

# Comparison between Deterministic and Probabilistic Stability Analysis, Featuring Consequent Risk Assessment

Isabela Moreira Queiroz

**Abstract**—Slope stability analyses are largely carried out by deterministic methods and evaluated through a single security factor. Although it is known that the geotechnical parameters can present great dispersal, such analyses are considered fixed and known. The probabilistic methods, in turn, incorporate the variability of input key parameters (random variables), resulting in a range of values of safety factors, thus enabling the determination of the probability of failure, which is an essential parameter in the calculation of the risk (probability multiplied by the consequence of the event). Among the probabilistic methods, there are three frequently used methods in geotechnical society: FOSM (First-Order, Second-Moment), Rosenblueth (Point Estimates) and Monte Carlo. This paper presents a comparison between the results from deterministic and probabilistic analyses (FOSM method, Monte Carlo and Rosenblueth) applied to a hypothetical slope. The end was held to evaluate the behavior of the slope and consequent risk analysis, which is used to calculate the risk and analyze their mitigation and control solutions. It can be observed that the results obtained by the three probabilistic methods were quite close. It should be noticed that the calculation of the risk makes it possible to list the priority to the implementation of mitigation measures. Therefore, it is recommended to do a good assessment of the geological-geotechnical model incorporating the uncertainty in viability, design, construction, operation and closure by means of risk management.

**Keywords**—Probabilistic methods, risk assessment, risk management, slope stability.

## I. INTRODUCTION

THE study to analyze the stability of the slope can be done through deterministic or probabilistic methods. Normally, a stability analysis is performed to determine the conditions for the project in such a way to ensure the required minimum security.

The probabilistic analysis differs from the deterministic methods, mainly because it considers the variability of the parameters. Most of the input data in a slope stability analysis are not known with precision. The variability is due to the dispersion of the results of tests or to the natural variability in the value of the grandeur that exists from one point to another in the slope. It is, therefore, a distribution of values for each parameter, which are considered random variables. Thus, it is concluded that the safety factor itself is a random variable which depends on many input variables and has its own distribution [13].

Isabela Moreira Queiroz is with Federal University of Minas Gerais, Belo Horizonte, MG 34000-000 Brazil (phone: (31) 98855-0727; e-mail: belamq@yahoo.com.br).

There are three commonly used probabilistic methods in geotechnical medium: FOSM (First-Order, Second-Moment), Rosenblueth (Point Estimates) and Monte Carlo.

The Monte Carlo method establishes that the probability distribution functions of the independent variables are initially known. In the absence of these, it usually adopts a normal distribution. This method has the advantage of obtaining the distribution of safety factor (dependent variable). As an example of a disadvantage, the method demands time, large computational effort, and specific programs [1].

The FOSM method (First-Order, Second-Moment) presented by [2], is based on the truncation of the Taylor series for determination of the dependent variable (safety factor). The average value of the dependent variable is calculated from the average values of the independent variables. The standard deviation is calculated from the input parameters and variances of the derivatives of the dependent variable for each independent variable. An advantage of this method is the quantification of the influence of each independent variable in the variance of the safety factor. As a disadvantage, this method does not obtain a distribution of the safety factor and so adopts assumptions on this distribution. In addition, the maximum probability of failure is not always related to the surface of failure for the minimum safety factor [3].

The last commonly used method would be the method of point estimates [4]. The Rosenblueth method requires the knowledge of the distribution functions of the independent variables, using only their values calculated in so-called point estimates (average standard deviation and mean less standard deviation). The dependent variable is calculated for these points, obtaining a sample of what one can calculate the corresponding average and standard deviation. The method is easy to apply. One must, however, take a distribution for the safety factor (usually normal) and it is assumed that the distribution of each independent variable is symmetric.

While the deterministic approach adopts the safety factor as stability index in the balance problem of the slope, the probabilistic methods adopt as the probability of failure, which is a data input on risk analysis.

The risk assessment process generally involves the scope definition and selection of the method of analysis, definition and identification of hazard conditions, estimate the probability of failure and consequence, risk estimation, documentation, verification, and analysis update. The methods of risk assessment can be qualitative or quantitative. The

qualitative methods rely on descriptive forms or sort numerical scales to describe the greatness of probability and consequence, whereas the set quantitative uncertainties are based, therefore, on numeric values of probability and consequence [2]–[5].

This paper compares the deterministic and probabilistic analysis applied to a hypothetical slope stability, and finally a risk assessment is carried out by quantitative method.

## II. CASE STUDY

The slope that was analyzed in this paper has about 40.0 m and an inclination of approximately 1V: 2H. The independent variables ( $X_i$ ) considered for the probabilistic analysis are ( $c'$ ) cohesion, friction angle ( $\phi'$ ), and the weight ( $\gamma$ ).

The parameters ( $\bar{X}_i$ ) used were referred to the deterministic

medium safety factor of about 1.5. Although fictitious values adopted are quite representative. The standard deviations ( $\sigma[X_i]$ ) have been determined based on the typical literary variation coefficients (Table I).

Table II summarizes the values adopted. Fig. 1 shows the geological-geotechnical model adopted, considering the reduced water level.

TABLE I  
TYPICAL VALUES OF COEFFICIENT OF VARIATION ACCORDING TO [6]

Parameter	Coefficient of variation (%)
Specific Weight	03 (02 to 08)
Cohesion	40 (20 to 80)
Effective Friction angle	10 (04 to 20)
Cohesion Not Drained	30 (20 to 50)

TABLE II  
GEOTECHNICAL PARAMETERS OF THE MATERIALS USED

Lithology	Cohesion (kPa)			Friction angle ( $^\circ$ )			Weight (kN/m <sup>3</sup> )		
	Mean	CV (%)	Standard deviation	Mean	CV (%)	Standard deviation	Mean	CV (%)	Standard deviation
Sandy Silt	5.0	40%	2.0	29.0	10%	2.9	21.1	3%	0.6
Pebble Sand	5.0	40%	2.0	34.0	10%	3.4	29.0	3%	0.9
Residual Soil	10.0	-	-	34.5	-	-	20.0	-	-
Saprolite	10.0	-	-	39.0	-	-	21.0	3%	-

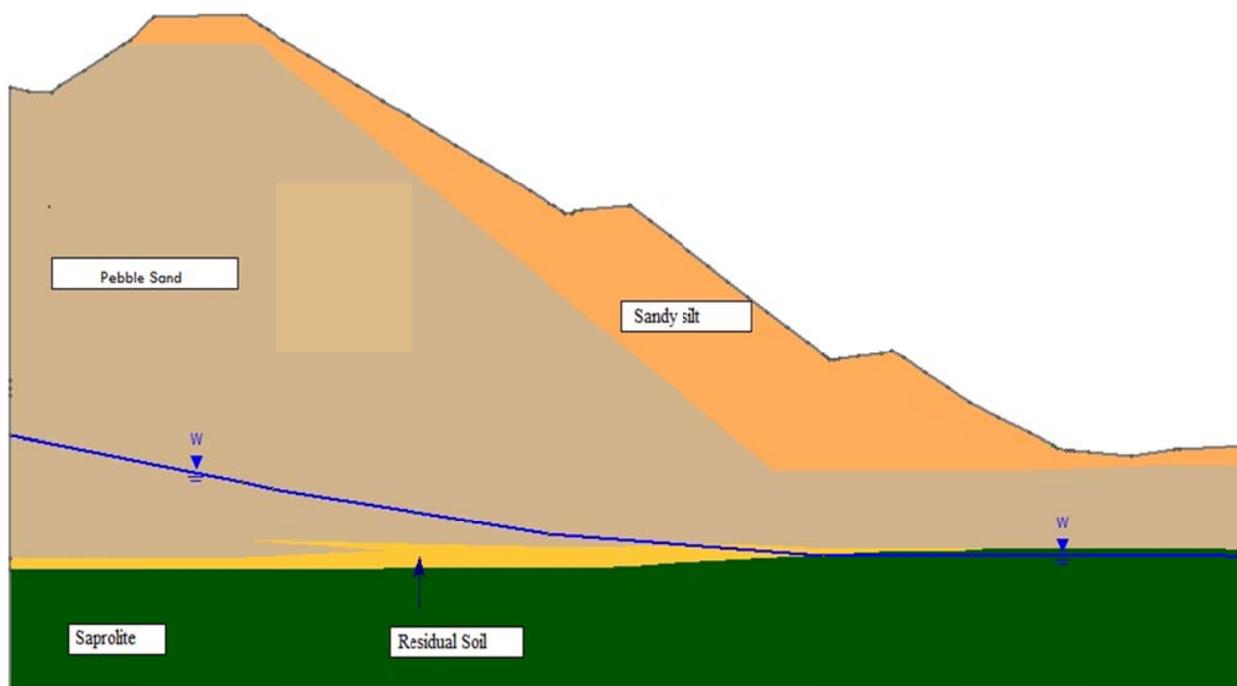


Fig. 1 Geological-geotechnical model used

## III. CALCULATION METHODOLOGY

Analyses were carried out with the assistance of Slide 6.0 program, developed by the company Rocscience. The safety factor was calculated by the method GLE/Morgenstern-Price.

The probability of failure was calculated by using probabilistic methods of Monte Carlo, FOSM (First-Order, Second-Moment) and Rosenblueth (Point Estimates).

The application of these three probabilistic methods requires the existence of a mathematical formulation (empirical, analytical or numerical) that relates the fault indicator (safety factor-dependent variable) with the input data (material properties, charging, water level-independent variables) [6].

Considering the safety factor as a random variable, the probability of failure can be computed as the probability of the

FS is less than or equal to 1, IE ( $P [FS \leq 1]$ ).

The probability of failure is being increasingly used as a criterion for acceptance in the last 35 years, although it is still viewed with varying degrees of enthusiasm and skepticism [7]. These authors point out the following advantages in calculating the probability of failure:

1) Allows the variation of the resistant forces and mobilizing

forces and helps to establish the level of confidence in the project. The reliability of the structure is its probability of success. Therefore, if the probability of failure of a slope is 20%, their reliability is 80%;

2) Is an essential parameter in the quantification of risk value (R), defined by Fig. 2.

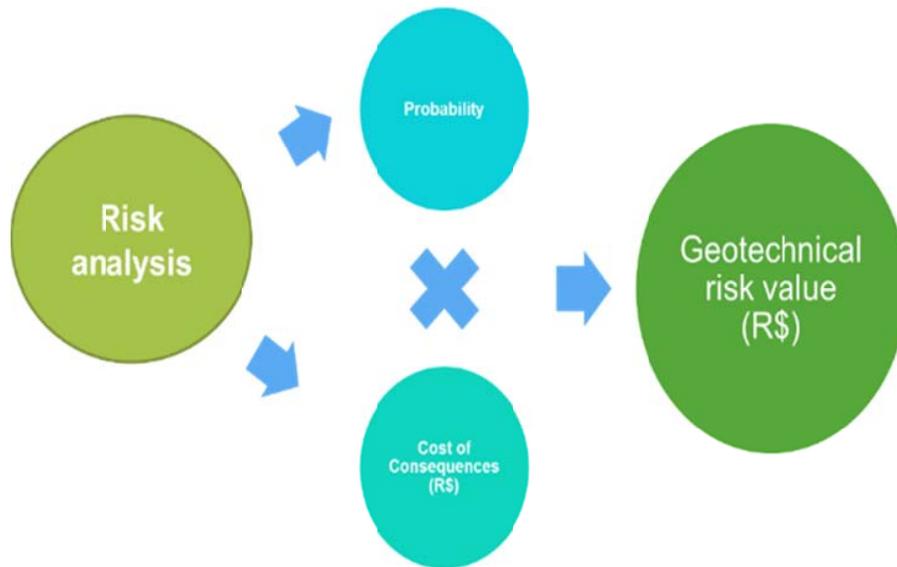


Fig. 2 Representation of the risk analysis

According to [5], risk assessment is based on the use of the information available to estimate the relative risk for individuals or populations, or properties, due to conditions of danger. It involves the breakdown or decomposition of the system and source of risk in its fundamental parts.

Technically, the risk assessments can be qualitative or quantitative. Qualitative analysis is used in a way that is descriptive or numerical sorting scales to present the magnitude of potential consequences and their probability of occurrence. The quantitative analysis is based on numeric values of the potential consequences and their probabilities, assuming that such values are a valid representation of the real magnitude of the consequences and the probability of the scenario studied [6].

The probability of failure is represented by the area under the curve of probability distribution, contained the left of  $FS = 1.0$ , as it is illustrated in Fig. 3. In this figure, the probability distributions of two cases, A and B, are represented. The case presents an average safety factor equal to 1.5 and a standard deviation of 0.25 and if B has an average safety factor of 2.0 and standard deviation of 0.85. According to Fig. , one can see that the probability of the case is lower than in case B, so, the project presents a lower possibility of undesirable events (break) [8].

The evaluation of the probability distribution of the safety factor (dependent variable) and the calculation of the probability of failure are carried out through the application of probabilistic methods, which are Monte Carlo methods,

FOSM, and method of Rosenblueth (Point Estimates).

*A. Monte Carlo Method*

The Monte Carlo method was first used as a research tool in the development of the atomic bomb during World War II [1]-[8]. It is an iterative procedure which covers four steps, described below [9]:

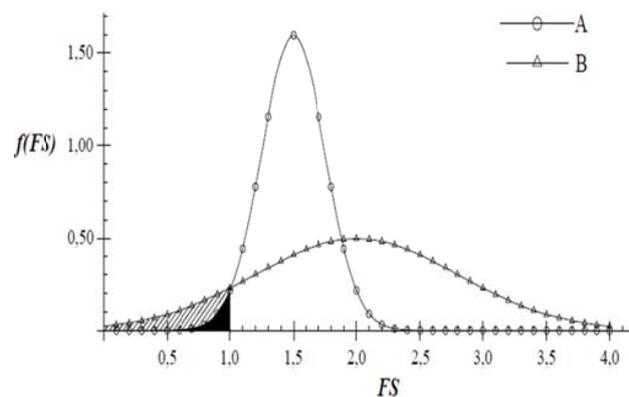


Fig. 3 Statistical distributions with different safety factors and standard deviation [8]

- 1) Estimate of the probability distributions for each of the input parameters considered as variables;
- 2) Generation of random values for each parameter;
- 3) Calculation of the operating forces;

- 4) Repeat the process N times (N > 100) and then determine the probability of failure  $P_f$  according to the following equation:

$$P_f = \frac{N-M}{N} \quad (1)$$

where M is the number of times that the resistant force exceeded the operating forces (i.e., the factor of safety is greater than 1.0).

**B. FOSM Method**

The FOSM method (First-Order, Second-Moment) was initially used in the steel industry projects [8]. It is a method that uses the first terms of the Taylor series expansion of the function of performance (i.e. safety factor) to estimate the expected value (mean) and the variance of the performance. It is called for a second time because the variance is a form of second moment, being the result of higher order statistics used in the analysis [2].

Assuming that multiple random variables are independent of each other, the equations of the FOSM method correspond to:

$$E[F] = F(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_N) \quad (2)$$

$$V[F] = \sum_1^n \left( \frac{\delta F_i}{\delta X_i} \right)^2 \times V[X_i] \quad (3)$$

where [F] = average value expected of F; V [F] = variance of

F which is equal to the square of its standard deviation;  $(\delta)$  F = variation that occurs when range from a value of  $(\delta)$ ;  $x_i$  each of n parameters  $x_i$ ;  $(\delta) X_i$  = rate of change of the variables involved in the study;  $V [X_i]$  = variance of each  $x_i$ .

If the number of random variables is equal to n, this method requires the making of n + 1 tests.

**C. Rosenblueth Method**

The method of estimate or Rosenblueth method was created in 1975. In this method, the distribution of a random variable  $X_i$  is concentrated on two particular points located by:

$$X_{i+} = \bar{X}_i + \sigma_i \quad (4)$$

$$X_{i-} = \bar{X}_i - \sigma_i \quad (5)$$

where:  $\bar{X}_i$  and  $\sigma$  are the mean value and the standard deviation of the distribution of the variable  $X_i$ , respectively.

“The method of estimation, for the case of n variables correlated, need  $2^n$  estimated values for each combination of the particular points  $X_{i-}$  and  $X_{i+}$ . After the completion of the combination of the particular points, starts the probabilistic procedure through deterministic calculations, for this group of  $2^n$  values. So, for each new probabilistic process step, the deterministic calculations are powered by a group of new values (data) input to be used in the next calculation “[10].

Fig. 4 illustrates the procedure for calculating the Rosenblueth method. It should be noted that this method assumes that the distribution of safety factor is normal and should be used when the random variables follow the normal distribution [11].

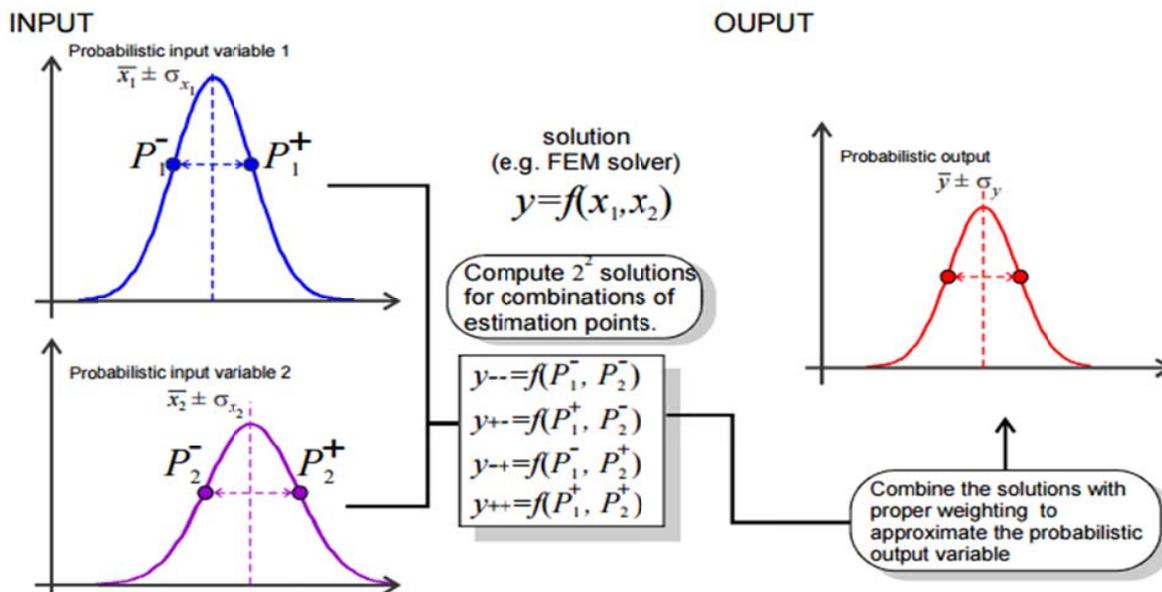


Fig. 4 Diagram of the procedure for calculating the Rosenblueth method [11]

## IV. RESULTS

## A. FOSM Method

The first analyses by the FOSM method uses the mean values of random variables, resulting in an average safety factor. The other analyses apply a rate of increase of 5% to 10% over the average value of each random variable. From the application of (3), the percentage of contribution of each random variable on the variance of the safety factor. For the calculation of the probability of failure, it was considered a normal distribution for the safety factor and the following equation in Microsoft Excel was used:

$$DIST. NORM. N(x; \text{average}; \text{standard deviation}; \text{cumulative}) \quad (6)$$

where the value of  $x$  corresponds to the minimum value adopted for the safety factor. There is also the logical value "cumulative" which must be set to true for a cumulative distribution function.

The results presented in this paper consider the calculation of probability of failure, that is,  $P_f[FS \leq 1.0]$ . Here, the results are obtained by applying the FOSM method.

Fig. 5 shows the critical surface of failure and the minimum safety factor obtained for the slope, and Fig. 6 illustrates the results of applying the FOSM method. Fig. 6 shows that the random variable that is mostly contributed to the variation of the safety factor was the friction angle of sandy silt (71.8%).

The cohesion of the sandy silt contributed 28.1%, and the specific weight contributed with only 0.1%.

The Table III displays the value of the probability of failure determined by the FOSM method, adopting a normal distribution for the safety factor.

## B. Monte Carlo Method

Large The Slide program 6.0 allows probabilistic analyses using the Monte Carlo method, where the user can choose to use the options "Global Minimum" or "Overall Slope".

For the analyses presented in this paper, it was opted for the "Global Minimum" option, where the probabilistic analysis is performed on the surface of slipping which presents the overall minimum safety factor, calculated by a deterministic stability analysis.

The safety factor is then calculated  $N$  times (where  $N$  is the number of iterations) to this failure surface, by using a different set of values (input data), generated randomly for each analysis. The probability of failure is the number of analyses that resulted in a safety factor less than 1.0 divided by the total number of iterations. The analysis option "Global Minimum" assumes that the probability of failure calculated for the critical surface with the minimum safety factor is representative of the probability of failure for the slope as a whole [11].

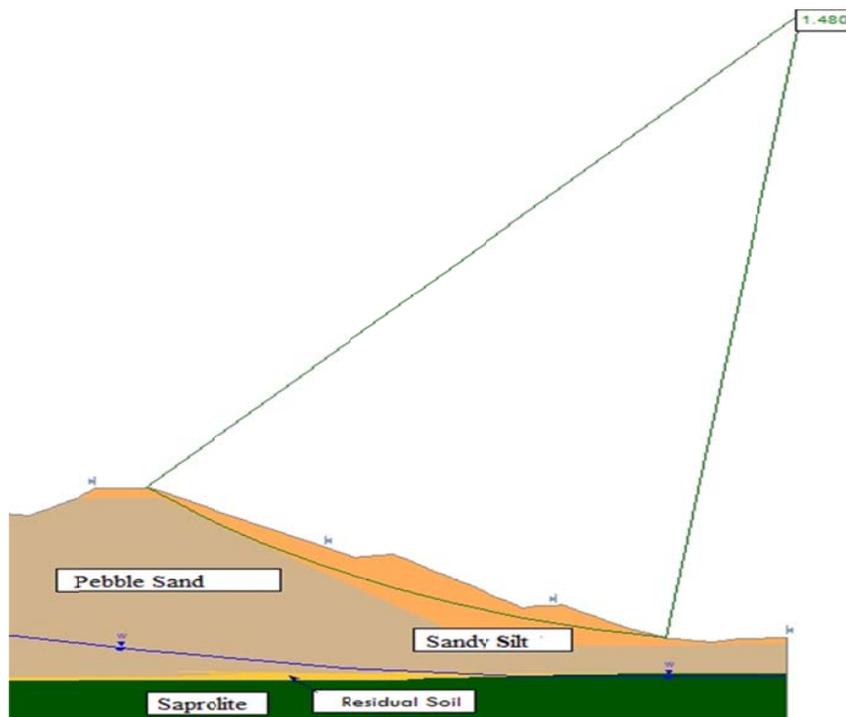


Fig. 5  $FS_{\text{medium}} = 1.48$  (Deterministic)

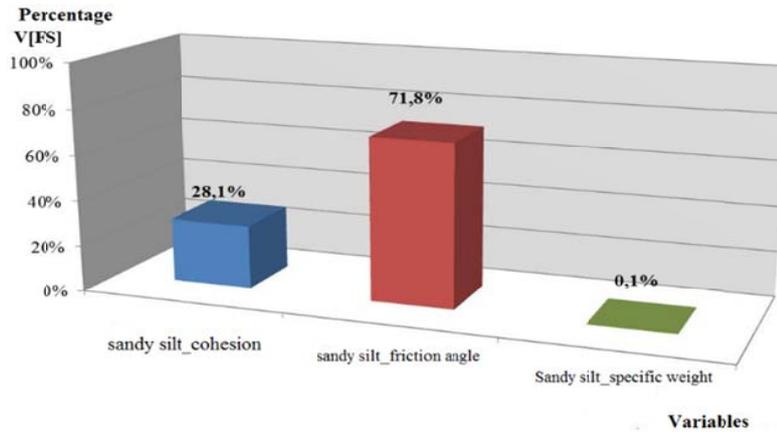


Fig. 6 Percentage of influence of random variables in the variation of the safety factor of slope

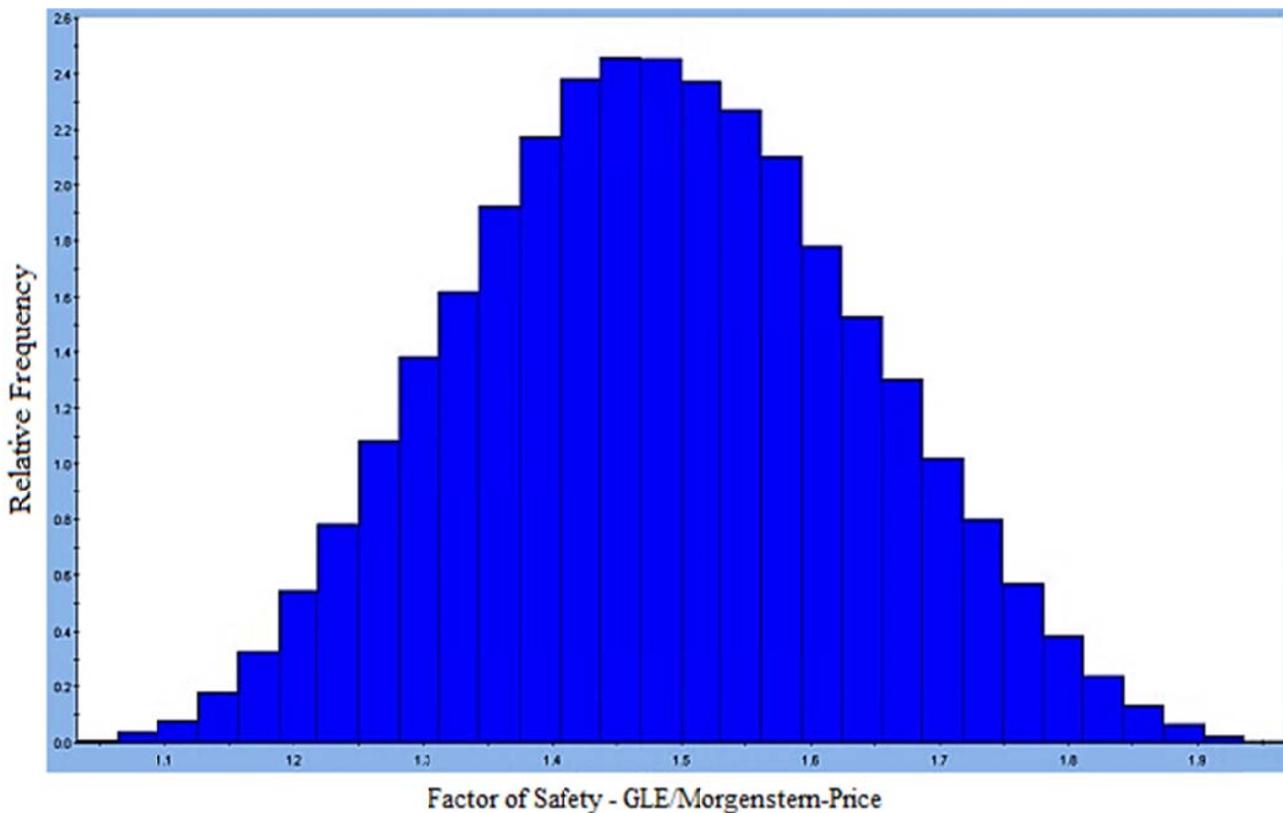


Fig. 7 Histogram of the safety factor

TABLE III  
SUMMARY OF THE RESULTS OF ANALYSIS BY APPLYING THE FOSM METHOD

FSmedium	$\sigma$ [FS]	$P_f$ [FS $\leq$ 1.0]	Statistical Distribution
1.48	0.15	$7 \times 10^{-4}$	Normal

where:  $\sigma$  [FS] = standard deviation of the safety factor;  $P_f$  [FS  $\leq$  1.0] = probability of failure.

The number of iterations performed to achieve the convergence by the Monte Carlo Method depends on the level of trust ( $\alpha$ ) admitted to the case study and can be estimated according to the following equation:

$$N = \left( \frac{Z_{\alpha/2}^2}{4\alpha^2} \right)^n \quad (7)$$

where, N = number of attempts; Z = parameter of reliability;  $\alpha$  = Tolerance; n = number of variables.

In this way, with the three random variables and a tolerance level ( $\alpha$ ) equal to 10%, the minimum required number of iterations by the Monte Carlo method, estimated by (7), would be approximately 310,000.

In cases where the number of iterations was insufficient for the convergence and/or none or very few safety factors, values

calculated during simulation by Monte Carlo Method, were less than 1.0, the area below the histogram of the distribution of safety factor was calculated by (6) presented earlier.

The Table IV shows the result obtained by Monte Carlo Method, and Fig. 7 shows the histogram of the resulting safety factor of iterations performed on Slide 6.0 program.

TABLE IV  
TEST RESULT BY APPLYING THE MONTE CARLO METHOD

FS medium	$\sigma$ [FS] <sup>(1)</sup>	$P_f$ [FS $\leq$ 1.0] <sup>(1)</sup>	Statistical Distribution
1.48	0.15	$8 \times 10^{-4}$	Normal

Notes: <sup>(1)</sup>  $\sigma$  [FS] = standard deviation of the safety factor;  $P_f$  [FS  $\leq$  1.0] = probability of failure.

C. Rosenblueth Method

TABLE V  
ROSENBLUETH METHOD-COMBINATIONS

Number of Calculation	Particular Points			Values			FS
	C' silt	( $\phi$ ) silt	$\gamma$ silt	C' silt (kPa)	( $\phi$ ) silt ( $^\circ$ )	$\gamma$ silt (kN/m <sup>3</sup> )	
1	+	+	+	7.0	31.9	21.7	1.638
2	-	+	+	3.0	31.9	21.7	1.550
3	+	-	+	7.0	26.1	21.7	1.398
4	-	-	+	3.0	26.1	21.7	1.247
5	+	+	-	7.0	31.9	20.5	1.635
6	-	+	-	3.0	31.9	20.5	1.558
7	+	-	-	7.0	26.1	20.5	1.412
8	-	-	-	3.0	26.1	20.5	1.254

For the application of the method of Rosenblueth, the three random variables used in previous analyses were considered,

resulting in eight combinations and values of FS presented in Table V.

The Table VI TABLE presents the result obtained by applying the method of Rosenblueth, adopting a normal distribution for the safety factor.

TABLE VI  
TEST RESULT BY APPLYING THE METHOD OF ROSENBLUETH

FSmedium	$\sigma$ [FS]	$P_f$ [FS $\leq$ 1.0]	Statistical Distribution
1.46	0.16	$2 \times 10^{-3}$	Normal

where:  $\sigma$  [FS] = standard deviation of the safety factor;  $P_f$  [FS  $\leq$  1.0] = probability of failure

TABLE VII  
SUMMARY OF THE RESULTS OBTAINED BY PROBABILISTIC METHODS

Method	E [FS]	$\sigma$ [FS]	$P_f$ [FS $\leq$ 1.0]	Distribution function
FOSM	1.48	0.15	$7 \times 10^{-4}$	Normal
Monte Carlo	1.48	0.15	$8 \times 10^{-4}$	Normal
Rosenblueth	1.46	0.16	$2 \times 10^{-3}$	Normal

V. SUMMARY OF THE RESULTS OF THE ANALYSES

Table VII shows a comparison between the results of the various probabilistic methods. It is observed that, in general, the probability values obtained by the three probabilistic methods, were quite close.

The values obtained for the FOSM and Monte Carlo methods, for example, have the same order of magnitude. Considering the Rosenblueth method, the order of magnitude of values is greater than the FOSM method and the Monte Carlo method.

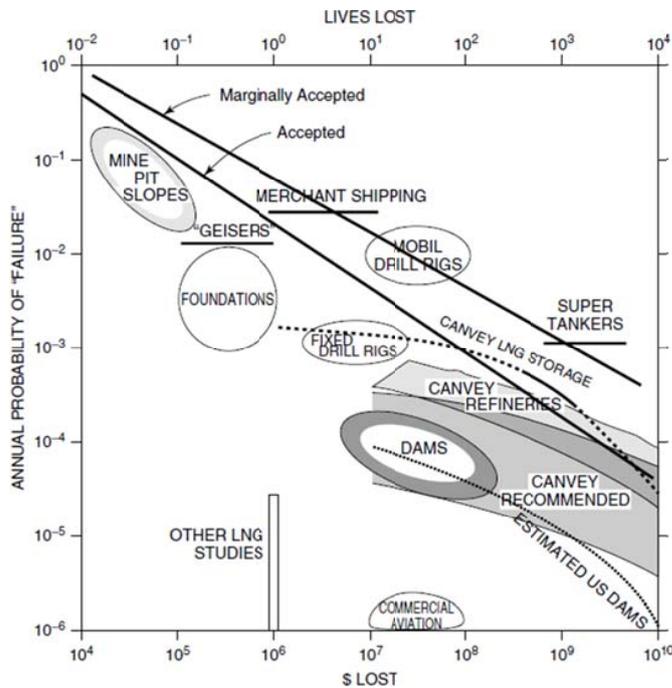


Fig. 8 Acceptable levels and marginally acceptable risk [2]

## VI. CONCLUSION

It should be emphasized that a great advantage of the probabilistic analysis is to reduce the level of conservatism, as it can be considered the variability of geotechnical properties of the materials and not just a single value.

A slope in which predominated the effect of friction angle of sandy silt was analyzed. The slope was reviewed by three probabilistic methods (FOSM, Monte Carlo, and Rosenblueth). In general, the following points can be highlighted.

The accurate determination of the standard deviation of each variable in the problem is important. Small errors in the coefficient of variation of a variable, can lead to significant errors in the probability of failure in a slope that depends strongly on this variable.

At first, the specific weight cannot be discarded. This parameter affects both resistive efforts, as active. In this way, it can positively or negatively affect the safety factor.

It is emphasizing the importance of strict determination of the surface of failure, once all other variables are calculated around this surface.

The random variables to be used in the other two methods (Monte Carlo and Rosenblueth) are obtained by using the FOSM method and they are the variables that contributed most in variation of the safety factor.

Considering all the cases analyzed, the one that presents the highest probability value corresponds to the method of Rosenblueth ( $P_f = 2 \times 10^{-3}$ ).

By this way, having assessed the probability of failure, it would be possible to perform the calculation of risk.

The acceptability of the individual risk of breach is on the order of  $10^{-6}$  the  $10^{-6}$  to  $10^{-8}$  per person per year. The individual risk, usually associated with the probability of loss of a human life, is a suggestive shape to represent the risk, as it allows their immediate comparison with different types of accident [12].

The acceptability of the societal risk is calculated by FN curves, established in terms of the number of victims, N, and the corresponding annual probability of failure (or frequency, F, accumulated by slope per year), with an expected value of victims equal to or greater than N. The criteria for acceptability and tolerability represent the maximum permissible limits for the risk and they are represented in the following graphic, where the annual probability of failure is on the vertical axis and the consequence, both in monetary cost and loss of life, on the horizontal [2].

The study of the consequences was not performed, so the risk assessment could not, effectively, be evaluated. Therefore, the risk was not appreciated in relation to any criterion of tolerance necessary to decide whether the risk should be mitigated.

Another important note is that values of probability of failure are annual and must be updated whenever changes occur in the conditions assumed in the analysis or a greater knowledge of the properties and geotechnical parameters.

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