARE DESIGNS OF ORDER 9

Source	df	ss
Sudoku squares	$g - 1$	SS_{ss}
Treatments	$k - 1$	SS_t
Rows	$g(k - 1)$	SS_r
Columns	$g(k - 1)$	SS_c
Sub-squares	$gk - cs - 1$	SS_s
Error	$gk(k - 3) - k + g + cs + 2$	SS_e
Total	$gk^2 - 1$	

Where SS_{ss} is the total sum of squares for Sudoku designs, SS_t is the total sum of squares for treatments, SS_r is the total sum of squares for rows, SS_c is the total sum of squares for columns, SS_s is the total sum of squares for sub-blocks and SS_e is the total error sum of squares.

The following linear model is proposed for Graeco Samurai design:

$$y_{(ij)lmx} = \mu + \theta_x + \alpha_i + \beta_j + \delta_{lx} + \gamma_{mx} + \phi_p + \varepsilon_{(ij)lmx} \begin{cases} i = 1, 2, \dots, k \\ j = 1, 2, \dots, w \\ l = 1, 2, \dots, k \\ m = 1, 2, \dots, k \\ x = 1, 2, \dots, g \\ p = 1, 2, \dots, k \end{cases}$$

where $y_{(ij)lm}$ is an observed value of the plot in the l th row and m th column, subjected to the i th treatment, j th box of the x th Sudoku design, μ is the grand mean, α_i , β_j , δ_l , γ_m , θ_x , are the main effects of the i th Latin letter, p th Greek letter, j th box, l th row, m th column, x th Sudoku design, respectively, ε_{ijm} is the random error.

TABLE II
ANOVA OUTLINE FOR GRAECO SAMURAI DESIGNS OF DATA FROM G
SUDOKU SQUARE DESIGNS ORDER 9

Source	df	ss
Sudoku squares	$g - 1$	$SSEE$
Latin letters	$k - 1$	SSL
Greek letters	$k - 1$	SSG
Rows	$g(k - 1)$	SS_r
Columns	$g(k - 1)$	SS_c
Sub-squares	$gk - cs - 1$	SS_s
Error	$gk(k - 3) + g - 2k + cs + 3$	SS_e
Total	$gk^2 - 1$	

Where SS_{ss} is the total sum of squares for Sudoku designs, SSL is the total sum of squares for Latin letters, SSG is the total sum of squares for Greek letters, SS_r is the total sum of squares for rows, SS_c is the total sum of squares for columns, SS_s is the total sum of squares for sub-blocks and SS_e is the total error sum of squares.

IV. CONCLUSION

In this paper, simple methods of constructing Samurai designs and orthogonal (Graeco) Samurai have been developed by cyclic permutation of row (or column) of matrix. Analyses of the designs were also discussed.

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