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Comparison of Meshing Stiffness of Altered Tooth Sum Spur Gear Tooth with Different Pressure Angles

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***Abstract*—**The estimation of gear tooth stiffness is important for finding the load distribution between the gear teeth when two consecutive sets of teeth are in contact. Based on dynamic model a C-program has been developed to compute mesh stiffness. By using this program position dependent mesh stiffness of spur gear tooth for various profile shifts have been computed for a fixed center distance and altering tooth-sum gearing (100 by ± 4%). It is found that the C-program using dynamic model is one of the rapid soft computing technique which helps in design of gears. The mesh tooth stiffness along the path of contact is studied for both 20° and 25° pressure angle gears at various profile shifts. Better tooth stiffness is noticed in case of negative alteration tooth-sum gears compared to standard and positive alteration tooth-sum gears. Also, in case of negative alteration tooth-sum gearing better mesh stiffness is noticed in 20° pressure angle when compared to 25°.

***Keywords*—**Altered tooth-sum gearing, bending fatigue, mesh stiffness, spur gear.

I. Literature Background and Motivation

SPECIFIC tooth load per unit tooth deflection in a meshed gear system is called *Mesh Stiffness*. While gears in operation, the points of contact are moving continuously and the load shared at these points vary in magnitude and direction. Due to this dynamic loading, the gear teeth were subjected to bending fatigue which also effect on stiffness and vibration character of the meshed gear tooth [1]. In this context many researchers are proposed number of numerical methods. Wang [2] and Yang [3] proposed and proved numerical methods to calculate dynamic loads. Moore [4] and Atanasiu et al. [5] discussed the bending and tooth surface fatigue at very high cyclic loads. They reported the time-varying mesh stiffness of gear tooth represents the main cause of undesired vibrations in gear train. In addition to dynamic loading effect, transmission error and sliding friction is also the major sources of noise and vibration in meshing gears [4], [7]. The difference between the effective and the ideal position of the output shaft with respect to the input shaft is called the transmission error [5], [6]. Dynamic loading, transmission error and sliding friction are sources of noise and vibration in meshing gears due to non-uniform motion in gear tooth mesh. This occurs due to adjacent pitch error, profile error, misalignment and lead errors [4]. Frolov and Kosarev [8] analyzed influence of mesh stiffness, forces, pitch errors and profile errors on vibration. Burgess [9] studied the influence of moment of inertia on mesh stiffness. Fakhfakh et al. [10] used a mathematical model to describe stiffness. Skrickij et al. [11] reported a mathematical model to show the center distance influence on mesh stiffness of spur gears. Similarly, Gill Jeong [12] used Fourier series to describe a mathematical model for mesh stiffness. Litak and Friswell [13], [14] computed the mesh stiffness based on tooth breaks and pitch errors by applying Chaos theory. Lin and Serag [15] reported a computer based procedure for the optimization of gear train for better stiffness.

By the above literature studies, it is found that many researchers investigated the mesh stiffness of gear tooth and its effect on performance of gear train for standard tooth-sum gears. But, altered tooth-sum method of gear design is more versatile and flexible. In addition to this, the moderate material surface strength can be utilized to a maximum extent by altered tooth-sum method of gear design. In this context few literatures were found on stiffness study of altered tooth-sum gears. Also, few researchers including authors of this paper have published a research work on contact stress of altered tooth-sum spur gear tooth using profile modification technique [16], [17]. In this investigation we made an attempt to develop a C-program based on Lin model [18] and used to compute the mesh stiffness of altered toothed-sum gear for various profile shifts.

II. Methods

A. Parameters Considered for Stiffness Computation

The geometric parameters considered for computing mesh stiffness are: 2 mm module, 20 mm face width, 20° and 25° pressure angles. Steel was taken as the material for the study with a Young's modulus of 200 GPa. The applied tangential load of 10 N per unit mm of face width was taken for computation.

B. Empirical Approach

The mesh stiffness between an engaged gear pair consists of local Hertzian deformation in addition to tooth bending deflection. The unit width Hertzian stiffness K_h resulting from the tooth surface contact was approximated by [3] as mentioned in (1):

$$K_h = \frac{\pi E}{4(1-\vartheta^2)} \quad (1)$$

where, E is the Young's modulus and υ is the poison's ratio of the material.

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As per the literature survey the unit width Hertzian stiffness is related only to the gear material which remains constant along the path of contact [15].

Based on the computational analysis results conducted by [19], the unit width stiffness $K_i(r)$ for a single tooth i at the loading position r approximated using:

$$K_i(r) = (A_0 + A_1X_i) + (A_2 + A_3X_i)r - \frac{R_i}{(1+X_i)m} \quad (2)$$

where, $K_i(r)$ is the bending stiffness per unit tooth width *(N-mm* $^{-1}$*)*, X_i is the addendum modification coefficient and N_i is the number of teeth of i gear, r is the radial distance (mm), R_i is the radius of pitch circle and m is the module *(mm)*. The coefficients for a steel gear are

$$A_0 = 3.867 + 1.612\,N_i - 0.02916N_i^2 + 0.0001553N_i^3 \quad (3)$$

$$A_1 = 17.060 + 0.7289N_i - 0.0178N_i^2 + 0.0000999N_i^3 \quad (4)$$

$$A_2 = 2.637 - 1.222N_i + 0.02217N_i^2 - 0.0001179N_i^3 \quad (5)$$

$$A_3 = -6.330 - 1.033N_i + 0.02068N_i^2 - 0.0001130N_i^3 \quad (6)$$

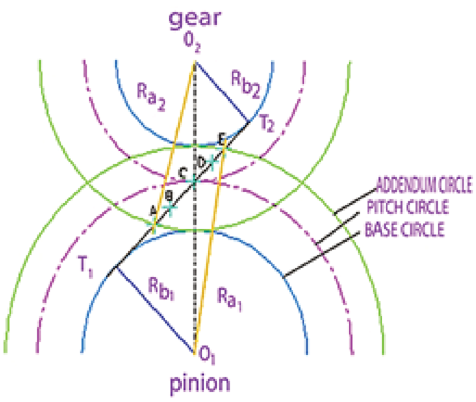

Fig. 1 Path of contact from beginning to end described as A, B, C, D and E (A -Beginning of contact, B_1-End of two pair mesh, B_2-Beginning of single pair mesh, C -Pitch point, D_1-Beginning of two pair mesh, D_2-End of single pair mesh, E -End of contact)

The single pair tooth stiffness K_A, K_B, K_D and K_E at contact points *A, B, D* and *E* (Fig. 1) is approximated by combining the unit width stiffness $K_1(r_1A)$, $K_2(r_2A)$, $K_1(r_1B)$, $K_2(r_2B)$, $K_1(r_1D)$, $K_2(r_2D)$, $K_1(r_1E)$, and $K_2(r_2E)$. The unit width Hertzian stiffness (K_h) of mating teeth computed as like springs connected in series as:

$$\frac{K_A}{F} = \frac{K_1(r_1A)K_2(r_2A)K_h}{K_1(r_1A)K_2(r_2A)+K_1(r_1A)K_h+K_2(r_2A)K_h} \quad (7)$$

$$\frac{K_B}{F} = \frac{K_1(r_1B)K_2(r_2B)K_h}{K_1(r_1B)K_2(r_2B)+K_1(r_1B)K_h+K_2(r_2B)K_h} \quad (8)$$

$$\frac{K_D}{F} = \frac{K_1(r_1D)K_2(r_2D)K_h}{K_1(r_1D)K_2(r_2D)+K_1(r_1D)K_h+K_2(r_2D)K_h} \quad (9)$$

$$\frac{K_E}{F} = \frac{K_1(r_1E)K_2(r_2E)K_h}{K_1(r_1E)K_2(r_2E)+K_1(r_1E)K_h+K_2(r_2E)K_h} \quad (10)$$

where, K_A, K_B, K_D and K_E represent the single tooth pair stiffness (N-mm^{-1}) of gears 1, 2 at mating points *A, B, D* and *E*. The *F* denotes the tooth width of spur gear. The mesh stiffness of engaged gear pair alternates with the change of contact position and the number of load sharing tooth pairs. The total mesh stiffness of two mating gears *(K)* was computed as per (11):

$$K = \frac{(\frac{K_A}{F} \times \frac{K_B}{F} x \frac{K_D}{F} x \frac{K_E}{F})}{(\frac{K_A}{F} + \frac{K_B}{F} + \frac{K_D}{F} + \frac{K_E}{F})} \quad (11)$$

III. Results and Discussions

The stiffness at various point of contacts such as, beginning point of contact, beginning of single pair mesh, end of single pair mesh and end of contact (point *A*, Point *B*, point *D* and point *E* respectively with reference to Fig. 1) have been computed by C-program for a tooth-sum of 100 and altered by ±4%. Fig. 1 shows the points along the length of path of contact referred as A, B, C, D and E [18], [19]. For external gear meshing, the pitch point C keeps constant position on the centerline of the gear, the points A, E and points B, D the single pair gearing segment changes their position along the theoretical gear segment T_1T_2. The maximum contact stresses and the shifting of the point C between points A, B and point D, E is going to occur at a particular value of profile shift [19].

Figs. 2 (a)-(c) show the stiffness at various points along the length of path of contact (K_A/F, K_B/F, K_D/F and K_E/F) for pinion pertaining to 48 teeth, 50 teeth, 52 teeth for various values of profile shift X_1 on the pinion for 25° pressure angle. Figs. 3 (a)-(c) show the stiffness for the pinion pertaining to the above mentioned tooth numbers for various values of profile shifts X_1 for 20° pressure angle.

From Fig. 2 (a), it is observed that the stiffness for 48 teeth at points B and D are higher compared to points A and E. In this case the stiffness value will lie between *24 N/mm to 27 N/mm*. The stiffness values increase initially with increased value of profile shift and after that starts decrease as the value of profile shift increases and observed same trend for at all points of contact (A, B, D and E). The stiffness values are crucial at initial and end point of contact. At these points the contact begins and ends between gears in mesh. At these points the stiffness values should be high due to that the gear tooth able to withstand the shock and vibration. It is observed from the plots that the stiffness remains same along the length of path of contact. This is occurring due to positive values of K_i(r) along the different points of contact (A, B, D and E). Also, it is noticed that the top land width of pinion and gear is 1.75 mm and 5.12 mm for profile shift X_1=0.3 as computed by C-program.

From Fig. 2 (b) it is observed that the stiffness values for 50 teeth at all the points of contact are less compared to previous tooth-sum. In this case the stiffness values at points B and D is higher compared to point A and E. Fig. 2 (b) it is noticed that the stiffness values are less in 100 tooth-sum compared to 96 tooth-sum. This occurs due to lesser K_i(r) at the points A, B, D and E along length of path of contact. The lesser K_i(r) is occurring due to negative value of profile shift on pinion compared to positive values of profile shift in 96 tooth-sums. Also, it is noticed that the top land width of pinion and gear is 1.46 mm and 5.18 mm for profile shift X_1= -0.9 as computed by C-program.

From Fig. 2 (c) it is observed that the stiffness is less compared to both the previous cases considered at all points of contact (A, B, D and E). In this case the stiffness values approximately remain constant at point B and D, whereas for points A and E the stiffness values fluctuate.

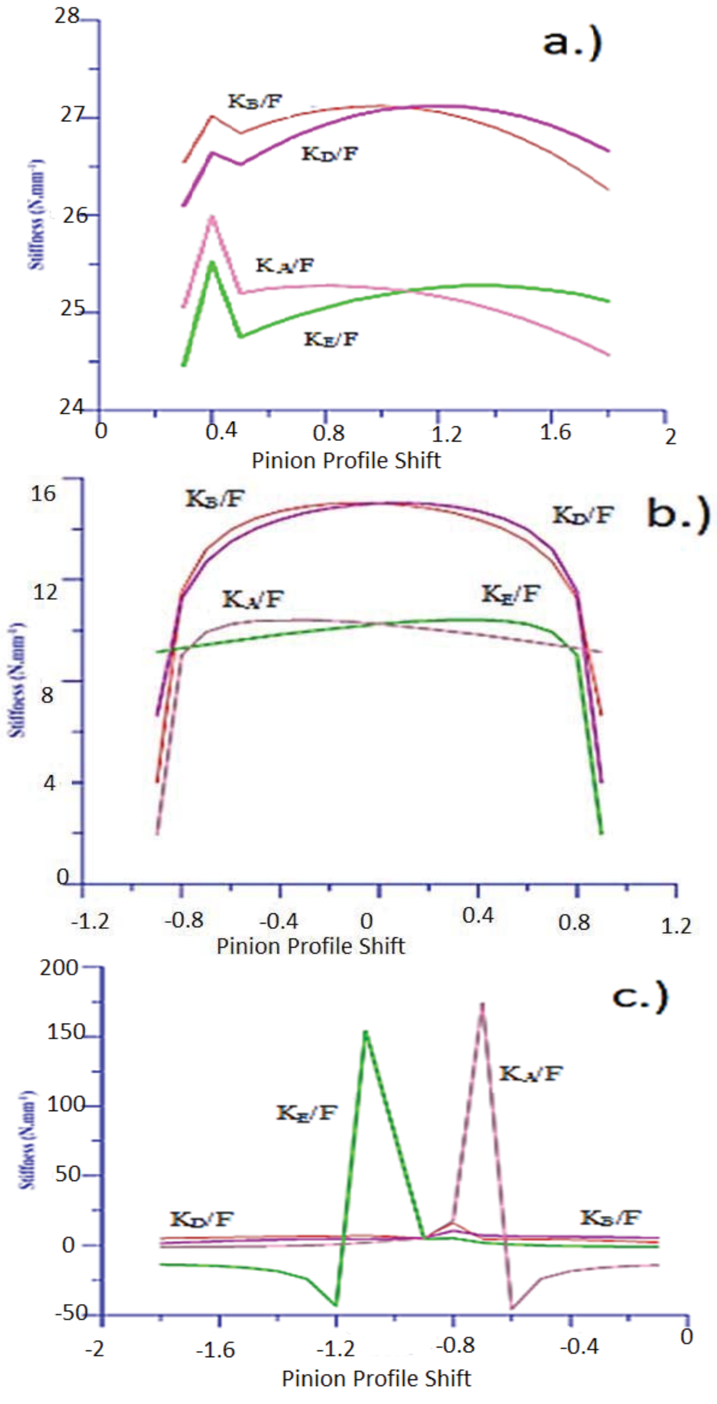

Fig. 2 Stiffness for 25° Pressure angle at different points along length of path of contact

It is observed that in all the cases considered the value of stiffness at point A continuously decreases as the tooth-sum increases from 48 teeth to 52 teeth. This remains same for other points of contact also. From this, it can be concluded that the negative alteration in tooth-sum has good stiffness compared to standard and positive alteration in tooth-sum.

In case of 25° pressure angle for 104 tooth-sum (Fig. 2 (c)) it is seen that the stiffness value for X1= -0.6 and X2=-1.202 at point A is -42.79 N/mm. This occurs due to negative value of K2(r2A) compared to all other points along path of contact. It is also observed that top land width of pinion and gear for this tooth-sum is 3.74 mm and 2.54 mm. From this it can be concluded that the top land width of the pinion is more compared to top land width of gear and hence the pinion is much stiffer compared to gear. The higher value of negative profile shift indicates that radius of curvature is more at root and hence tooth root is very weak for gear. This occurs for large negative value of profile shift on gear. This negative value of stiffness can be seen in positive alteration in tooth-sum and the range of negative stiffness is more in +4 alterations in tooth-sum.

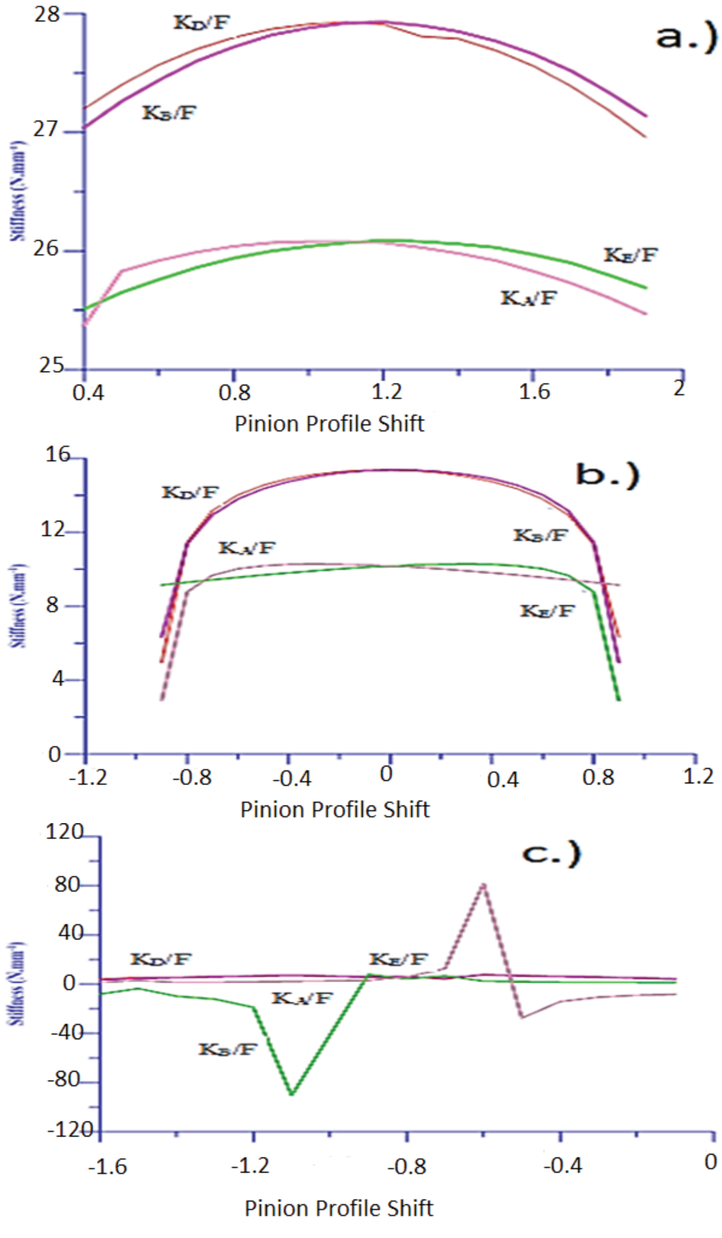

Fig. 3 Stiffness for 20° Pressure angle at different points along length of path of contact

Similarly, mesh stiffness values for 48 teeth, 50 teeth and 52 teeth for various values of profile shift for 20° pressure angle are plotted as shown in Figs. 3 (a)-(c). Fig. 3 (a) shows the mesh stiffness for 48 teeth for various values of profile shift. The above analysis holds good for 20° pressure angle also in the same way as 25° pressure angle. The negative alteration in tooth-sum is better due to smooth dividing of the carried force between gear teeth pairs in turn smother change of stiffness compared to standard and positive alteration. Further, it is noticed that the mesh stiffness is better in negative alteration in tooth-sum, standard and positive alteration in tooth-sum for 20° pressure angle compared to 25° pressure angle.

In case of 96 tooth-sums the profile shift will vary from 0.3 to 1.8. But, in case of 100 tooth-sums the profile shift will vary from -0.9 to 0.9. The percentage difference in mesh stiffness has been calculated by considering the range of profile shift from 0.3 to 0.9 which is common to both the cases. Fig. 4 (a) shows percentage difference in stiffness at different points along length of path of contact for 96 tooth-sums and 100 tooth-sums for 25° pressure angle. It is observed that the percentage difference at point KA/F for 96 tooth-sums is on an average around 1.64 times of 100 tooth-sums. This means that negative alteration in tooth-sums has more than 1.64 times stiffer compared to 100 tooth-sums. Similarly, for 96 tooth-sums at different points KB/F, KD/F and KE/f the stiffness 1.31, 1.63 and 2.94 times stiffer compared to 100 tooth-sums. Similarly, percentage of stiffness between 100 tooth-sums and 104 tooth-sums are as shown in Fig. 4 (b). It is observed that the percentage change in stiffness at different points along KA/F, KB/F, KD/F and KE/F will be 1.54, - 0.48, -0.46 and 0.88 times. In this case standard tooth-sum performs better compared to positive alteration in tooth-sum.

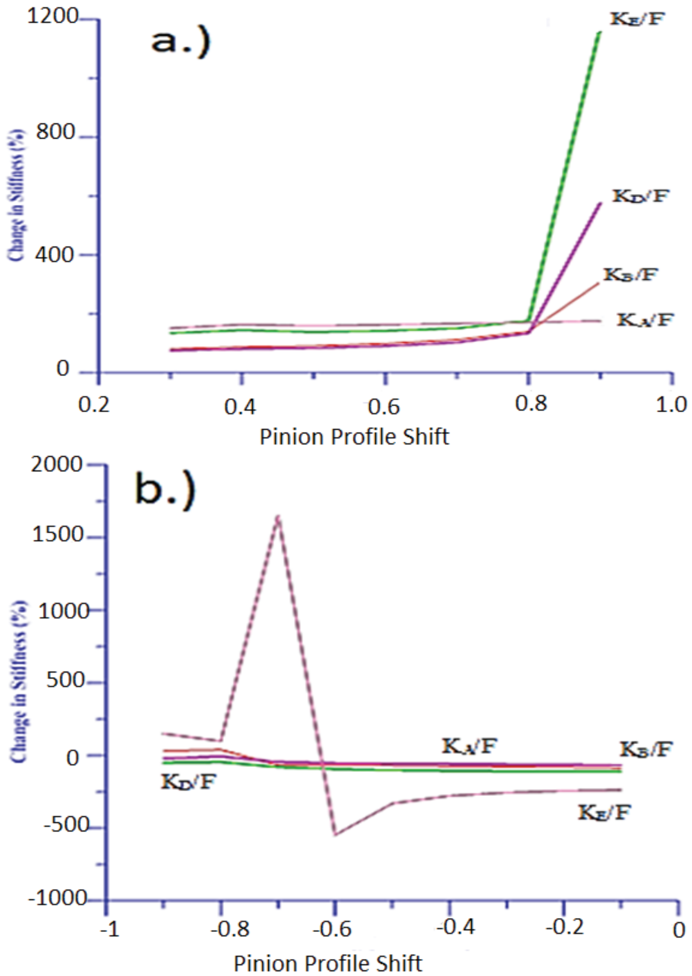

Fig. 4 (a) Percentage difference in stiffness for 25° between 96 and 100 tooth-sum 4. (b) 104 and 100 tooth-sum

Fig. 5 (a) shows percentage difference in stiffness at different points along length of path of contact for 96 tooth-sums and 100 tooth-sums for 20° pressure angle. In case of 96 tooth-sums the profile shift will vary from 0.4 to 1.9 whereas for 100 tooth-sums the profile shift will vary from -0.9 to 0.9. The percentage difference in mesh stiffness has been calculated by considering the range of profile shift from 0.4 to 0.9 which is common to both the cases. It is observed that the percentage difference between K_A/F for 96 tooth-sums and 100 tooth-sums is on an average around 1.72 times. The negative alteration in tooth-sums has more than 1.72 times stiffer compared to 100 tooth-sums. Similarly, for 96 tooth-sums at different points K_B/F, K_D/F and K_E/F are 1.45, 1.62 and 2.69 times stiffer compared to 100 tooth-sums. Similarly, the percentage difference between 100 tooth-sums and 104 tooth-sums is shown in Fig. 5 (b). It is observed that the percentage change in stiffness at different points along K_A/F, K_B/F, K_D/F and K_E/F will be -0.50, -0.50, -0.52 and -0.65 times. In this case standard tooth-sums exhibits better compared to positive alteration in tooth-sum.

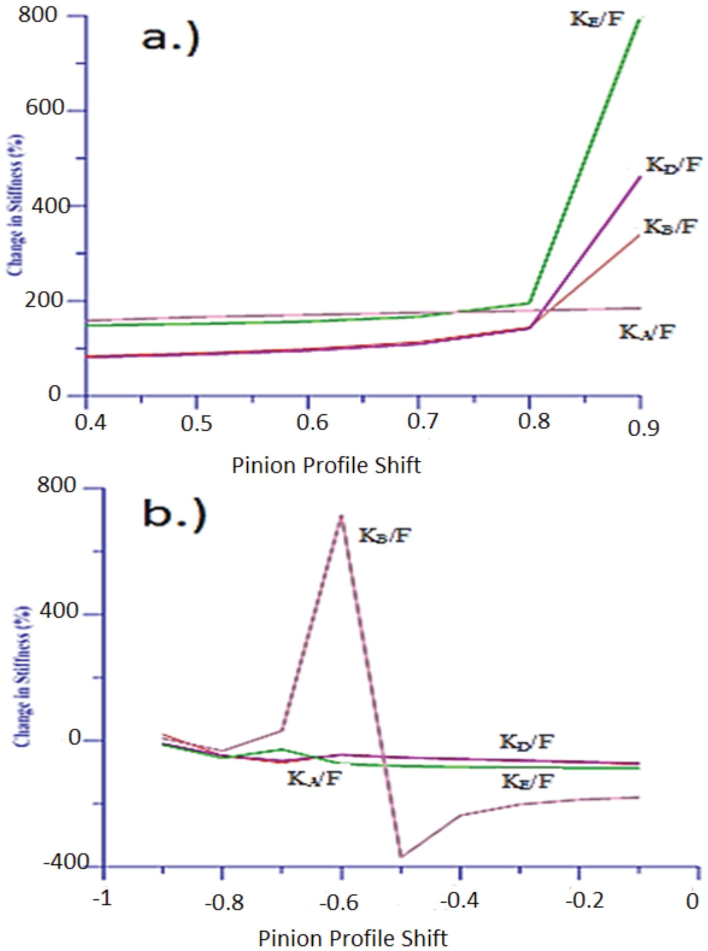

Fig. 5 (a) Percentage difference in stiffness for 20° between 96 and 100 tooth-sum 5. (b) 104 and 100 tooth-sum

IV. Conclusions

The dynamic model proposed by Lin was used to develop a C-program successfully and used to compute mesh stiffness for altered tooth-sums of 100 teeth altered by ±4%. The mesh stiffness at critical points A, B, D and E has been studied. Based on this study the following conclusions were drawn.

The mesh stiffness depends on unit width stiffness $K_i(r)$ which increased at different points along length of path of contact. Also, mesh stiffness depends upon the top land width of pinion and gear. For lower top land width of gear the value of $K_i(r)$ at points A will be low and in turn the stiffness will be low. Also, It is observed that the $K_i(r)$ at point A for gear will be negative which indicates that the gear is much more weaker compared to pinion. This is due to negative value of profile shift.

The mesh stiffness for negative alteration in tooth-sum at various points of contact was better compared to standard and positive alteration in tooth-sum in both 25° and 20° pressure angles. The mesh stiffness in negative alteration in 20° pressure angle gear tooth is better compared to 25° pressure angle.

The increase in mesh stiffness in negative alteration (96 tooth-sums) at initial point of contact and end of point of contact has 1.64 and 2.94 times stiffer respectively compared to standard tooth-sum in 25° pressure angle. Similarly the mesh stiffness in negative alteration (96 tooth-sums) at initial point of contact and end of point of contact has 1.72 and 2.69 times stiffer compared to standard tooth-sum in 20° pressure angle.

Nomenclature

K_h Hertzian Stiffness (N/mm)
E_1 Young's Modulus (Pinion) N/mm^2
E_2 Young's Modulus (Gear) N/mm^2
ϑ_1 Poisson's Ratio (Pinion)
ϑ_2 Poisson's Ratio (Gear)
X_i Addendum Modification Coefficient
N_i Number of teeth
m Module (mm)

R_i Pitch circle radius (mm)
F Tooth Width (mm)

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