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Computational Simulation of Imploding Current Sheath Trajectory at the Radial Phase of Plasma Focus Performance

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Abstract**—**When the shock front (SF) hits the central electrode axis of plasma focus device, a reflected shock wave moves radially outwards. The current sheath (CS) results from ionization of filled gas between two electrodes continues to compress inwards until it hits the out-going reflected shock front. In this paper the Lagrangian equations are solved for a parabolic shock trajectory yielding a first and second approximation for the CS path. To determine the accuracy of the approximation, the same problem is solved for a straight shock.

Keywords**—**Radial compression, Shock wave trajectory, Current sheath, Slog model.

I. Introduction

IN plasma focus generators the magnetic energy is stored behind the moving current sheath [1]. A portion of this energy is converted into plasma energy during the rapid collapse of the current sheath towards the axis beyond the end of the central electrode. Electrical breakdown generates some initial plasma configuration through which the discharge current can flow and at very low pressure a discharge can develop within the whole inter-electrode volume. The current sheath formed at the end of the breakdown phase is accelerated by Lorentz force towards the open end of the inner electrode and then the current sheath sweeps around the end of the anode electrode and finally implodes due to the inward J×B force. When the current sheath reaches the end of the central electrode, it reverses over itself and collapses radially inward, heating the pinching plasma enclosed in it. The radial compression of CS is open at one end. Hence a gas dynamic shock is propagated ahead of the CS into the undisturbed filling gas [2]. The snowplow model is used for axial acceleration of CS to obtain axial trajectory, CS speed and current profile. As the CS is assumed to be infinitesimally thin, no information of density is contained in the physics of the equation of motion, although an estimate of density may be obtained by invoking shock wave theory [3]. Fig. 1 shows the configuration of the 2D cylindrical geometry shock wave and variation of shock wave accompanied with inward motion of current sheath at the radial compression. The path of CS and shock wave are correlated together and if we know the path of the shock at the radial compression of CS, we can simulate the trajectory of imploding CS.

II. Simulation Model

In the plasma focus model a radially implosive plasma slug is formed above the anode in the radial compression of CS. This slug is driven by the radial inward magnetic piston. The plasma gas inside the slug is compressed and heated by the shock wave. The motion of the plasma slug can be described by the cylindrical geometry 2D shock wave equations. Let us consider an ideal cylindrical magnetic piston of argon plasma. Suppose that the radius of this piston decreases so rapidly that a strong shock is driven in front of the wall toward the axis of the cylinder. We suppose that two particles that are located at different radii in the cylindrical CS, their respective radii will always the same. In Fig. 2 the path of current sheath CS (t), the path of shock wave S (t), and the path of any particle P(ξ,t) are shown. ξ corresponds to the mass between the CS and considered particle and t' refers to the time when the shock passes over the particle. Thus t' may be regarded as a function of ξ and we define $\xi = \pi\rho_0\left(a^2 - S(t')^2\right)$ in which ρ_0 and a are undisturbed gas density and radius of the central electrode [4].

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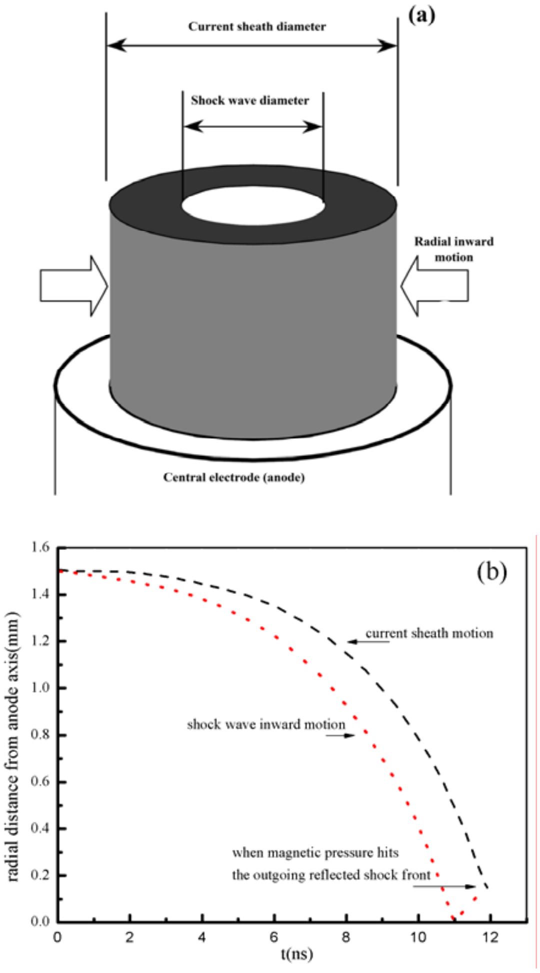

Fig. 1 (a) - configuration of the 2D cylindrical geometry shock wave, (b) - formation of shock wave driven by CS inward motion

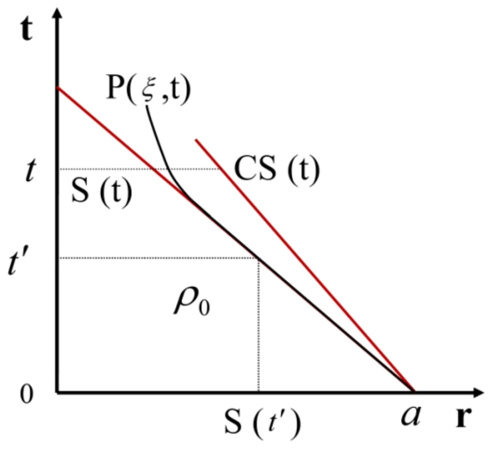

Fig. 2 The path of current sheath CS (t), the path of shock wave S (t), and the path of any particle P(ξ ,t)

The momentum equation is seen to be $\rho \frac{\partial^2 r}{\partial t^2} = -\frac{\partial P}{\partial r}$ where P is the pressure acting on the particle to accelerate it. The pressure is assumed to be entirely isentropic except for a jump in entropy as the shock crosses the particle's pass. Therefore the ratio $\frac{P}{\rho^{\gamma}}$ is a constant for each particle as it travels from the shock toward the axis of cylinder. γ is the ratio of specific heats($\gamma = 1.667$ for Ar as filling gas). The pressure and density of a particle immediately after the shock (P_i and ρ_i) can be found by using the shock relations in conjunction with the perfect gas law as $P_i(t') = \rho_0(1-\varepsilon)\dot{S}^2(t')$ and $\rho_i = \rho_0 / \varepsilon$ in which $\varepsilon = \frac{\gamma - 1}{\gamma + 1}$ and $\dot{S}(t')$ is the velocity of the shock as it crosses the particle $\zeta(t')$. At any later time, ζ may be found by taking the integral $\xi = -2\pi \int_{CS(t)}^{P(\zeta,t)} \rho(r,t) r dr$ in which $\rho(r,t)$ is the density at any point on the (r,t) plane. Thus we can conclude that

$$P^2(\zeta,t) = S^2(t) + \frac{1}{\pi}\int_{\zeta}^{\zeta_s} \{\rho_0^{1-\gamma}(1-\varepsilon)\varepsilon^{\gamma}\dot{S}^2(t')\}^{\frac{1}{\gamma}} P^{-\frac{1}{\gamma}} d\zeta$$

where ζ_s is the value of ζ at the shock at any time t . From the equation of momentum we find that $\frac{\partial P}{\partial r} dr = \frac{1}{2\pi r}\frac{\partial^2 r}{\partial t^2} d\zeta$. Upon integration from the shock to any particle this becomes $P(\zeta,t) = P_i(t) + \frac{1}{2\pi}\int_{\zeta_s(t)}^{\zeta} \frac{\partial^2 r(\zeta,t)}{\partial t^2}\frac{d\zeta}{r(\zeta,t)}$. Let us putting this information back into equation of $P^2(\zeta,t)$ and nondimensionalize the quantities appearing in the equation as $x = \frac{r}{a}, x_s = \frac{S}{a}, Z = \frac{\zeta}{\pi a^2}, \tau = \frac{t}{t_0}$. t_0 is chosen so that τ is in μs . Most interesting cases may be covered by assuming a parabolic shock trajectory as $x_s = 1 - \alpha\tau - \beta\tau^2$ in which α is nondimensional velocity of the shock and β is the shock's constant acceleration or deceleration toward the axis depending upon β is positive or negative. Substituting this information into equation of $P^2(\zeta,t)$, we find

$$x^2(Z,\tau) = x_s^2(\tau) + \varepsilon \int_{Z}^{1-x_s^2(\tau)} \left[\frac{\alpha^2 + 4\beta(1-\sqrt{1-Z})}{(\alpha+2\beta)^2 - \frac{1}{2(1-\varepsilon)}\int_{Z}^{1-x_s^2} \frac{\partial^2 x}{\partial \tau^2}\frac{dZ}{x(Z,\tau)}} \right]^{\frac{1}{\gamma}} dZ$$

. This equation implies that a particle Z is, at a time τ , at a position away from the shock by a distance equal to ε multiplied by an integral. To simplify the equation it may be assumed that the pressure doesn't change much between the shock and CS. In effect we conclude that

$$x^2(Z,\tau) = x_s{}^2(\tau) + \frac{\varepsilon}{(\alpha+2\beta\tau)^{2/\gamma}} \int_{Z}^{1-x_s^2(\tau)} (\alpha+4\beta-4\beta\sqrt{1-Z})^{\frac{1}{\gamma}} dZ \ .$$

we invoke a program to perform a numerical integration to obtain path of CS respect to strong shock wave trajectory [5].

III. Numerical Simulation Results and Discussion

Fig. 3 shows a constant velocity shock ($\beta = 0$) with the CS path computed for $\gamma = 1.667$ and $\gamma = 1.1$. The equation of CS path for the first approximation becomes $x_{first}^2 = x_s^2 + \varepsilon(1-x_s^2) - \varepsilon Z$. In order to find the second approximation, we must substitute the second derivative of $x_{first}(Z,\tau)$ with respect to τ into the subintegral of $x^2(Z,\tau)$ equation. In Fig. 4 CS trajectory for accelerating shock and in Fig. 5 CS trajectory for decelerating shock ($\gamma = 1.667, 1.1$) simulated.

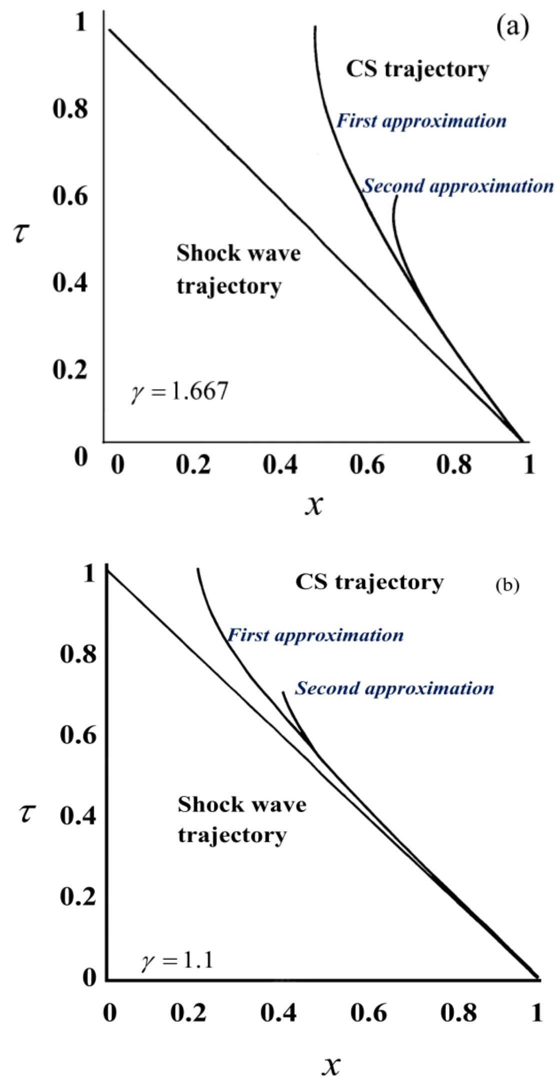

Fig. 3 CS trajectory for constant velocity shock

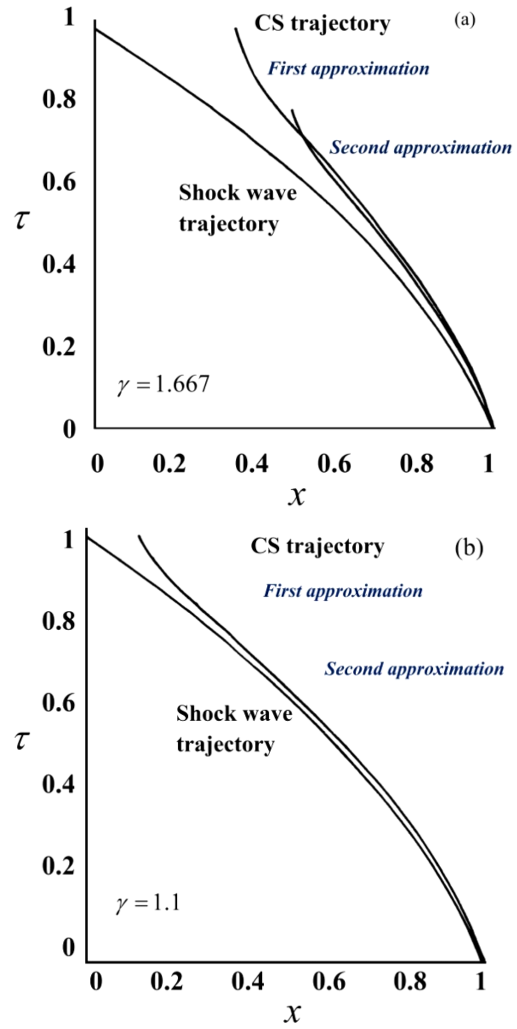

Fig. 4 CS trajectory for accelerating velocity shock

In the pinch phase of focused plasma much of the energy available is absorbed in the ionization process. Here the real value of γ when argon is used as working gas may be expected would be closer to 1.1 than to 1.667 . As it shown in fig.3, for $\gamma = 1.667$ it is seen that the first and second approximations for the CS trajectory are very close together until the CS reaches a radial position 0.75 . At this point the second approximation diverges from the first approximation and ultimately turns back toward its initial position. Physically a decrease in the denominator corresponds to a decrease in pressure at the CS. It is logical the pressure decrease from the shock to the CS at a given time because in this region of the flow, there is quasi-steady supersonic flow into a converging channel which implies a decrease in velocity and a corresponding adverse pressure gradient. Since the conditions behind the shock are fixed, the pressure at the CS must be steadily decreasing as the gap between the shock and CS widens. As we see in fig.4, for $\gamma = 1.1$ there is no difference large enough to be seen between the first and second approximations until the second approximation reaches the zero pressure limit. This occurs at $x(0,\tau) = 0.27$ that is much smaller than the final radius of the CS for the constant shock. This fact implies that the CS pushing an accelerating shock has control over the shock for a longer time than the CS pushing a constant velocity shock. For $\gamma = 1.667$, the accelerating shock has a piston path given by the second approximation that is closer to the center than the first approximation. That is, in order to accelerate the flow, the pressure at the piston must be greater than the pressure at the shock.

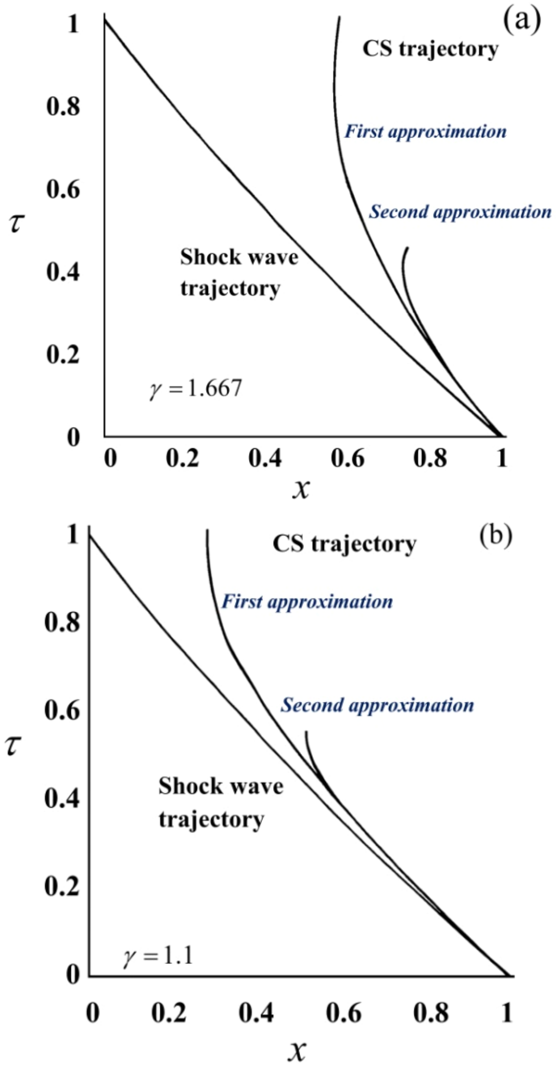

Fig. 5 CS trajectory for decelerating velocity shock

For the first approximation, this pressure difference is neglected. In the second approximation it is included. This effect is also present for the $\gamma = 1.1$ case; however, it is so small that it cannot be seen on the scale of Fig. 4. Fig. 5 shows the case of a decelerating shock. There is little new on this graph except that the piston turns back even sooner than it does for the constant velocity shock.

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