Comparison of Newton Raphson and Gauss Seidel Methods for Power Flow Analysis

H. Abaali, T. Talbi, R. Skouri

Abstract—This paper presents a comparative study of the Gauss Seidel and Newton-Raphson polar coordinates methods for power flow analysis. The effectiveness of these methods are evaluated and tested through a different IEEE bus test system on the basis of number of iteration, computational time, tolerance value and convergence.

Keywords—Convergence time, Gauss-Seidel Method, Newton-Raphson Method, number of iteration, power flow analysis.

NOMENCLATURE

Var	volt-ampere reactive
$\overline{Y} = Y\angle\theta = G + jB$	complex nodal admittance matrix
Z	nodal impedance matrix
J	Jacobian matrix
G	conductance
B	susceptance
N-bus	number of buses in system
s	slack bus index
N_{PV}	number of generator buses
N_{PQ}	number of load buses
$\overline{V}_i = V_i\angle\delta_i$	complex nodal voltage at i^{th} bus
I_i	complex nodal injected current at i^{th} bus
P_i	active power at i^{th} bus
Q_i	reactive power at i^{th} bus
$\overline{S}_i = P_i + jQ_i$	complex nodal injected power at i^{th} bus
$\Delta P_i = P_{ci} - P_i$	active power mismatch
$\Delta Q_i = Q_{ci} - Q_i$	reactive power mismatch

I. INTRODUCTION

THE power flow studies, commonly referred to as load flow, are the backbone of power system analysis and design. They are necessary for planning future expansion, control, economic scheduling, and management of power systems as well as in determining the best operation of existing systems. In addition, power flow analysis is required for many other analyses such as transient stability and contingency studies.

The network equations can be formulated systematically in a variety of forms. However, the node-voltage method is commonly used for power system analysis. The network equations which are in the nodal admittance form result in complex linear simultaneous algebraic equations in terms of node currents [1]. When node currents are specified, the set of linear equations can be solved for the node voltages. However,

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in a power system, powers are known rather than currents. So, the steady-state performances of an interconnected power system can be modeled from a polynomial system equation in several variables called power flow equations become nonlinear. To approximate the solutions of these nonliner equations, the numerical methods should be used [2]. The algorithms applied to solve the power flow equations can be classified into two broad categories: coordinate category methods such as Gauss Seidel [1] and gradient category methods such as Newton-Raphson [4]-[6]... The power flow results give the bus voltage magnitude and phase angles and hence the power flow through the transmission lines, line losses and injection at all the buses.

The power system is assumed to be operating under balanced condition and can be represented by a single-phase network. The power system network contains hundreds of buses and branches with impedances specified in per-unit on a common MVA base.

In this work, we present a comparative study of these two categories iterative solution techniques for power flow analysis [7]. The performances of these methods are assessed from four aspects: Number of iterations, computational time, tolerance value and convergence. The two methods are tested on five IEEE standard 3-Bus, 5-Bus, 14-Bus, 30-Bus and 57-Bus test systems. The simulation is carried out using MATLAB version 7.8.0.347 (R2008a) in Intel® Atom ™ CPU N450 @1,66GHz.

This paper is organized as follows: After the introduction, the modelling and power flow problem formulation of an electric power system was briefly discussed and deals with the steady-state analysis of interconnected power system during normal operation in Section II. Section III describes the load flow solution using Gauss Seidel and Newton Raphson polar coordinates methods. Section IV introduces the simulation results and discussion. Finally Section V presents the conclusion.

II. POWER FLOW PROBLEM FORMULATION

A. Bus Classification

In a power flow study, a bus is defined as the vertical line at which the several components such as generators, loads and transformers are connected. Each bus is associated with four variables: Magnitude of voltage, phase angle of voltage, active and reactive power. Two of the four variables are specified and the other two variables are unknown. The two unknown variables are determined through the solution of the nonlinear power flow equations. The buses are classified into three categories:

- Load (PQ bus): No generator is connected to the bus. The load drawn by these buses are defined by real power $-P_{Li}$ and reactive power Q_{Li} in which the negative sign accommodates for the power flowing out of the bus. The objective of the load flow is to find the bus voltage magnitude V_i and its angle δ_i.
- Generator bus: The input power P_{Gi} and the bus voltage V_i are kept constant. We have to find the unknown angle δ_i of the bus voltage and the generated reactive power Q_{Gi} through the load flow solution.
- Slack (swing) bus: Usually this generator bus is numbered 1 for the load flow studies. This bus sets the angular reference for all the other buses. Since it is the angle difference between two voltage sources that dictates the real and reactive power flow between them, the particular angle of the slack bus is not important. However, it sets the reference against which angles of all the other bus voltages are measured. For this reason the angle of this bus is usually chosen as $\delta_1=0$. Furthermore, it is assumed that the magnitude of the voltage V_1 of this bus is known.

The bus classification is summarised in Table I.

B. Network Models

Power network can be operating under balanced or unbalanced conditions. The normal procedure for a load flow problem formulation is to assume balanced equations at the buses of the system and to use a single-phase representation equivalent to the positive sequence network and selecting a proper subset of these equations which will provide the minimum number of simultaneous equations in terms of an equal number of state variables (equal number of equations and unknowns). The solution of these equations provides the system state.

The power flow model consists of a set of nonlinear algebraic equations. These equations formulate active and reactive line flows based on bus voltage magnitudes and phase angles. In most power system studies, transmission lines are represented by a π-equivalent circuit [8].

A systematic way of writing the power flow equations for any bus is given by (9), followed by the selection process. The power flow equations can be developed with reference to Fig. 1 illustrating a general circuit between any two buses i and j buses.

In general, one or more circuits may be connected to a bus. In addition, an admittance y_{ii} may be also connected to a bus (a capacitor, a reactor, etc.). It is assumed that electric current I_{gi} is injected to bus i from the generators connected to this bus. Also, electric current I_{di} is absorbed from the electric load connected to this bus. One or both of these currents may be absent from a bus. The voltage of bus i is assumed to be V_i and the voltage of bus j is assumed to be V_j.

TABLE I
BUS CLASSIFICATION

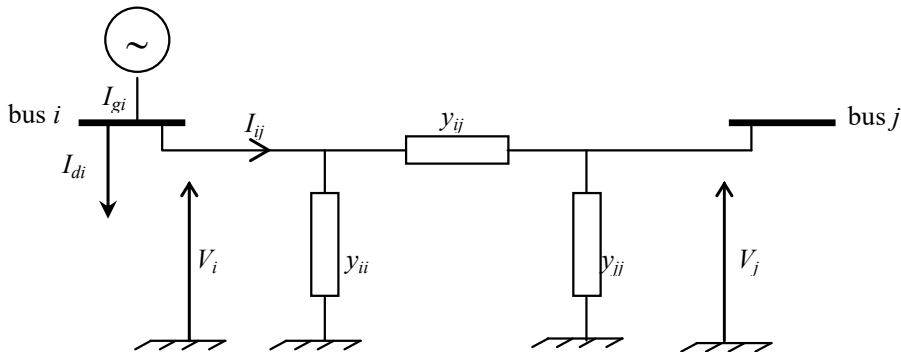| Bus Type	Bus code	Known variables	Unknown variables
Slack (Swing) bus	1	V∟δ	P and Q
Generator (PV) bus	2	P and V	Q and δ
Load (PQ) bus	3	P and Q	V∟δ

Fig. 1 Symbolic representation of a line model of an electric power system

C. Bus Admittance Matrix

The bus admittance matrix, or Y matrix, is a numerical model of the power system that characterizes the behaviour of the network components such as power lines, transformers and loads using the complex injected currents at the nodes and their relation to the complex node voltages, based on Kirchoff's current law [1].

Generally, for a network with N independent buses, we can write the following N nodal equations for a power system network applying the Kirchoff's current law to bus i will yield:

$$\bar{I}_i = \sum_{j=1}^{N} \bar{Y}_{ij}\bar{V}_j \quad i = 1, 2,...N \tag{1}$$

In matrix representation:

$$[\bar{I}] = [\bar{Y}][\bar{V}] \tag{2}$$

where: $[\bar{I}]$: the bus current injection vector; $[\bar{V}]$: the bus voltage vector; $[\bar{Y}]$: the bus admittance matrix; $\bar{Y}_{ii}$: the diagonal elements of bus admittance matrix, are called the self-admittance of bus i, which equals the sum of all branch

admittances connecting to bus i. $\overline{Y}_{ij}$: the off-diagonal elements, of the bus admittance matrix, are the negative of the admittances buses i and j ($\overline{Y}_{ij} = -\overline{Y}_{ji}$). The off-diagonal elements are equal to zero if there is no line between buses i and j, obviously, the bus admittance matrix is a sparse matrix.

D. Real and Reactive Power Injected in a Bus

The complex power delivered to i^{th} bus is given by:

$$\overline{S}_i = \overline{V}_i \overline{I}_i^* \quad \text{with } i=1, 2, 3 \dots N \tag{3}$$

and

$$\overline{S}_i^* = \overline{S}_{gi}^* - \overline{S}_{di}^* = (P_{gi} - P_{di}) - j(Q_{gi} - Q_{di}) \tag{4}$$

where $\overline{S}_{gi}$: the complex power injection vector generator connecting to bus i; $\overline{S}_{di}$: the complex power load vector connecting to bus i; P_{gi}: the real power output of the generator connecting to bus i; Q_{gi}: the reactive power output of the generator connecting to bus i; P_{di}: the real power load connecting to bus i; Q_{di}: the reactive power load connecting to bus i.

The bus current can be represented by bus voltage and power:

$$\overline{I}_i = \frac{\overline{S}_{gi}^* - \overline{S}_{di}^*}{\overline{V}_i^*} = \frac{(P_{gi} - P_{di}) - j(Q_{gi} - Q_{di})}{\overline{V}_i^*} \tag{5}$$

Substituting (5) into (1), we have:

$$\overline{I}_i = \frac{(P_{gi} - P_{di}) - j(Q_{gi} - Q_{di})}{\overline{V}_i^*} = \sum_{j=1}^{N} \overline{Y}_{ij} \overline{V}_j \quad i = 1, 2, \dots N \tag{6}$$

In the power flow problem, the load demands are known variables. We define the following bus power injections as:

$$P_i = P_{gi} - P_{di} \tag{7}$$

$$Q_i = Q_{gi} - Q_{di} \tag{8}$$

Substituting the above two equations into (6), we can get the general form of power flow equation as

$$P_i - jQ_i = \overline{V}_i^* \sum_{j=1}^{N} \overline{Y}_{ij} \overline{V}_j \quad i = 1, 2, \dots N \tag{9}$$

If we divide (9) into real and imaginary parts, we can get two equations for each bus with four variables, that are, voltage amplitudes V and phases δ at load PQ buses, reactive power Q and voltage phases V at generator PV buses and active and reactive power (P, Q) at the slack bus, as given in Table I. The slack bus variables δ_l and V_l are omitted, since they are already known. To solve the power flow equations,

two of these should be known for each bus (Table I). Assuming the system consists of N buses. The 1th bus is the slack bus, buses 2 to M are PQ buses and buses $(M+1)$ to N are PV buses. The variables and equations are twice the number of the network buses, then $2(N-1)-(M-1)$ unknowns variables.

If the bus voltage is expressed using the polar coordinate system, the complex voltage, and admittance elements can be written as:

$$\overline{V}_i = V_i \angle \delta_i = V_i(\cos \delta_i + j \sin \delta_i) \tag{15}$$

$$\overline{Y}_{ij} = Y_{ij} \angle \theta_{ij} \quad \text{with } i, j = 1, 2, 3 \dots N$$

$$\overline{Y}_{ij} = Y_{ij}(\cos \theta_{ij} + j \sin \theta_{ij}) = G_{ij} + jB_{ij} \tag{16}$$

where $\theta_{ij} = \theta_i - \theta_j$, which is the angle difference between buses i and j.

The power flow equations in polar coordinate are based on the nonlinear power flow equation gives by (9). The terms V, P and Q are represented in per-unit and δ terms are represented in degrees. For each PV or PQ bus, we have the following real and reactive powers mismatch equations given by (17) and (18):

$$\Delta P_i = P_{ic} - P_i = P_{ic} - V_i \sum_{j=1}^{N} V_j [G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}] = 0 \tag{17}$$

$$\Delta Q_i = Q_{ic} - Q_i = Q_{ic} - V_i \sum_{j=1}^{N} V_j [G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}] = 0 \tag{18}$$

where P_{ic}, Q_{ic} are, respectively, the calculated bus real and reactive power injections.

The power flow equations (17) and (18) are nonlinear and cannot be explicitly inverted. To solve these nonlinear equations the numerical iterative techniques are used [2].

III. LOAD FLOW SOLUTION

The mathematical recalls of the Gauss Seidel method and Newton Raphson polar coordinates method are given in this section.

A. Gauss Seidel Method

The Gauss-Seidel iterative method [3] is the simplest of all the iterative methods. From (9), we get

$$\overline{V}_i = \frac{1}{\overline{Y}_{ij}} \left[\frac{P_i - jQ_i}{\overline{V}_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{N} \overline{Y}_{ij} \overline{V}_j \right] \quad i = 1, 2, \dots N \tag{10}$$

According to the Gauss-Seidel method, the iteration formula (10) can be written as:

$$\overline{V}_i^{k+1} = \frac{1}{\overline{Y}_{ij}} \left[\frac{P_i - jQ_i}{\overline{V}_i^{k*}} - \sum_{j=1}^{i-1} \overline{Y}_{ij} \overline{V}_j^{k+1} - \sum_{j=i+1}^{N} \overline{Y}_{ij} \overline{V}_j^{k} \right] \tag{11}$$

Equation (11) is the formulation for the iterative solution of power flow problem by Gauss Seidel method; for the *PQ* bus, the real and reactive powers are known. Thus, if the initial bus voltage $\dot{V}_i^0$ is given, we can use (11) to perform the iteration calculation.

For the PV bus, the bus real power and the magnitude of the voltage are known. It is necessary to give the initial value for bus reactive power. The bus reactive power will then be computed by iterative calculation. That is:

$$Q_i^k = \text{Im}\left[\overline{V}_i^k I_i^{k*} \right] = \text{Im}\left[\overline{V}_i^k \left(\sum_{j=1}^{i-1} \overline{Y}_{ij}^* \overline{V}_j^{k+1*} + \sum_{j=i+1}^{N} \overline{Y}_{ij}^* \overline{V}_j^{k*} \right) \right] \tag{12}$$

After the iteration is over, all bus real and reactive power, as well as the voltage, are obtained. The power of the slack bus can be obtained by solving:

$$P_1 + jQ_1 = \overline{V}_1 \sum_{j=1}^{N} \overline{Y}_{1j}^* \overline{V}_j^* \tag{13}$$

The line complex power flow can be also obtained as:

$$\overline{S}_{ij} = P_{ij} + jQ_{ij} = \overline{V}_i \ I_{ij}^* = \overline{V}_i^2 \overline{Y}_{i0}^* + \overline{V}_i \ (\overline{V}_i^* - \overline{V}_j^*) \overline{Y}_{ij}^* \tag{14}$$

where Y_{ij} is the admittance of the branch *i-j* and Y_{i0} is the admittance of the ground branch at the end *i*.

B. Newton Raphson Method

According to the Newton method [4]-[6], the power flow equations (17) and (18) can be expanded into Taylor series and the following first-order approximation. The result is a linear system of equations that can be expressed as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V / V \end{bmatrix} \tag{19}$$

where:

$$\Delta V / V = V_D^{-1} \Delta V$$

$$\Delta P = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_N \end{bmatrix} \cdot \Delta Q = \begin{bmatrix} \Delta Q_2 \\ \vdots \\ \Delta Q_M \end{bmatrix} \quad \cdot \quad \Delta \delta = \begin{bmatrix} \Delta \delta_2 \\ \vdots \\ \Delta \delta_N \end{bmatrix} \cdot \quad \Delta V = \begin{bmatrix} \Delta V_2 \\ \vdots \\ \Delta V_M \end{bmatrix}$$

$$V_D^{-1} = \begin{bmatrix} V_2 & \\ & V_M \end{bmatrix}$$

The partial derivatives in each Jacobian matrix block, derived from (17) and (18) are given by:

J_1 is an (*N*-1)x(*N*-1) matrix, and its element is $J_{1ij} = \dfrac{\partial \Delta P_i}{\partial \delta_j}$

J_2 is an (*N*-1)x*M* matrix, and its element is $J_{2ij} = V_j \dfrac{\partial \Delta P_i}{\partial V_j}$

J_3 is an *M*x(*N*-1) matrix, and its element is $J_{3ij} = \dfrac{\partial \Delta Q_i}{\partial \delta_j}$

J_4 is an *M*x*M* matrix, and its element is $J_{4ij} = V_j \dfrac{\partial \Delta Q_i}{\partial V_j}$

If $i{\neq}j$ the expressions for the elements in Jacobian matrix are as:

$$J_{1ij} = V_i V_j [G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}] \tag{20}$$

$$J_{2ij} = V_i V_j [G_{ij} \cos \delta_{ij} - B_{ij} \sin \delta_{ij}] \tag{21}$$

$$J_{3ij} = -V_i V_j [G_{ij} \cos \delta_{ij} - B_{ij} \sin \delta_{ij}] \tag{22}$$

$$J_{4ij} = V_i V_j [G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}] \tag{23}$$

If $i=j$, the expressions for the elements in the Jacobian matrix are as:

$$J_{1ii} = -V_i^2 B_{ii} - Q_i \tag{24}$$

$$J_{2ii} = V_i^2 G_{ii} + P_i \tag{25}$$

$$J_{3ii} = -V_i^2 G_{ii} + P_i \tag{26}$$

$$J_{4ii} = -V_i^2 B_{ii} - Q_i \tag{27}$$

The steps for calculation of the Newton power flow solutions are as follows [1], [2]:

1 Given input data.
2 Form bus admittance matrix.
3 Assume the initial values of bus voltage.
4 Compute the power mismatch according to (17) and (18). Check whether the convergence conditions are satisfied.

$$Max \left| \Delta P_i^{(k)} \right| < \varepsilon_1 \tag{28}$$

$$Max \left| \Delta Q_i^{(k)} \right| < \varepsilon_2 \tag{29}$$

If (28) and (29) are met, stop the iteration, and calculate the line flows and real and reactive powers of the slack bus. If not, go to next step.

5 Compute the elements in the Jacobian matrix (19)–(27).
6 Compute the corrected values of the bus voltage using (30).
   Then compute the bus voltage.

$$V_i^{k+1} = V_i^k + \Delta V_i^k \qquad (30)$$

$$\delta_i^{k+1} = \delta_i^k + \Delta \delta_i^k \qquad (31)$$

7   Return to Step (4) with new values of the bus voltage.

The Jacobian matrix gives the linearized relationship between small changes in angle and voltage magnitude with the small changes in real and reactive power. This method begins with initial guesses of all unknown variables (voltage magnitude and angles at load buses and voltage angles at generator buses). The good initial guess is needed to start the iterative process [4]. Typically a flat start is an acceptable initial guess. The algorithm stops if the variable increments are lower than a given tolerance or the number of iterations is greater than a given limit. In the latter case, the algorithm has likely failed to converge. In this case it will be seen that $J_1=J_4$ and $J_2=-J_3$. Or, in other words the symmetry is restored. The number of elements to be calculated for an N-dimensional Jacobian matrix are only $N + N^2/2$ instead of N^2, thus again saving computer time and storage.

IV.   SIMULATION RESULTS AND DISCUSSION

A. Simulation Results

The simulation results of the comparative study of Gauss Seidel (GS) and Newton-Raphson (NR) methods for power flow analysis are presented for different standard IEEE bus test systems [7]. The performances of this comparative study are assessed from number of iterations, computational time, tolerance value and convergence. The simulation is carried out using MATLAB version 7.8.0.347 (R2008a) in Intel® Atom ™ CPU N450 @1,66GHz.

The standard IEEE 3-bus test system diagram is given in Fig. 1. The circuit data and the bus data of the standard IEEE 3-bus test system are respectively given in Tables II and III.

The NR and GS comparative results of power flow analysis for a simple standard IEEE 3-bus test system are respectively given in Tables IV and V. The line flow and losses of a standard IEEE 3-bus test system are given in Table VI.

The comparison results of convergence, number of iterations and computational time using NR and GS methods are given respectively in Figs. 2, 3 and 4. The summary of the obtained results are given in Table VII.
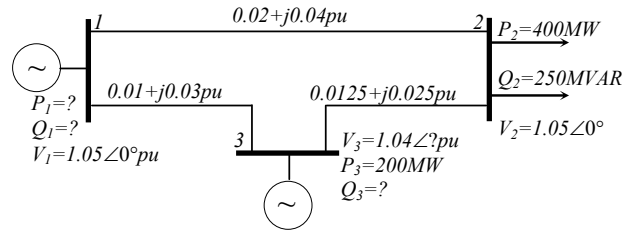

Fig. 1 Standard IEEE 3-bus test system diagram

TABLE II
STANDARD EEE 3-BUS TEST SYSTEM CIRCUIT DATA

From bus°	To bus	Resistance R pu	Reactance X pu	Susceptance B/2 pu	Transformer TAP
1	2	0.02	0.04	0.03	1
1	3	0.01	0.03	0.02	1
2	3	0.0125	0.025	0.025	1

TABLE III
STANDARD IEEE 3-BUS TEST SYSTEM BUS DATA

Bus N°	Bus type	Voltage		Power generated MW MVAR		Load MW MVAR			
		$|V|$	δ	P_{Gi}	Q_{Gi}	P_{Li}	Q_{Li}	Q_{min}	Q_{max}
1	1	1.05	0	0	0	0	0	0	0
2	3	1.00	0	0	0	400	250	-40	50
3	2	1.04	0	200	0	0	0	0	40

TABLE IV
NR LOAD FLOW ANALYSIS

Bus N°	$|V|$ pu	δ deg	Power Injection MW P	MVAR Q	Power Generation MW P	MVAR Q	Load MW P	MVAR Q
1	1.05	0.00	219.85	236.17	219.85	236.17	0.00	0.00
2	0.96	-2.51	-400	-250	0.00	0.00	400	250
3	1.02	-0.67	200	39.18	200	39.18	0.00	0.00
Total			**19.85**	**25.35**	**419.85**	**275.35**	**400**	**250**

TABLE V
GS LOAD FLOW ANALYSIS

Bus N°	$|V|$ pu	δ deg	Power Injection P MW	Q MVAR	Power Generation P MW	Q MVAR	Load P MW	Q MVAR
1	1.05	0.00	219.82	235.28	219.82	235.28	0.00	0.00
2	0.96	-2.51	-399.99	-249.99	0.01	0.01	400	250
3	1.02	-0.70	199.99	40.00	199.99	40	0.00	0.00
Total			**19.82**	**25.29**	**419.82**	**275.29**	**400**	**250**

TABLE VI
LINE FLOW AND LOSSES OF STANDARD IEEE 3- BUS TEST SYSTEM (NR)

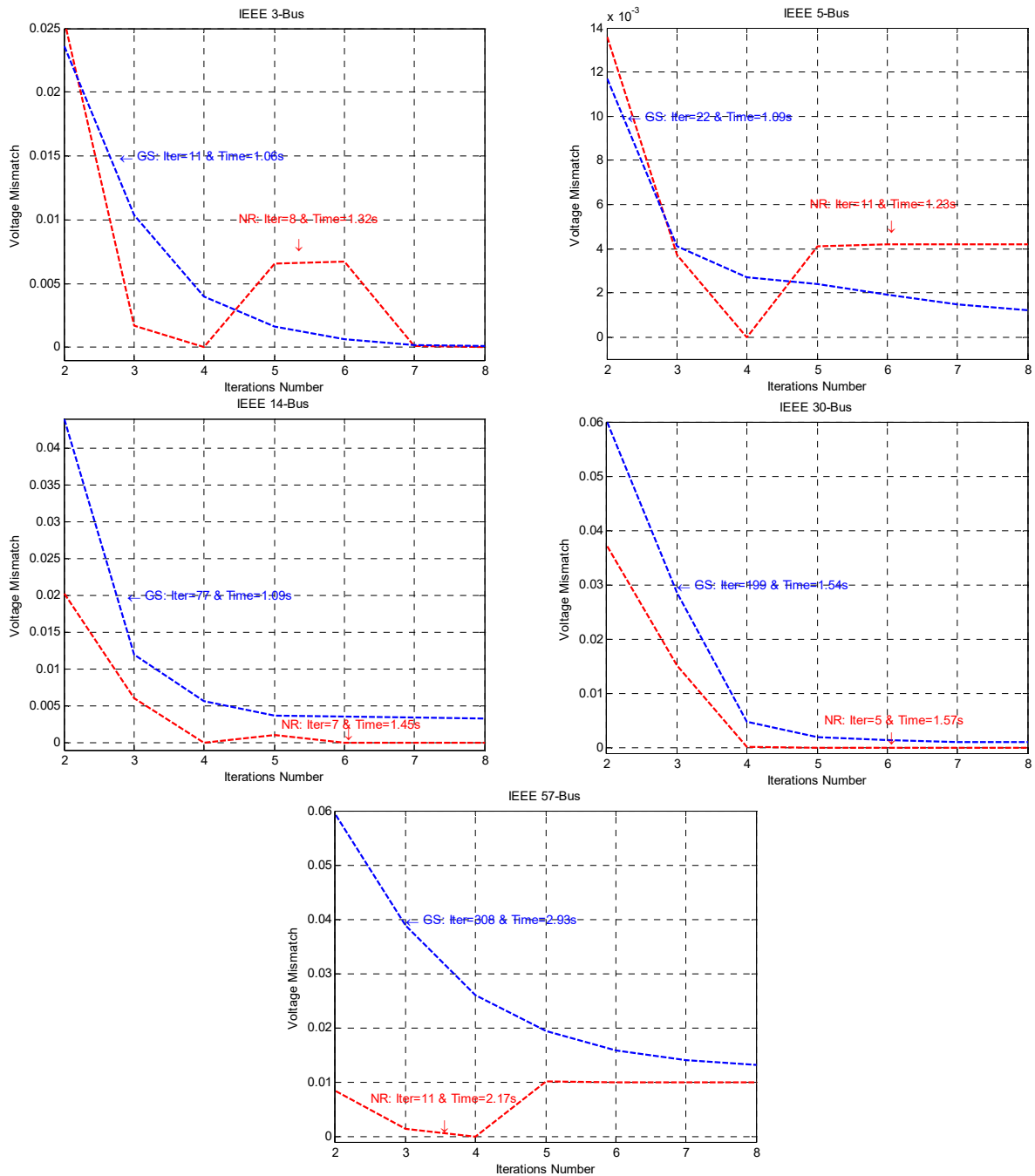| From Bus	To Bus	P(MW)	Q (MVar)	From Bus	To Bus	P(MW)	Q(MVar)	Line Loss P(MW)	Q(MVar)
1	2	184.58	148.14	2	1	-174.42	-127.82	10.16	20.32
1	3	35.24	92.65	3	1	-34.35	-89.98	0.89	2.68
2	3	-225.57	-117.11	3	2	234.35	134.66	8.77	17.55
Total Loss								19.83	40.54

Fig. 2 Comparison of convergence for different standard IEEE test bus system using NR method and GS method
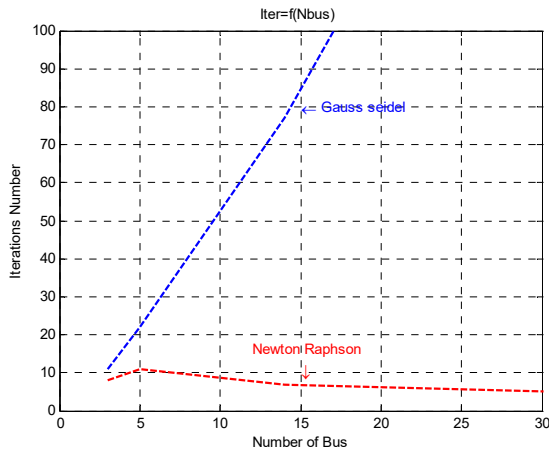
Fig. 3 Comparison of number of iterations for different standard bus test system using NR method and GS method
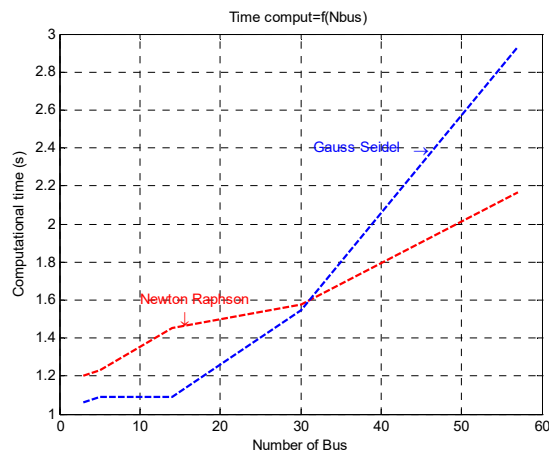

Fig. 4 Comparison computational time for different standard IEEE test bus system using NR method and GS method

which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR method. They require about 10 iterations for convergence in large systems.

V. CONCLUSION

This paper has described a comparison of NR and GS methods for Power Flow Analysis. Five different standard IEEE bus test systems are considered to investigate the effectiveness of the proposed methods. The compared results show that the NR is the most reliable method because it has the least number of iteration and converges faster.

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[8]

TABLE VII
PERFORMANCES OF THE COMPARED STUDY

IEEE Bus System test	3	5	14	30	57
GS					
Number of Iterations	11	22	77	199	308
Computation Time (s)	1.06	1.09	1.09	1.54	2.93
NR					
Number of Iterations	8	11	7	5	11
Computation Time (s)	1.32	1.23	1.45	1.57	2.17

B. Discussion

The GS method is relatively easy to program however programming of NR method is more involved and becomes more complicated mostly if the buses are randomly numbered and the storage requirements are more (Jacobian elements).

The time per iteration in NR method is larger than in the GS method. In NR method, the time per iteration increases directly as the number of buses.

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method