

Comparison of Compression Ability Using DCT and Fractal Technique on Different Imaging Modalities

Sumathi Poobal, and G. Ravindran

Abstract—Image compression is one of the most important applications Digital Image Processing. Advanced medical imaging requires storage of large quantities of digitized clinical data. Due to the constrained bandwidth and storage capacity, however, a medical image must be compressed before transmission and storage. There are two types of compression methods, lossless and lossy. In Lossless compression method the original image is retrieved without any distortion. In lossy compression method, the reconstructed images contain some distortion. Direct Cosine Transform (DCT) and Fractal Image Compression (FIC) are types of lossy compression methods. This work shows that lossy compression methods can be chosen for medical image compression without significant degradation of the image quality. In this work DCT and Fractal Compression using Partitioned Iterated Function Systems (PIFS) are applied on different modalities of images like CT Scan, Ultrasound, Angiogram, X-ray and mammogram. Approximately 20 images are considered in each modality and the average values of compression ratio and Peak Signal to Noise Ratio (PSNR) are computed and studied. The quality of the reconstructed image is arrived by the PSNR values. Based on the results it can be concluded that the DCT has higher PSNR values and FIC has higher compression ratio. Hence in medical image compression, DCT can be used wherever picture quality is preferred and FIC is used wherever compression of images for storage and transmission is the priority, without losing picture quality diagnostically.

Keywords—DCT, FIC, PIFS, PSNR.

I. INTRODUCTION

A common characteristic of most images is that they contain some redundant information. Image compression is necessary to reduce the number of bits needed to represent an image by removing the redundancies as much as possible. Lossless compression is completely reversible but the compression ratio obtained is low. In lossy compression methods, irreversible, the decompressed images contain degradation compared to the original image, because the method completely discards redundant information.

In this paper the lossy compression methods DCT and Fractal image compression are used. However, lossy methods are capable of achieving much higher compression with visually lossless decompressed images.

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In medical images, using lossy compression, the issues of image quality are to be considered so that the reconstructed image has not lost diagnostically relevant information. In this work DCT and FIC are applied on different imaging modalities like CT Scan, Ultrasound, Angiogram, X-ray and Mammogram their compression ability and image quality are analysed.

II. FRACTAL IMAGE COMPRESSION

Fractal compression is ideally suited to real world image compression, due to the inherent fractal nature of many natural images [6][4]. FIC uses the property of self similarity of fractal objects. A fully automatic Fractal based image compression technique of digital monochrome images was first proposed by Jacquin [1]. The encoding process of an image is given by approximating the smaller range blocks, from the larger domain blocks until an affine contractive map is found. A domain is a region where the transformation maps from and a range is a region where it maps to. The number of transformations required to approximate the given image depends on image complexity.

FIC is obtained using Quadtree partitioning, one of the PIFS methods. Fractal encoding is a mathematical process to encode any given image as a set of mathematical data that describes the properties of the image. Fractal encoding relies on the fact that all objects contain information in the form of similar, repeating patterns called attractor. The encoding process has intense computations, since large number of iterations is required to find the fractal patterns in the image. Fractal compression provides a high degree of compression. Compressed image size does not generally increase linearly with increase in image size; it increases at a lower rate. Compression is non symmetrical. i.e. the time taken for encoding process is more than the decoding. The decoding process is much simpler as it interprets the fractal codes into the image. Decompression takes significantly less time than compression. [3][7][10].

A. Partitioned Iterated Function Systems

Normally, the natural images are not self similar. They do not contain affine transformation of itself. By partitioning the images into pieces, partitioned self-similarity can be achieved.

In PIFS, a fixed point, called an attractor, of an image f that satisfies $W(f) = f$; that is, when the transformations are applied to the image, the original image can be obtained. The contractive mapping theorem says that the fixed point of W will be the image obtained, when the sequence $W(f_0), W(W(f_0)), W(W(W(f_0))) \dots$, is computed, where f_0 is any image. The affine transformations can skew, stretch, rotate, scale and translate an input image.

The contractive mapping theorem can be applied to W^{om} , so it is sufficient for W^{om} to be contractive. [6]. The transformation W will be contractive, if the distance between any two points P_1, P_2 , is

$$d(w(P_1), w(P_2)) < s \cdot d(P_1, P_2), \text{ for } s < 1,$$

where s is the scaling factor and d is the distortion metric.

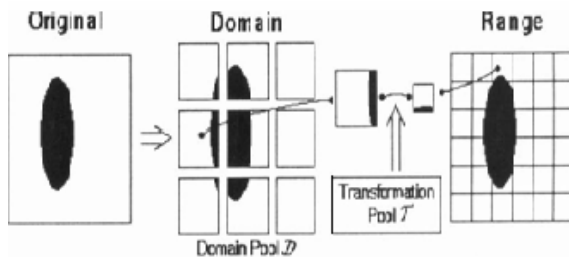


Fig. 1 Encoding process

Encoding phase of fractal image compression is illustrated in Fig 1. An image is partitioned into a number of disjoint blocks of size $B \times B$ called range blocks and a number of blocks of size $2B \times 2B$ called domain blocks. The encoding of each range block consists of finding the best affine transformation by searching a pool of domain (D) blocks. For each range block $R = (r_{ij})$, the domain pool searched to find the domain block $\hat{D} = \hat{d}_{ij}$ and the transformation τ , such that $\tau(\hat{D})$ provides best matching, that has low root mean square (RMS) error when mapped to R . The domain blocks are shrunk either by sub-sampling or by pixel averaging to match the range block size. Data compression is achieved by storing only the set of transformations, i.e. the fractal code, which contains the complete information required to reconstruct the image.

The decoding is done by iterating W from any initial image. After several iterations the decompressed image is closer to a fixed point.

B. Quad-Tree Partitioning

The most popular partitioning mechanism is partitioning the image in a tree structure. A quad-tree partitioning is a representation of an image as a tree in which each node corresponding to a square portion of the image contains four sub-nodes corresponding to the four quadrants of the square, the root of the tree being the initial image as shown in Fig. 2.

The selection of ranges is as follows: The squares at the nodes are compared with domains in the domain pool D , which are twice the range size. The pixels in the domain are

averaged in groups of four so that the domain is reduced to the size of range and the affine transformation of the pixel values is found that minimizes the root mean square (RMS) difference between the transformed domain pixel values and the range pixel values. Apart from offset and scaling, a domain block has eight isometric orientations to match a given range block. If the depth of the quadtree is less than an assumed maximum depth and if optimal RMS difference is larger than a tolerance factor, the range block will break up into squares, thereby creating additional ranges with corresponding transformation codes. The process is repeated until the optimal RMS is less than the tolerance factor. The set of transformations and domains are stored and the encoding process is completed [3][7][9][10].

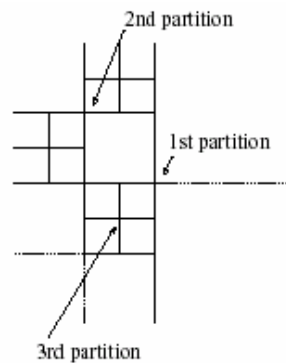


Fig. 2 Quadtree partitioning

Decoding process is done by iterating the set of transformations on an arbitrary initial image and the quadtree partition is used to determine the range in the image. For each range block, the size of the domain block that maps to it is shrunk by 2×2 pixels averaging. The pixel values of the shrunk domain block are then placed in the location in the range determined by the orientation information after scaling and offsetting. Computing all the range blocks constitutes one decoding iteration. After several iterations, the decompressed image will be very close to the original image.

As eyes are very sensitive to discontinuities at boundaries, the pixel values at block boundaries are smoothed by post processing.

III. DISCRETE COSINE TRANSFORM

The discrete cosine transform (DCT) is a popular compression scheme with an orthogonal basis and high energy compactness.

DCT has been successfully used in many coding systems due to its energy compactness in the frequency domain. That is, the original signals can be represented within a relatively narrow range of frequencies. Fig. 3 illustrates the coefficient distribution after 8×8 DCT transformation. All the coefficients have been rounded to the nearest integers. It is quite obvious that most of the energies are concentrated into the regions of low frequency, which appear in the upper left corner of the DCT, discarding the higher frequency

components that do not give rise to salient perceptual distortion after inverse DCT operation. DCT has excellent energy compaction for highly correlated data. It transforms a signal or image from the spatial domain to the frequency domain.

A. DCT Encoding

The 2D DCT for input data size $M \times N$ is defined as follows: Given a time domain data sequence $f(i,j)$, where $0 \leq i \leq M-1$, $0 \leq j \leq N-1$, the transformation of the sequence into a frequency domain data sequence $F(u,v)$, where $0 \leq u \leq M-1$, $0 \leq v \leq N-1$ is defined by the function in the equation for DCT as

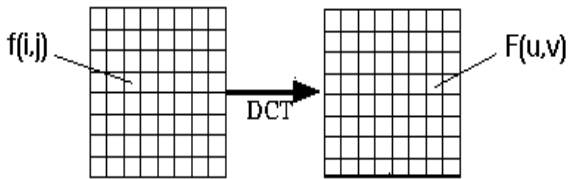


Fig. 3 Illustration of the coefficient distribution after 8x8 DCT transformation

$$F(u,v) = \frac{2}{\sqrt{MN}} C(u)C(v) \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(i,j) \cos\left(\frac{(2m+1)u\pi}{2M}\right) \cos\left(\frac{(2n+1)v\pi}{2N}\right) \quad \text{---(1)}$$

$$\text{Where } c(x) = \begin{cases} 1/\sqrt{2} & \text{for } x = 0; \\ 1 & \text{otherwise.} \end{cases}$$

M = No. of rows in the input data set,
 N = No. of columns in the input data set.
 i = Row index in the time domain
 $0 \leq u \leq M-1$,
 j = Column index in the time domain
 $0 \leq v \leq N-1$.
 $f(i,j)$ = Time domain data
 u = Row index in the frequency domain
 v = Column index in the frequency domain
 $F(u,v)$ = frequency domain coefficient

Decoding the compressed image is done by using Inverse DCT (IDCT), which is given as

$$f(i,j) = \frac{2}{\sqrt{MN}} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} C(u)C(v)F(u,v) \cos\left(\frac{(2m+1)u\pi}{2M}\right) \cos\left(\frac{(2n+1)v\pi}{2N}\right) \quad \text{--- (2)}$$

In this work M and N are considered to be the same as 256.

The equations 1 and 2 define $N \times N$ point DCT function. In image compression the input data set is usually partitioned into basic square blocks of data. The 8x8 DCT is most commonly used in compression applications [2] [8][11].

The DCT input is an 8 by 8 array of integers. This array contains each pixel's gray scale level; 8 bit pixels have levels from 0 to 255. Therefore an 8 point DCT would be

computationally easier to implement and more efficient to regard the DCT as a set of basis functions which, given a known input array size (8 x 8) can be pre computed and stored. The result is an 8×8 transform coefficient array in which the (0,0) element is the DC (zero-frequency) component and entries with increasing vertical and horizontal index values represent higher vertical and horizontal spatial frequencies. Smoother a function is, the fewer terms in DCT are required to represent it accurately, and the more it can be compressed. Compression is achieved by quantizing the transform coefficients. The quantization is done to compress the image without introducing visible artifacts. The 64 (8 x 8) DCT basis functions are illustrated in Fig. 4.

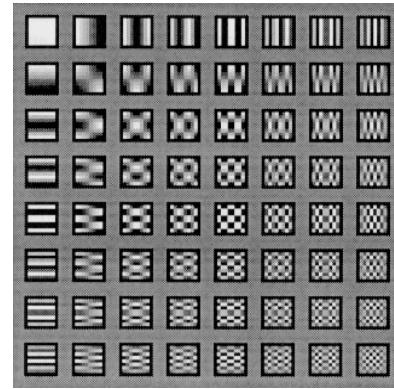


Fig. 4 Basis function of DCT

IV. IMAGE QUALITY

Signal to Noise Ratio (SNR) is a quantitative measure that estimates the quality of a reconstructed image with the original image [3][5][7]. PSNR has been accepted as a widely used quality measurement in the field of image compression.

Considering $(f(i,j))$ as original image that contains $N \times N$ pixel, values ranges between black (0) and white (255) and $(F(i,j))$ as reconstructed image the mean square error is given as

$$MSE = \sum \frac{[f(i,j) - F(i,j)]^2}{N^2} \quad (3)$$

The quality of the image is obtained by computing the peak signal- to- noise ratio (PSNR), of the reconstructed image, which is given by

$$PSNR = 20 \log_{10} \left(\frac{N}{RMSE} \right) \text{dB}, \quad (4)$$

where N is the largest possible value of the pixel in the image and RMSE is the root mean square error. Typical PSNR values range between 20 and 40 for good quality reconstructed image [5].

V. COMPRESSION RATIO

Data compression is the process of encoding information using fewer bits than an unencoded representation. The essential figure of merit for data compression is the compression ratio, or ratio of the size of a compressed file to the original uncompressed file.

VI. METHODOLOGY

Fractal Image compression using quad-tree partitioning and DCT are applied on grey scale images. The images of Uterus in Ultrasound, Chest wall in CT Scan, Brain in Angiogram, Chest in X-Ray and Mammogram of the size 256 x 256, with 8 bit grey scale are considered for the analysis.

The quad-tree partitioned compression technique is applied on the images. The standard parameters of FIC like scaling factor, offset factor, maximum and minimum tree partition depth, and tolerance factor are kept constant for all these images.

The quad-tree partitioned compression technique is applied on the images with the parameters of FIC like Tolerance factor T_{\max} , Minimum tree depth m , Maximum tree depth M , number of bits used for scaling factor s_i and offset factor o_i are fixed as 8, 4, 6, 5 and 7 respectively for all modalities of images.

A. FIC - Quadtree Encoding Algorithm

Mammogram images, chest images of Ultrasound, CT Scan, Angiogram, X-Ray, of size 256x256 of 8 bit gray scale is considered for the analysis. The tolerance criterion T_{\max} is varied from 1,2,...10. The parameters minimum tree depth m , maximum tree depth M , bits used for scaling factor s_i and offset factor o_i of quad tree partitioning are fixed as 4, 6, 5 and 7 respectively. Image is partitioned into four subnodes and is compared with domains from the domain pool D . The pixels in the domain are averaged, in groups of four so that the domain is reduced to range size. The root mean square (rms) value between the transformed domain pixel values and the range pixel values is found out as, $\text{rms} = \sqrt{\min E(R, D) / n}$, where n is the number of pixels in the range R . If the $\text{RMS} \geq T_{\max}$ and $\text{depth} \leq M$, image is partitioned further into four subnodes and is compared with domains from the domain pool D . The pixels in the domain are averaged, in groups of four so that the domain is reduced to range size. If the $\text{RMS} \leq T_{\max}$, the domain is mapped as w_i . The collection of all such maps is given as $W = \bigcup w_i$, where W is the encoded image.

B. Decoding

Decoding an image consists of iterating W from any initial image. For each range R_i , the domain D_i that maps is shrunk by two averaging non-overlapping groups of 2x2 pixels. The shrunken domain pixel values are then multiplied by s_i added to o_i and placed in the location in the range determined by the orientation information. This iteration is done until the fixed point is approximated by maximum number of iterations.

1. Post Processing Procedure

The pixels at the boundary of the range blocks are modified using the weighted average of their values. If the pixel values of a boundary are a and b then they are replaced by $w_1a + w_2b$ and $w_2a + w_1b$ with $w_1 + w_2 = 1$. Ranges that occur at the maximum depth of the quadtree are averaged with weights of $w_1 = 2/3$ and $w_2 = 1/3$. These values are largely heuristic but the results are satisfactory.

C. Steps Involved in DCT

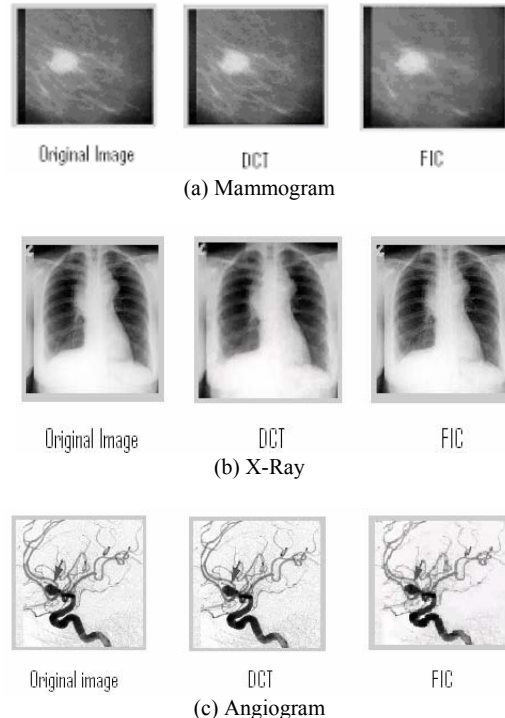
The image is split into 8x8 blocks. DCT is applied to each block. Each block is compressed through quantization. The quantization factor is fixed as 20. The array of compressed blocks that constitute the image blocks, which constitute the image, is stored in a drastically reduced amount of space. The compressed image is reconstructed through decompression using IDCT[2][8][11].

D. Computation of PSNR

The quality of the reconstructed image is arrived by computing the peak signal- to- noise ratio (PSNR) using (3) and (4).

VII. RESULTS

FIC and DCT are applied on images of Ultrasound, CT Scan, Angiogram, X-Ray, Mammogram and Compression ratio and PSNR values are computed as in Table I and analyzed.





(d) CT Scan

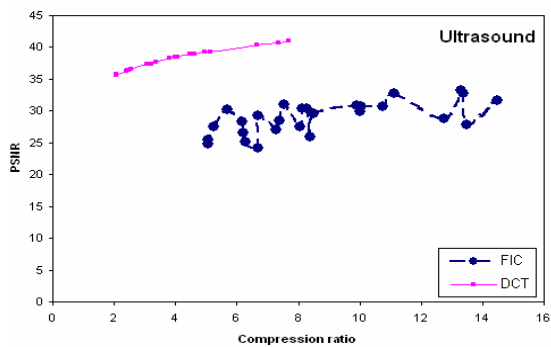


(e) Ultrasound

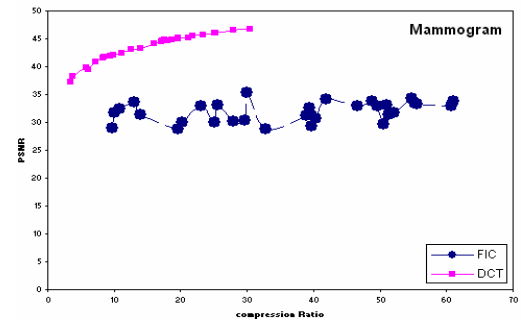
Fig. 5 The original images and the decompressed images of different modalities obtained by DCT and FIC

TABLE I
COMPRESSION RATIO AND PSNR VALUES OF FRACTAL IMAGE COMPRESSION
AND DCT ON DIFFERENT IMAGING MODALITIES

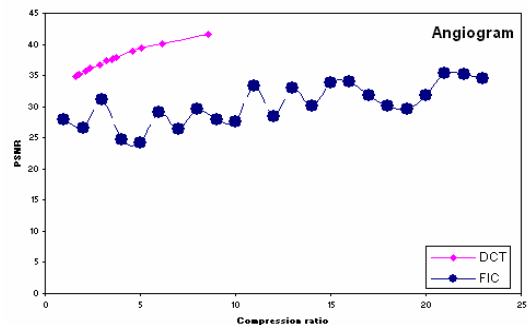
S.No	Modality of Image	Discrete Cosine Transform		Fractal Image Compression	
		Comp. Ratio	PSNR in dB	Comp. Ratio	PSNR in dB
1	Ultra-sound	3.11	37.09	8.66	28.65
2	CT Scan	3.97	38.11	10.08	27.32
3	Angiogram	4.24	38.36	16.03	30.26
4	X-Ray	11.10	42.22	19.71	31.95
5	Mammogram	19.70	44.82	35.74	31.90



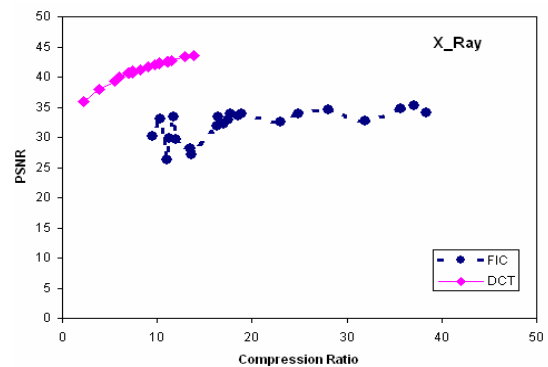
(a)



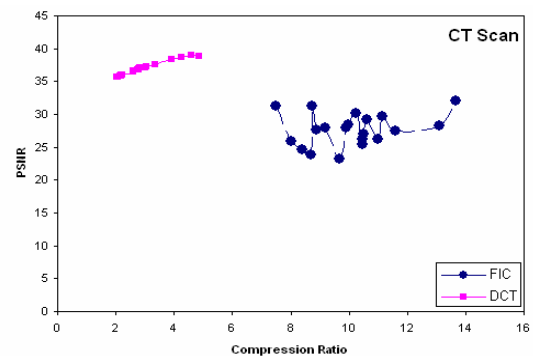
(b)



(c)



(d)



(e)

Fig. 6 (a-e) The graphs between compression ratio and PSNR in dB for different modalities of images like for FIC and DCT

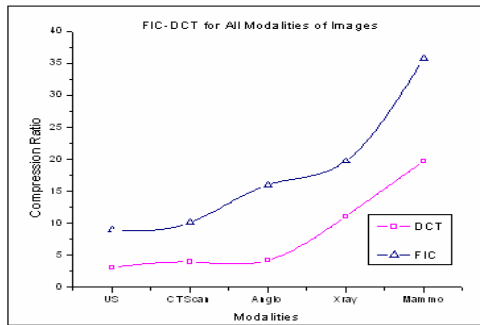


Fig. 7 The graph indicating compression ratio for all modalities of medical images like Ultrasound, CT Scan, Angiogram, X-ray and Mammogram

VIII. CONCLUSION

From the values computed and presented in Table I, it can be concluded that, Compression ratio for Discrete Cosine Transform (DCT) varies from 3.11 to 19.07 and in the case of Fractal Image Compression it varies from 8.66 to 35.74, with Ultrasound images being the lowest and Mammogram being the highest. From the Fig. 6 (a-e) and Fig. 7, it can be concluded that,

1. For all modalities of images considered in this work, in FIC the compression ratio is higher than DCT.
2. PSNR computed for FIC is lower than DCT, but within allowable range that is between 20dB and 40dB for a quality reconstruction of the image.
3. DCT is preferred wherever picture quality is required and FIC is preferred wherever higher compression for transmission and storage without losing picture quality diagnostically.
4. The compression ratio in Ultrasound images is the lowest in both FIC and DCT as the image has complicated contents.
5. Whenever the image is uniform like in the case of mammogram, the compression ratio is higher in both FIC & DCT compared to other imaging modalities.

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He was awarded Brammiah Sastri Memorial Award & Dr. E. Balakrishnan Award for contribution in the area of Biomedical Engineering.