Comparative Study of Static and Dynamic Bending Forces during 3-Roller Cone Frustum Bending Process

Mahesh K. Chudasama, Harit K. Raval

Abstract—3-roller conical bending process is widely used in the industries for manufacturing of conical sections and shells. It involves static as well dynamic bending stages. Analytical models for prediction of bending force during static as well as dynamic bending stage are available in the literature. In this paper bending forces required for static bending stage and dynamic bending stages have been compared using the analytical models. It is concluded that force required for dynamic bending is very less as compared to the bending force required during the static bending stage.

Keywords—Analytical modeling, cone frustum, dynamic bending, static bending.

NOMENCLATURE

- \( C \) = strength coefficient for dynamic bending in N/mm²
- \( E \) = Young’s modulus (N/mm²)
- \( K \) = strength coefficient for static bending (N/mm³)
- \( m \) = strain rate sensitivity index
- \( n \) = strain hardening exponent
- \( P \) = Vertical load at the top roller and bending plate interface (N)
- \( R \) = radius of curvature of the bent plate (mm)
- \( r_i \) = radius of bottom roller (mm)
- \( t \) = thickness of the plate (mm)
- \( U \) = Vertical distance travelled by the top roller for first stage of static bending (mm)
- \( x \) = half the horizontal distance of the bottom roller centers (mm)
- \( y \) = distance of fiber from neutral plane (mm)
- \( y_p \) = distance of the fiber upto which elasticity \( E \) is constant (mm)
- \( \beta \) = bottom roller inclination angle
- \( \gamma \) = Shear strain
- \( \epsilon \) = strain
- \( \dot{\epsilon} \) = strain rate (m/s)
- \( \zeta \) = stress ratio
- \( \theta \) = Angle between frictional force and horizontal plane at the roller plate interface (radians)
- \( \mu \) = coefficient of friction at roller plate interface
- \( \nu \) = Poisson’s ratio
- \( \sigma_i \) = yield stress (N/mm²)
- \( \tau \) = Shear stress

I. INTRODUCTION

The 3-roller bending is a continuous bending operation in which a metal sheet or plate is passed through sets of three rollers. Metal plate is kept between a top roller and two bottom rollers as shown in Fig. 1. The bending is achieved by lowering the top roller and then rotating the bottom rollers.

Fig. 1 Plate kept between Top and Bottom Rollers

The conical shells or sections widely used in process industries are manufactured by 3-roller bending machines. The mechanics of bending forces involved in the processing of conical sections or shells by 3-roller bending machines is complex and 3-dimensional force pattern is observed [1]. Roll bending process is widely used process for producing cylindrical as well as conical sections and shells. Various researchers have analyzed the processes of roll bending, with the assumption of single pass bending analytical models for the moment and spring-back prediction for the roller bending of plates for cylindrical bending were developed [2]-[4]. Various researchers have developed analytical models of force or moment prediction for cylindrical bending [5]-[7]. Cone bending using compatible rollers have been investigated considering geometry using FEA methods [8]. Reaction force over the rollers using Finite Element Analysis has been reported and the results have been used to study the effects of temperature [9]. From the reviewed literature it is also found that work related to machine setting for required geometry and force for 3-roller bending has been reported [10]. 3-roller conical bending process for various cone geometries has been investigated considering the machine setting parameters [11], [12]. The 3-roller conical bending process is completed in of four stages: Static bending, forward rolling, backward rolling and unloading the plate from the rollers. The top roller of the 3-machine applies the force to bend the plate during the static bending stage. Analytical models for bending force prediction during static as well as dynamic bending stages have been reported [13]-[15]. The external bending moment required by the roller was equated with the internal bending moment developed in the plate to resist the bending to develop the analytical model of force predictions. Internal bending moment induced in the plate has been formulated assuming the simplified stress conditions. Based on the stress conditions, three cases can be assumed namely, 1. Major stress along an axis only, 2. Principal stress coinciding with the normal axis, and 3. Shear stresses along with the normal stresses. It is observed that case 3, i.e. considering shear stresses along with the normal stresses gives better...
approximation of the bending force as compared to first two cases. Analytical models of bending force for static and dynamic bending stages are reproduced as (1) and (2) respectively:

\[
P_{\text{static}} = \frac{2}{\pi} \int_{0}^{\pi} \left( \frac{\tan y}{2} + \tan y \right) \frac{1}{1 - \sin \theta} \sqrt{1 - \cos^2 \theta} \, dy + \int_{0}^{\pi} \frac{2}{\pi} \left( \frac{\tan y}{2} + \tan y \right) \frac{1}{1 - \sin \theta} \sqrt{1 - \cos^2 \theta} \, dy
\]

where \( E'' = \frac{E''}{1 + \nu} \); \( y'' = y'' \); \( \nu' = \nu' \); \( K' = \frac{K^2}{1 + \nu} \).

\[
P_{\text{dynamic}} = \frac{2}{\pi} \int_{0}^{\pi} \left( \frac{\tan y}{2} + \tan y \right) \frac{1}{1 - \sin \theta} \sqrt{1 - \cos^2 \theta} \, dy + \int_{0}^{\pi} \frac{2}{\pi} \left( \frac{\tan y}{2} + \tan y \right) \frac{1}{1 - \sin \theta} \sqrt{1 - \cos^2 \theta} \, dy
\]

where, \( x' = x - r_1 \sin \theta \); \( a' = a - r_1 \sin \theta \); \( U' = U - r_1 (1 - \cos \theta) \).

For comparison of static bending force and dynamic bending force, geometrical and material parameters (as per Table I), have been inserted in the analytical models of static and dynamic bending force. As the stress conditions considering case 3 are giving better approximation, analytical models considering case 3 for static as well as dynamic bending are used for comparison. Analytical results of bending force for static and dynamic bending stage have been plotted on the same graph as shown in Fig. 2 considering different thicknesses and different bottom roller inclinations.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Material Property Parameters Taken for Analytical Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate Thickness, ( t ) (mm)</td>
<td>Yield Stress, ( \sigma_y ) (N/mm²)</td>
</tr>
<tr>
<td>5.84</td>
<td>350</td>
</tr>
<tr>
<td>7.88</td>
<td>278</td>
</tr>
<tr>
<td>8.86</td>
<td>320</td>
</tr>
<tr>
<td>11.84</td>
<td>346</td>
</tr>
<tr>
<td>13.96</td>
<td>270</td>
</tr>
</tbody>
</table>

For metal forming processes the range of the coefficient of friction from 0.2 to 0.3. [16]. Hence the range of the values of friction coefficient \( \mu \) is 0.2 to 0.3 is taken for the analytical calculation. Vertical Displacement of top roller \( U \) for the first pass is 20 mm for plate thicknesses 5.84 mm, 7.87 mm and 8.85 mm, while \( U \) is 15 mm and 10 mm for plate thicknesses of 11.84 mm and 13.97 mm respectively. Bottom roller radius \( r_1 \) is 80 mm and value of angle \( \theta \) is calculated from the geometry of the setup and thickness of the plate. The value for strength coefficient \( 'C' \) ranges from 1000 to 2000 N/mm² for structural steels [17]. Average value for the strength coefficient \( 'C' \) as1500 N/mm² is taken for dynamic bending force calculations. Similarly the value of strain rate \( 'e' \) is taken to be 0.001 and the value of strain rate sensitivity index \( 'n' \) is taken as 0.02 [17].

Dynamic bending force as percentage of static bending force has been calculated and tabulated as shown in Table II.

It can be observed from Fig. 2 and Table II that dynamic bending force required to bend the same thickness e.g. to bend 5.84 mm thickness plate is low. The similar results have been reported by Gajjar et al. for cylindrical bending with 3-roller bending process [18]. Dynamic bending force is almost 10 to 20 % of the static bending force for the same thickness of the plate. Figs. 3 (a) and (b) show the bending phenomena occurring during static as well as dynamic bending. During static bending the whole plate length between the bottom rollers has to be bent by the bending force applied by the top roller. In case of dynamic bending, the bending line progresses through the plate length between bottom rollers and at instantaneous time the plate material near the bending line is to be bent and not the whole plate material between the bottom rollers as shown in Fig. 3 (b). The bending progresses through the plate length gradually and it will require less dynamic bending force as compared to static bending, wherein static bending the bending takes place throughout the material simultaneously and it will require higher bending force.

Again value of static bending forces is in the range of 10 to 20 times the dynamic bending forces because of the friction phenomenon. The friction at roller-plate interface opposes the bending force in case of static bending as shown in Fig. 4 (a) while the friction at roller-plate interface is in favor of the bending force in case of dynamic bending as shown in Fig. 4 (b). Hence, the bending force during static bending is quite higher as compared to the bending force required during the dynamic bending.
Fig. 2 Comparison of static and dynamic bending force
It can also be observed from the Table II that as the bottom roller inclination increases dynamic bending force is relatively more with respect to static bending force, e.g. for 7.88mm thick plate and $\beta = 0$ the percentage of dynamic bending force is 9.33%, while for the same plate thickness and $\beta = 3.71$ it is 27.12%. It means that relatively higher dynamic bending force is required for higher bottom roller inclination for the constant plate thickness. It can be because of the shear stresses. During dynamic conditions shear stresses affects the dynamic bending force more as compared to affecting the static bending force.

It can also be observed from Table II that there is no set pattern of variation for either static bending force or dynamic bending force with respect to bottom roller inclination. In general it can be said from Table II that the static bending force decreases as the bottom roller inclination increases while the dynamic bending force increases as the bottom roller increases.

REFERENCES