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Conjugate Heat transfer over an Unsteady Stretching Sheet Mixed Convection with Magnetic Effect

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Abstract**—**A conjugate heat transfer for steady two-dimensional mixed convection with magnetic hydrodynamic (MHD) flow of an incompressible quiescent fluid over an unsteady thermal forming stretching sheet has been studied. A parameter, M, which is used to represent the dominance of the magnetic effect has been presented in governing equations. The similar transformation and an implicit finite-difference method have been used to analyze the present problem. The numerical solutions of the flow velocity distributions, temperature profiles, the wall unknown values of f''(0) and $\theta'(0)$ for calculating the heat transfer of the similar boundary-layer flow are carried out as functions of the unsteadiness parameter (S), the Prandtl number (Pr), the space-dependent parameter (A) and temperature-dependent parameter (B) for heat source/sink and the magnetic parameter (M). The effects of these parameters have also discussed. At the results, it will produce greater heat transfer effect with a larger Pr and M, S, A, B will reduce heat transfer effects. At last, conjugate heat transfer for the free convection with a larger G has a good heat transfer effect better than a smaller G=0.

Keywords**—**Finite-difference method, Conjugate heat transfer, Unsteady Stretching Sheet, MHD, Mixed convection.

I. Introduction

THE boundary layer flow and conjugate heat transfer in a quiescent fluid driven by a continuous thermal forming stretching sheet with magnetic effect is significance in a number of industrial engineering processes, such as the drawing of a polymer sheet or filaments extruded continuously from a die, the cooling of a metallic plate in a bath, the extrusion of a polymer sheet from a die or in the drawing of plastic films, annealing and tinning of copper wires, the wire and fiber coating, etc. During the processes, mechanical properties are greatly dependent upon the rate of cooling.

Sakiadis [1] was the first study in the boundary layer flow generated by a continuous stretching surface moving with a constant velocity. Several authors [2–5] investigated the heat transfer problem in a stretching sheet with a linear or non-linear surface velocity and a uniform or different surface temperature condition. Abo–Eldahab and Aziz [6] extended the problem to involve a space-dependent exponentially decaying with internal heat generation or absorption. Abel et al. [7] and Bataller [8] presented the effects of non-uniform heat source on viscoelastic fluid flow and heat transfer over a stretching sheet. Moreover, Mukhopadhyay et al. [9], Pantokratoras [10] and Mukhopadhyay and Layek [11] extended to consider the effects of variable fluid properties or specific dimensionless parameters on the flow over a stretching sheet. The related boundary layer flow in the non-linear mechanics were studied by Rajagopal et al [12,13]. In all these above studies, the flow and temperature fields have been considered to be at steady state.

Some authors [14–17] studied the problem for unsteady stretching surface condition by using a similarity method to transform governing time-dependent boundary layer equations into a set of nonlinear ordinary differential equations. Some methods have been used to analysis the related unsteady stretching sheet problems, such as Sajid et al. [18], Mehmood et al.[19] and Liu and Andersson [20] that have used series solution method, homotopy analysis method, respectively. Recently, most authors [21-24] toward the second grade or viscoelastic fluids over an unsteady stretching sheet with heat transfer or others related effects by similar and non-similar analysis methods with numerical solution methods to solve such kinds of problems. Most recently, Tsai et al. [25] have studied a quiescent fluid flow and heat transfer over an unsteady stretching surface with non-uniform heat source by using similarity method and solved numerically by Chebyshev finite difference method (ChFD), but not consider the magnetic effect.

From above, provide the motivation for the present analysis to study the flow and conjugate heat transfer in an incompressible quiescent fluid caused by mixed convection with magnetic effect on a thermal forming stretching sheet. It is a point of view to examining the influence of flow and heat transfer characteristics phenomena. The magnetic force and buoyancy force is important in the present problem due to the difference among the previous studies. A similar derivation technique has been used and the resulting non-linear similar equations were solved by using the finite-difference method.

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II. Theoretical and Analysis

It was consider the unsteady two-dimensional magnetohydrodynamic (MHD) laminar flow of an incompressible quiescent fluid over a thermal forming stretching sheet. A constant magnetic field of strength **B0** and **g** is applied perpendicular to the thermal forming stretching sheet, which is a valid assumption on a laboratory scale under the assumption of small magnetic Reynolds number. Under the usual boundary layer assumptions and in the absence of pressure gradient, the unsteady basic boundary layer equations governing the MHD flow of quiescent fluid. Let us consider the unsteady, incompressible, two-dimensional MHD flow of a quiescent thin liquid film of uniform thickness h (t) over the horizontal thermal forming stretching sheet. The fluid motion within the film is due to stretching of the elastic sheet. The geometry of the problem is shown in Fig. 1.

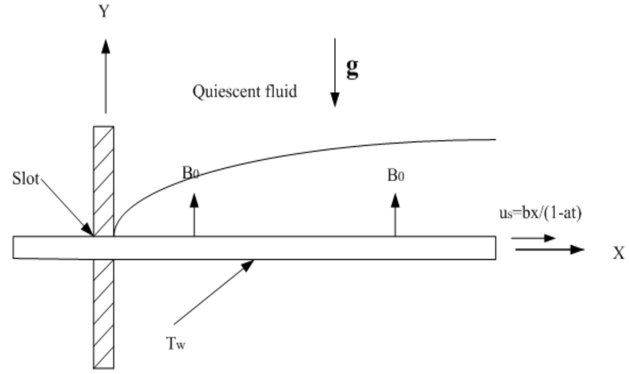

Fig. 1 A sketch of the physical model for unsteady mixed convection flow with magnetic effect of an incompressible quiescent fluid over a thermal forming stretching sheet

The conducting fluid is permeated by an imposed uniform magnetic field $B = (0, B_0, 0)$ which acts in the positive y-direction. The magnetic Reynold number is assumed small enough so that the induced magnetic field can be neglected. The magnetic force **J**×**B** under these assumptions becomes $\sigma(\mathbf{V}\times\mathbf{B})\times\mathbf{B} = -\sigma B_0^2\mathbf{V}$. The well-known Boussinesq approximation is used to represent the buoyancy mixed term. The fluid flow is modeled as an unsteady, two dimensional, incompressible viscous laminar flow on a horizontal thin elastic sheet that issues from a narrow slot at the origin and is continuous stretching with a velocity $u_s = bx/(1-at)$ [14] (where a and b are positive constants, and t<1/a) in the positive x-direction. The fluid is considered as a quiescent Newtonian liquid with constant properties at temperature T_∞ . The thermal forming stretching sheet is assumed to have a non-uniform internal heat generation/absorption and the sheet temperature varies with the coordinate x and time t. The governing conservation equations of mass, momentum and energy at unsteady state can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u + g\beta(T - T_{\varpi}) \tag{2}$$

$$\rho c_p\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2} + \quad .$$

$$\left(\frac{ku_s}{x\nu}\right)[A(T_w - T_\infty)e^{-\eta} + B(T - T_\infty)] \tag{3}$$

The correspondence boundary conditions are:

$$u = u_s(x,t),\ v = 0,\ T = T_w(x,t),\ \text{at } y=0 \tag{4}$$

$$T = T_s \text{ at } y = 0;\ \frac{\partial T}{\partial y} = 0 \text{ at } y = h\,. \tag{5}$$

where u_s and T_w are the velocity and temperature of the stretching sheet at the surface y = 0, A is the space-dependent parameter, B is temperature-dependent parameter, respectively. The flow is induced due to stretching at y = 0 which moves in the x-direction with the velocity:

$$u_s = \frac{bx}{1-at}, \tag{6}$$

in which a and b are positive constants with dimension $(\text{time})^{-1}$ .It can be noted from Eq. (6) that the effective stretching rate $b/(1-at)$ increases with time since a > 0. The surface temperature T_w of the sheet is:

$$T_w = T_0 - T_{ref}\left[\frac{bx^2}{2\nu}\right](1-at)^{-3/2}, \tag{7}$$

where T_0 and T_{ref} are the temperature at the slit and reference temperature respectively. Expression (7) reflects that the sheet temperature decreases from T_0 at the slot in proportion to x^2 and temperature reduction increases with an increase in $(1-at)$. But it should be noticed that Eqs. (6) and (7), which are responsible for the whole analysis, are valid only for time $t < 1/a$. The following dimensionless parameters are introduced

$$\eta = \sqrt{\frac{b}{\nu}}(1-at)^{-\frac{1}{2}}y,\ \psi = \sqrt{b\nu}\,x(1-at)^{-\frac{1}{2}}f(\eta), \tag{8}$$

and the stream function $\psi(x,y)$ through:

$$u = \frac{\partial\psi}{\partial y} = \frac{bx}{1-at}f'(\eta), \tag{9}$$

$$u = -\frac{\partial\psi}{\partial x} = -\sqrt{\frac{b\nu}{1-at}}f'(\eta), \tag{10}$$

the continuity equation (1) is identically satisfied and dimensionless problems of flow and temperature are:

$$f''' - f'^2 + ff'' - S\left(f' + \frac{1}{2}\eta f''\right) - M^2 f' + G\theta = 0\,, \tag{11}$$

$$\theta'' - P_r\left[\frac{1}{2}S\left(3\theta + \eta\theta'\right) + 2f'\theta - f\theta'\right] + Ae^{-\eta} + B\theta = 0\,, \qquad (12)$$

And the associated with boundary conditions become:

$$f'(0) = 1, \quad f(0) = 0, \quad \theta(0) = 1 \qquad (13)$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0 \qquad (14)$$

Here $S = a/b$ is the unsteadiness parameter, $M^2 = \sigma B_0^2(1-at)/\rho b$ is the dimensionless quiescent and magnetic parameters, $G = g\beta[\frac{(1-at)^2}{b^2}]$ is the free convection parameter, respectively. Here primes indicate the differentiation with respect to η. The skin-friction coefficient C_f and the Nusselt number Nu are defined as:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_s^{\,2}} = -2G^{-1/4}f''(0)\,, \qquad (15)$$

$$Nu = \frac{hx}{k} = -G^{1/4}\theta'(0)\,, \qquad (16)$$

where Re_x is the local Reynold number and C_f is the skin-friction coefficient.

The formulation of the first analysis principle for free convection along a stretching sheet involves the energy conservation for the stretching sheet and the boundary layer equations for the flow. For a slender stretching sheet, ample evidence based on finite difference solutions shows that a one-dimensional model is adequate. The stretching sheet temperature at any x location serves as the wall temperature for the adjacent fluid and has denoted as $T_f(x)$. The energy equation for the stretching sheet may be written in two different forms, depending on how the coupled-fin/boundary-layer problem is solved. The stretching sheet energy equation can be expressed as:

$$\frac{d^2T_f}{dx^2} = \frac{q}{k_f t} \qquad (17)$$

or

$$\frac{d^2T_f}{dx^2} = \left(\frac{h}{k_f t}\right)(T_f - T_e) \qquad (18)$$

in which, k_f is the thermal conductivity of the stretching sheet. For the solutions of either equation (17) or (18) at a given cycle of the iterative procedure, h and q can be regarded as known quantities. At first glance, it appears advantageous to use equation (18) rather than equation (17) because it is easier to solve; however, equation (18) has been employed in the solution scheme. Equation (18) recasts in a dimensionless form by the substitutions

$$X = x/L,\ Y = y/L,\ \theta_f = (T_f - T_e)/(T_0 - T_e) \qquad (19)$$

where T_0 is the base temperature of the stretching sheet, so that:

$$\frac{d^2\theta_f}{dx^2} = \hat{h}\,N_{cc}\theta_f \qquad (20)$$

with boundary conditions:

$$\theta_f = 1 \ \text{(X=0)}, \quad \frac{d\theta_f}{dX} = 0 \ \text{(X=1)} \qquad (21)$$

where N_{cc} is the conduction-convection number and is defined as:

$$N_{cc} = (kL/k_f t)G^{1/4} \qquad (22)$$

The quantity $\hat{h}$ is a dimensionless form of the average convective heat transfer coefficient and can be written as:

$$\hat{h} = \left(\frac{hL}{k}\right)G^{-1/4} \qquad (23)$$

III. Numerical Technique

In the present problem, the set of similar equations from (11) to (14) and (20) to (23) are solved by a finite difference method. These ordinary differential equations are discretized by a second-order accurate central difference method [26], and a computer program has been developed to solve these equations. Vajravelu [27-29] and Hsiao et al. [31-34] are also using analytical and numerical solutions to solve the related problems. So, some numerical technique methods will be applied to the same area in the future. In this study, the program to compute finite difference approximations of derivatives for equal spaced discrete data. The code employ centered differences of O(h^2) for the interior points and forward and backward differences of O(h) for the first and last points, respectively. See Chapra and Canale, Numerical Methods for Engineers [30]. To ensure the convergence of the numerical solution to exact solution, the step sizes $\Delta\eta$ and have been optimized and the results presented here are independent of the step sizes at least up to the fourth decimal place. The convergence criteria based on the relative difference between the current and previous iteration values of the velocity and temperature gradients at wall are employed. When the difference reaches less than 10^{-6} for the flow fields, the solution is assumed to have converged and the iterative process is terminated. The sequence of equations of above was expressed in difference form using central difference scheme in η-direction. In each iteration step, the equations were then reduced to a system of linear algebraic equations.

IV. Results and Discussion

The objective of the present analysis is to study the heat transfer of a quiescent fluid cooled or heated by a high or low Prandtl-number, quiescent fluid with various parameters. An extension of previous works has then performed to investigate the heat transfer of a quiescent fluid pass a thermal forming stretching sheet with magnetic effect and free convection effect, are included. The model for quiescent fluid has been used in the momentum equations. Effects of dimensionless parameters, the unsteadiness parameter (S), the Prandtl number (Pr), the space-dependent parameter (A) and temperature-dependent parameter (B) for heat source/sink and the magnetic parameter (M) are main interests of the study. Flow and temperature fields of the quiescent fluid flow have analyzed by utilizing the boundary layer concept to obtain a set of coupled momentum equations and energy equations. A similarity transformation has then used to convert the nonlinear, coupled partial differential equations to a set of nonlinear, coupled ordinary differential equations. A generalized derivation is used to analyze an unsteady flow has been studied. A second-order accurate finite difference method used to obtain solutions of these equations.

Comparing $-\theta'(0)$ to results of [2] for an unsteady quiescent fluid flow (S =0, A=0 , M=0) showed a good agreement and these values have listed in Table I. Table II is shown a mixed convection and unsteady quiescent fluid flow field results (A=4) for different B, Pr, S, and with different magnetic parameters M. Table III shows that the different values of physical parameters for free convection flow M=0.1, A=1, B=-1, Pr=1, S=0.1 and for mixed convection (G=0 and G=10) flow its $-\theta'(0)$ for different values of physical parameters.

TABLE I
A COMPARISON OF -Θ'(0) FOR AN UNSTEADY QUIESCENT FLUID FLOW (S =0, A=0 , M=0)

B	Pr	-θ'(0) Ref.[2]	Present Solution	Errors
-1	1	1.710937	1.710935	0.00002
-2	2	2.486000	2.485991	0.00009
-3	3	3.082179	3.082152	0.00027
-4	4	3.585194	3.585137	0.00057
-5	5	4.028535	4.028511	0.00024

TABLE II
MIXED CONVECTION FOR AN UNSTEADY QUIESCENT FLUID FLOW FIELD (A=4)

B	Pr	S	M	f''(0)	θ'(0)
-1	1	0.1	0.1	-0.8328	-1.5328
-2	2	0.2	0.2	-0.7206	-1.7782
-3	3	0.3	0.3	-0.6473	-1.6953
-4	4	0.4	0.4	-0.6000	-1.5243
-5	5	0.5	0.5	-0.5700	-1.3582

TABLE III
VALUES OF $-\theta'(0)$ FOR DIFFERENT VALUES OF PHYSICAL PARAMETERS
M=0.1, A=1, B=-1, Pr=1, S=0.1

$-\theta'(0)$	$-\theta'(0)$	$\hat{h}$	$\hat{h}$
(G=0)	(G=10)	(G=0)	(G=10)
1.5329	1.7736	9.0537	10.4753

The main contribution of this study considers the conjugate heat transfer magnetic effect in a mixed convection for a quiescen fluid flow past a thermal forming stretching sheet hybrid heat transfer system. From the figures provide more physical insights, as follow:

Fig. 2 is shown dimensionless velocity gradient f' vs. η as S=0.01-0.5. It is represent the fluid flow phenomenon toward the flow filed. The numerical calculation results are satisfied the boundary layer conditions at the figure. The momentum was interacting with each other and the figure curves are all having a strong varies with η along the boundary layer for different unsteadiness parameter S. When the S value is larger and the dimensionless velocity gradient f' is larger, so that the unsteadiness effect will increase the momentum force and the flow will move quickly with the whole flow field. On the other hand, when the S value is lower and the dimensionless velocity gradient f' is also lower too, so that the unsteadiness effect is not good for a lower unsteadiness parameter S.

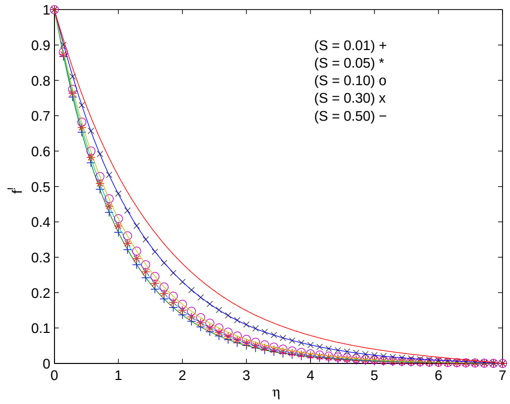

Fig. 2 Dimensionless velocity gradient f' vs. η as S = 0.01, 0.05, 0.1, 0.3, 0.5

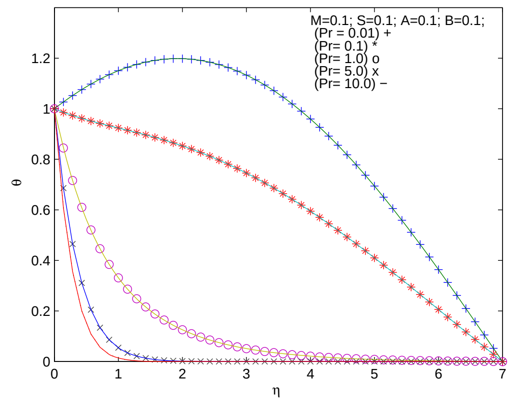

Fig. 3 dimensionless temperature profiles θ vs. η as M=0.1, S=0.1, A=0.1, B=0.1 and Pr=0.01, 0.1, 1.0, 5.0, 10.

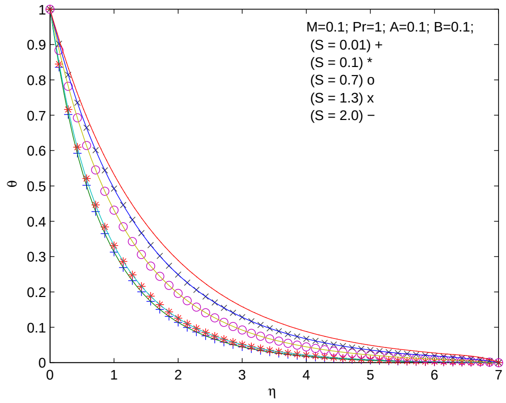

Fig. 4 Dimensionless temperature profiles θ vs. η as M=0.1, Pr=1, A=0.1, B=0.1 and S=0.01,0.1,0.7,1.3,2.

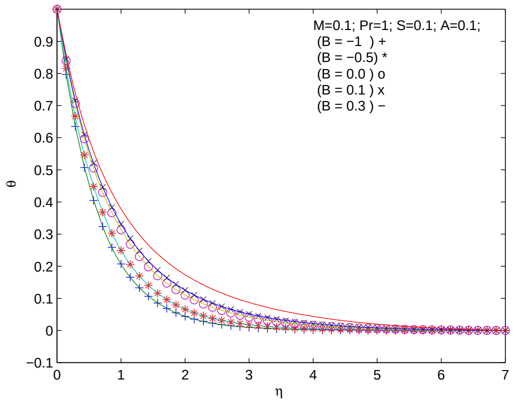

Fig. 6 Dimensionless temperature profiles θ vs. η as M=0.1, Pr=1, S=0.1 and B=-1, -0.5, 0, 0.1, 0.3

For discussing the result for Figs. 3, 4, 5, 6 and 7 some numerical calculations have carried out for dimensionless temperature profiles for different values of S, M, E, Pr A and B.

Fig. 3 is shown dimensionless temperature profiles θ vs. η as M=0.1, S=0.1, A=0.1, B=0.1 and Pr=0.01-10. The dimensionless temperature profiles are a parabolic type curve satisfied with the boundary conditions. The Prandtl number is larger, when the dimensionless temperature profile is lower. So that, the Prandtl number can remove the heat from the fluid and its effect is good for a higher The Prandtl number.

Fig. 4 is shown dimensionless temperature profiles θ vs. η as M=0.1, Pr=1, A=0.1, B=0.1 and S=0.01-2.The dimensionless temperature profiles are a parabolic type curve satisfied with the boundary conditions. The unsteadiness parameter is larger, when the dimensionless temperature profile is higher. So that, the unsteadiness force can not remove the heat from the fluid and its effect is not good for a higher magnetic parameter.

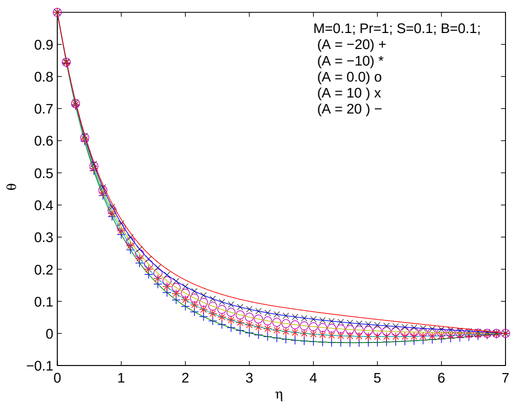

Fig. 5 Temperature profiles θ vs. η as M=0.1, Pr=1, S=0.1, B=0.1 and A=-20, -10, 0, 10, 20

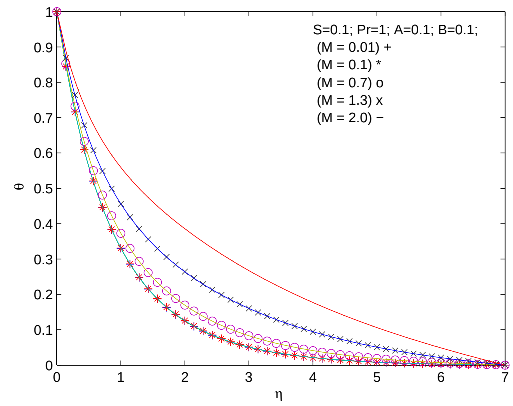

Fig. 7 Dimensionless temperature profiles θ vs. η as Pr=1, S=0.1, A=0.1, B=0.1 and M=0.01, 0.1, 0.7, 1.3, 2.0

Fig. 5 is shown dimensionless temperature profiles θ vs. η as M=0.1, Pr=1, S=0.1, B=0.1 and A=-20-20. The dimensionless temperature profiles are a parabolic type curve satisfied with the boundary conditions. The parameter A is smaller, when the dimensionless temperature is lower. On the contrary, the parameter A is larger, when the dimensionless temperature is larger. So that, the heat transfer effect is not good for a higher parameter A.

Fig. 6 is shown dimensionless temperature profiles θ vs. η as M=0.1, Pr=1, S=0.1, A=0.1 and B=-1-0.3. The dimensionless temperature profiles are a parabolic type curve satisfied with the boundary conditions. The parameter B is larger, when the dimensionless temperature profile is higher. So that, the parameter B can not remove the heat from the fluid and its effect is not good for a higher parameter B.

Fig. 7 is shown dimensionless temperature profiles θ vs. η as Pr=1, S=0.1, A=0.1, B=0.1 and M=-1-0.3.The dimensionless temperature profiles are a parabolic type curve satisfied with the boundary conditions. The magnetic parameter is larger, when the dimensionless temperature profile is higher. So that, the magnetic force can not remove the heat from the fluid and its effect is not good for a higher magnetic parameter.

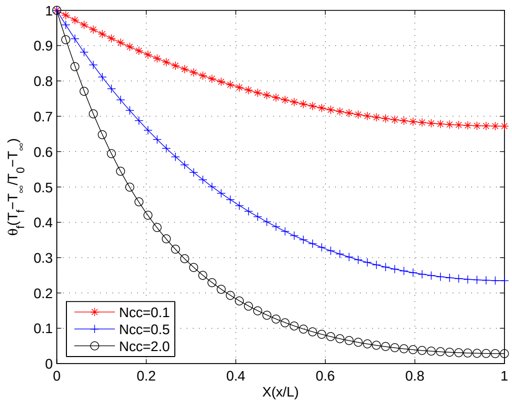

Fig. 8 Conjugate stretching sheet temperature distributions for M=0.1, A=1, B=-1, Pr=1, S=0.1, G=0, $\hat{h}$=9.0537

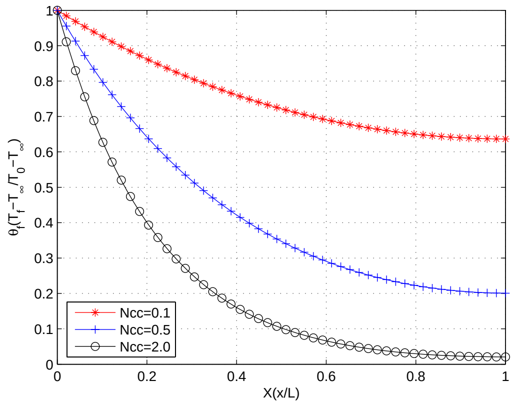

Fig. 9 Conjugate stretching sheet temperature distributions for M=0.1, A=1, B=-1, Pr=1, S=0.1, G=10, $\hat{h}$ = 10.4753

Fig. 8 depicts conjugate stretching sheet temperature distributions for M=0.1, A=1, B=-1, Pr=1, S=0.1, G=0, $\hat{h}$=9.0537, Ncc=0.1, 0.5, 2. Fig. 9 depicts conjugate stretching sheet temperature distributions for M=0.1, A=1, B=-1, Pr=1, S=0.1, G=5, $\hat{h}$ = 10.4753, Ncc=0.1, 0.5, 2. Figs. 8 and 9 show the conjugate stretching sheet temperature distributions. The results obtained from the present computation for different Ncc values and different viscoelastic coefficients E by centered finite difference methods. From the results, we find that the larger Ncc parameters reduce the stretching sheet temperature effect is better than the lower Ncc parameters. On the other hand, compared the two Figs. 8 and 9, find that the larger parameters $\hat{h}$ reduce the stretching sheet temperature effect is better than the lower $\hat{h}$ parameters. From Figs. 8 and 9, we find an important result that the free convection fluid heat transfer effect is better than the mixed convection fluid flow for about 16% at current conditions, and produce a good effect in heat conduction for about 5% to 15% at these conditions.

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