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Buckling of Plates on Foundation with Different Types of Sides Support

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***Abstract*—**In this paper the problem of buckling of plates on foundation of finite length and with different side support is studied.

The Finite Strip Method is used as tool for the analysis. This method uses finite strip elastic, foundation, and geometric matrices to build the assembly matrices for the whole structure, then after introducing boundary conditions at supports, the resulting reduced matrices is transformed into a standard Eigenvalue-Eigenvector problem. The solution of this problem will enable the determination of the buckling load, the associated buckling modes and the buckling wave length.

To carry out the buckling analysis starting from the elastic, foundation, and geometric stiffness matrices for each strip a computer program FORTRAN list is developed.

Since stiffness matrices are function of wave length of buckling, the computer program used an iteration procedure to find the critical buckling stress for each value of foundation modulus and for each boundary condition.

The results showed the use of elastic medium to support plates subject to axial load increase a great deal the buckling load, the results found are very close with those obtained by other analytical methods and experimental work.

The results also showed that foundation compensates the effect of the weakness of some types of constraint of side support and maximum benefit found for plate with one side simply supported the other free.

***Keywords*—**Buckling, Finite Strip, Different Sides Support, Plates on Foundation.

I. Introduction

THE problem of buckling of plates on elastic foundation received attention of many researcher in the past, some of them approached the problem by using the principal of minimum of total potential energy for unit width [1], others derived the governing differential equation starting from the expression of applied bending moments relationship with deflection [2], with obvious difficulty of selecting proper displacement function and introducing different boundary .

In this paper the problem is solved using the Finite Strip Method for local stability. The plate is loaded in its plane with distributed axial load on its width b, it is made of metallic material with Young's modulus E, Thickness t, and length a, the analysis considered plate with length to width ratio a/b =9, and b/t=64.

The foundation material is linearly elastic and idealized as closely spaced springs [3] and it is completely glued to the skin, has young's modulus E_C in the direction normal to loading to provide adequate stiffness, to oppose deflection in that direction, the resulting spring modulus coefficient K_f is computed from

$$K_f = E_C / h, \tag{1}$$

where h is the foundation depth.

In this work the value selected for the foundation depth h=0.4band the values of E_C =0.0 to 50 N/mm^2.

Finite Strip Method [4] is used to find buckling load for plate on foundation with the following side supports:

a) One side simply supported the other free.
b) One side clamped the other free.
c) Both sides clamped.

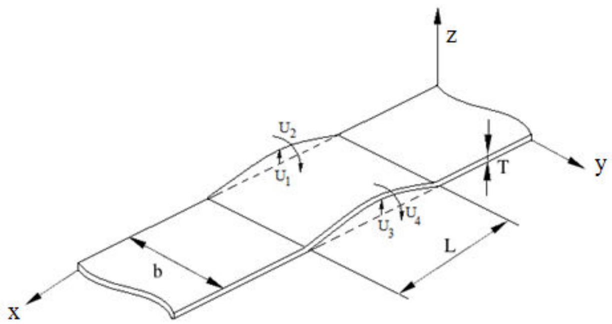

Fig. 1 Typical finite strip with nodal displacements

II. Finite Strip for Structural Stability

The structural stability problem of plate supported by elastic medium which forms the foundation is based on expression the elastic stiffness of the plate as sum of elastic stiffness $\mathbf{K_e}$, foundation stiffness $\mathbf{K_f}$ and geometric matrices $\mathbf{K_g}$.

Each of the overall matrices mentioned above is formed from the matrices of the single strips as in a standard finite element procedure, the element matrices will be derived as follow:

A. Finite Strip Elastic Stiffness Matrix:

A single strip is as shown in Fig. 1, with two nodal displacements at each edge, U_i for out of plane displacement and U_j for rotations, the derivation follow a standard Finite Element Method procedure, the assumed displacement function [4];

$$w = \{c_1 Y^2 + c_2 Y + c_3\}\sin(\pi x/L) \tag{2}$$

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where Y=y/b, b plate width, L is half wave length of buckling and C_1,C_2,C_3 are constants.

In terms of shape functions the displacement function w(x,y) becomes:

$$w=\{N1 \quad N2 \quad N3 \quad N4\}.\sin(\pi x/L)U \tag{3}$$

where the shape functions N_i are expressed as follow:

$$N1=(1-3Y^2+2Y^2),\quad N2=(Y-2Y^2+Y^3)b,\quad N3=(2Y^2-2Y^3),\quad N4=(-Y^2+Y^3)b$$

and **U**={U1 U2 U3 U4} is column of nodal displacement.

Then from strains as second order partial derivatives:

$$\begin{aligned} \epsilon_x &= -z\delta^2 w/\delta x^2 \\ \epsilon_y &= -z\,\delta^2 w/\delta y^2 \\ \epsilon_{xy} &= -2z\delta^2 w/\delta xy \end{aligned} \tag{4}$$

which in matrix form

$$\boldsymbol{\epsilon} = \mathbf{b}\,\mathbf{U} \tag{5}$$

and from the expression of elastic stiffness for single strip

$$k_e = \iiint \mathbf{b}^{\mathbf{T}}\mathbf{E}\mathbf{b}\,dV, \tag{6}$$

where the matrix **E** is the elastic modulus matrix [7], V is the volume of element.

We obtain the finite strip elastic stiffness matrix $\mathbf{k_e}$ given below:

$$k_e = k_{e1} + k_{e2} + k_{e3} \tag{7}$$

where

$$K_{e1} = \frac{\pi^4 Ebt^3}{10080(1-v^2)L^3}\begin{bmatrix} 156 & & & \\ 22b & 4b^2 & Sym. & \\ 54 & 13b & 156 & \\ -13b & -3b^2 & 1-22b & 4b^2 \end{bmatrix}$$

$$K_{e2} = \frac{\pi^2 \ Et^3}{360(1-v^2)bL}\begin{bmatrix} 36 & & & \\ (3+15v) & 4b^2 & Sym. & \\ -36 & -3b & 36 & \\ -13b & -b^2 & -(3+15)b & 4b^2 \end{bmatrix}$$

$$K_{e3} = \frac{ELt^3}{10080(1-v^2)L^3}\begin{bmatrix} 12 & & & \\ 6b & 4b^2 & Sym. & \\ -12 & -6b & 12 & \\ 6b & 2b^2 & -6b & 4b^2 \end{bmatrix}$$

B. Foundation Stiffness Matrix

The foundation is assumed to be formed by elastic isotropic material with elastic modulus $=E_c$, to be perfectly glued to the plate, depth of the foundation is assumed to be =h.

Under critical load the plate buckles into a number of half waves and the core material glued to the plate wrinkle the same way, under the first half-wave it will be pressed down and under the second half wave it will be pulled upward and so on.

At distant h from the plate the core remains undisturbed. In practice the core is thick enough for this to be true.

We can assume the spring constant of the foundation modulus be computed from

$$k_f = E_c/h \tag{8}$$

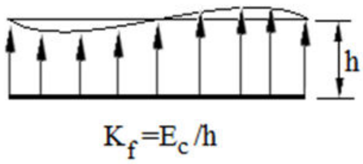

Fig. 2 Foundation idealization

The derivation of foundation stiffness matrix [5] can be as follow:

From the assumed displacement function written above, the strain energy expression resulting from the elastic stiffness of the foundation U_{EF} can be written as:

$$U_{EF} = \frac{k_f}{2}\iint w^2 dxdy \tag{9}$$

and substituting with the expression of the displacement function w written in matrix form

$$U_{EF} = \frac{k_f}{2}\boldsymbol{U}^T\left(\int_0^a\int_0^b \boldsymbol{N}^T.\boldsymbol{N}\,S^2 dxdy\right)\boldsymbol{U} \tag{10}$$

where b is plate width, a plate length, N is shape function matrix, and U column of displacement functions and

$$s = \sin(\pi x/L) \tag{11}$$

Then each element of the foundation matrix $\mathbf{k_f}$ is given from:

$$k_f(i,j) = \delta^2 U_{EF}/\,\delta U_i \delta U_j \tag{12}$$

where U_i, and U_j are nodal displacements. Foundation matrix is given;

$$\mathbf{kf} = \frac{Kfab}{840}\begin{bmatrix} 156 & & & \\ 22b & 4b^2 & Sym. & \\ 54 & 13b & 156 & \\ -13b & -3b^2 & 1-22b & 4b^2 \end{bmatrix} \tag{13}$$

C. Geometric Stiffness Matrix for Finite Strip Element:

The geometric matrix for a strip is derived [4] by assuming middle plane constant stress σ_x acting in x direction the geometric matrix will be given from

$$\mathbf{kg} = \int_0^L\int_0^b\int_{-t/2}^{t/2} N'^T N'\ \mathrm{dxdydz} \tag{14}$$

where N′is first derivative of N relative to x and equal to =(π/L) N(y)cos(π x/L).

The geometric matrix for single strip$\mathbf{k_g}$ is given as follow:

$$\mathbf{kg} = \frac{\sigma\pi^2 bt}{840L}\begin{bmatrix} 156 & & & \\ 22b & 4b^2 & Sym. & \\ 54 & 13b & 156 & \\ -13b & -3b^2 & 1-22b & 4b^2 \end{bmatrix} \tag{15}$$

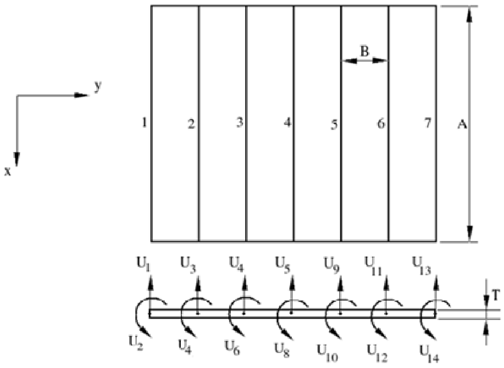

Fig. 3 Idealization of the plate

D. The Buckling Equation for the Assembled Structure

The displacement–load equation for the assembled structure constructed from the matrices of single elements Fig. 3 is given from [7]:

$$U = (k_e + k_f + k_g)^{-1} F \tag{16}$$

instead of the column matrix **F** we substitute (λ**F′**) where **F′** is the relative magnitude of the applied load column matrix and λ is a constant of proportionality or (load factor) of **F,** and since the geometric stiffness is proportional to the applied load, it can be written as $\lambda k'_g$, with k'_g is the geometric stiffness matrix for unit value of λ.

For small displacement $\mathbf{K_e}$ can be considered constant then the general equation can be written as follow:

$$U = (k_e + k_f + \lambda k'_g)^{-1} \lambda F' \tag{17}$$

It follows that for buckling with displacement tending to infinity the determinant=0, or

$$|k_e + k_f + \lambda k'_g| = 0 \tag{18}$$

This determinant is stability equation used to find the buckling loads and buckling modes.

The lowest root (Eigen-value) and the associated eigenvector will be the critical buckling load and buckling mode.

E. Analysis Procedure:

In this study since the elastic, foundation, and geometric matrices are functions of buckling wave length which is unknown, the buckling load and the associated buckling mode are found by an iteration procedure.

For the analysis the plate is divided into six finite strips each of length equivalent to half wave length, Fig. 1 shows typical finite strip of the plate with nodal displacements, and Fig. 2 shows the idealization of the full plate.

The assembly matrix for full plate is of order 14×14, the reduced matrix which obtained by introducing the boundary condition at sides of plate will be of order 13×13 for side simply supported the other free, for clamped free plate the reduced stiffness matrix will be of order 12×12, and the clamped-clamped plate the reduced matrix will be of order 10×10.

III. Flow Chart and FORTRAN List

The flow chart and the FORTRAN list are given in Appendix. Fig. 7 illustrates the flow chart of the plate buckling program.

The program steps are as follow:

1) RUN=1, geometric stiffness is computed for one strip element
2) Assembly matrix is initiated and receives each element matrix in the proper space
3) Boundary condition is introduced and reduced geometric matrix is formed.
4) Run=2, steps 1-,2-,3-, are repeated for elastic stiffness matrix and reduced elastic stiffness matrix for full plate is derived.
5) To Find Eigen-value and Eigenvectors Since subroutine require positive definite matrix to be introduced first, positions of matrices $\mathbf{K_e}$ +$\mathbf{K_f}$, and $\mathbf{K_g}$ are interchanged in the characteristic determinant as follow:

$$\left|\boldsymbol{k'_g} + (\mathbf{1}/\boldsymbol{\lambda})(\boldsymbol{k_e} + \boldsymbol{k_f})\right| = \mathbf{0} \tag{19}$$

$$\left|\boldsymbol{k'_g} + \boldsymbol{\lambda'}(\boldsymbol{k_e} + \boldsymbol{k_f})\right| = \mathbf{0} \tag{20}$$

where $\boldsymbol{\lambda'}$ is the required Eigen-value

6) A number of half wave lengths are used in the iteration procedure and the minimum of the curve will give the buckling load required.
7) The subroutines used for solving the characteristic equation for each step of the iteration process is as follow [8]:

- To find Eigen-values and eigenvectors for the problem in the form **Ax=Bx**.
- The second matrix **B** is decomposed into Land $\mathbf{L^t}$ $\mathbf{B=LL^t}$
  Then (CALL CHOLDC{K_e, N, n})
  Note $\mathbf{K_e}$ +$\mathbf{K_f}$≡B, and $\mathbf{K_g}$≡A
- The equation **Ax=Bx.** Becomes $\mathbf{(L^{-1}\ A\ L^{-1})(L^{-1}x)=(L^{-1}x)}$.
- which can be written as $\mathbf{P_y=y}$ where $\mathbf{P=L^{-1}\ A\ L^{-1}}$ is the symmetric matrix

(CALL PMAT{KEI,KG,KEIT,P,N}).

- Householder's method is used to transform the matrix **P** into tridiagonal matrix.

(CALL TRIDIAG{P,N,DP,EP})

- QL Algorithm is used to find eigenvalues and eigenvectors

(CALL tgli{dP,eP,n,N,z}

Note: The eigenvectors are related to the original x by the relations=L^t x.

IV. Analysis and Presentation of Results

The analysis is based on study of relatively long plate of steel with young's modulus E equal 210,000 N/mm^2,

σ_ε=600N/mm^2 with length a=457mm, and b=50.8mm, thickness t=0.79 mm, foundation height h=0.4b and foundation modulus of elasticity E_c vary from 0.0 to 50.0 Nmm^2, and the corresponding elastic spring modulus $K_f=E_c/(0.4b)$.

Three boundary conditions column matrices NB(I) are as follow:

- One side simply supported the other free

$$NB(1)=[1] \tag{21}$$

- One side clamped the other free

$$NB(2)=\begin{bmatrix}1\\2\end{bmatrix} \tag{22}$$

- Two sides clamped

$$NB(4)=\begin{bmatrix}1\\2\\13\\14\end{bmatrix} \tag{23}$$

and nodal numbering matrix for the plate is as follows

$$ND^T(6,2)=\begin{bmatrix}1 & 2 & 3 & 4 & 5 & 6\\2 & 3 & 4 & 5 & 6 & 7\end{bmatrix} \tag{24}$$

Ends simply supported in all cases.

The Fortran list and flow chart used are given in Appendix, the results of analysis are given as follow:

Fig. 4 shows results of critical buckling stress versus values of K_f

Fig. 5 shows results of critical buckling stress coefficient K versus values of K_f. where

$$k = \sigma/E\left(\frac{t}{b}\right)^2 \tag{25}$$

Fig. 6 shows variation of buckling wave length to plate width ratio $\lambda L_{cr}/b$ for the three cases considered versus K_f.

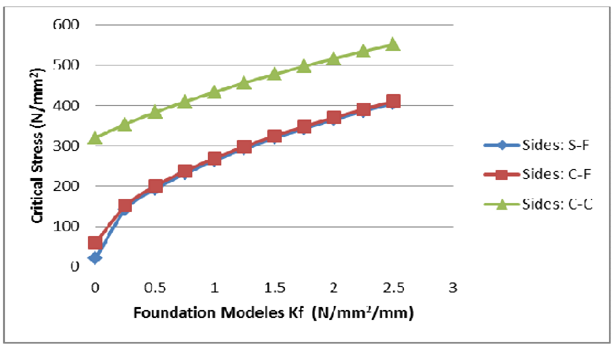

Fig. 4 Critical buckling stress for plates on foundation

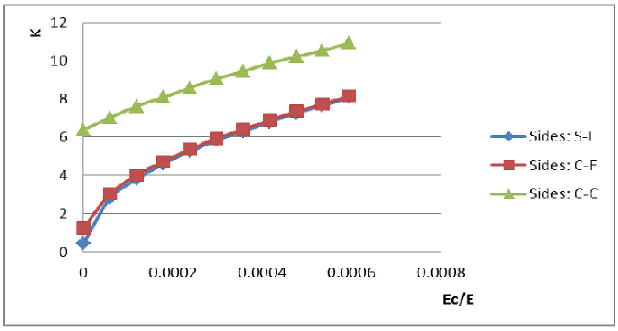

Fig. 5 Critical buckling stress coefficient for plates on foundation

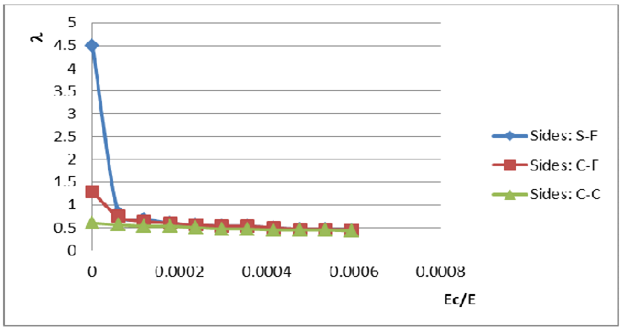

Fig. 6 Critical wave length ratio for plates on foundation

V. Conclusion

1) Critical buckling stress increases with increase in foundation stiffness and wave length decrease
2) The plate with one side simple supported and one side free benefit most from increase in stiffness of the foundation.
3) For all types of side constraint values of buckling stress coefficients tend to get closer at high values of foundation spring modulus K_f.
4) Comparison of results

- Comparison with theoretical analysis For clamped-free plate not on foundation K_f=0.0, from ESDU [6]σ_x= 57.4N/mm^2, L_{cr} =90N/mm^2

In this study for the same plate σ_x=59.3N/mm^2, L_{cr}=65.5mm

- Comparison with experimental results[5] for long steel plate on foundation b=42 mm, and a/b=10.7, t=0.79mm:
- Test 1

K_f=1.3N/mm^2 /mm, σ_x= 260 N/mm^2, L_{cr}=30.mm

In this study for the same K_f and for a/b=8.85, σ_x=298.5N/mm^2, L_{cr}=27mm.

- Test 2

K_f= 1.5N/mm^2, σ_x= 277N/mm^2, L_{cr}=30.mm

In this study for the same K_f and for a/b=8.85, σ_x=324N/mm^2, L_{cr}=27mm.

As it can be seen from the comparison, relatively good results are obtained.

APPENDIX

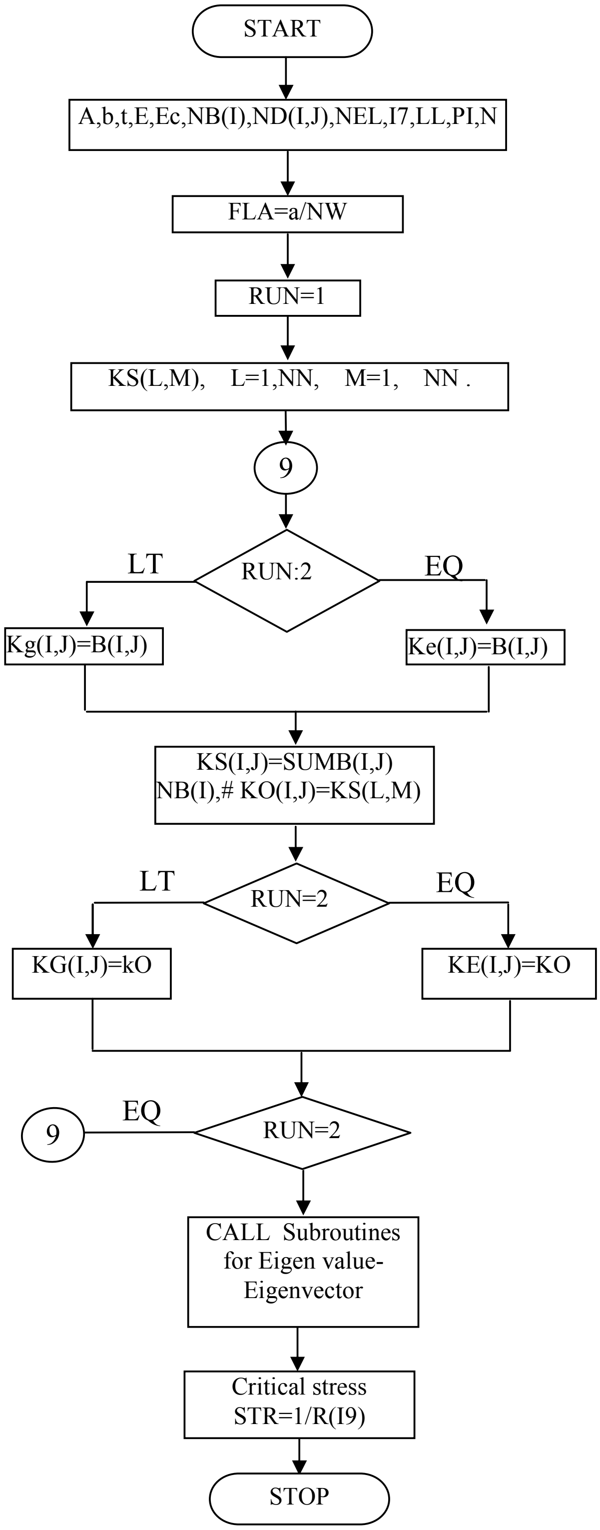

Fig. 7 Flow chart of plate buckling program

```
C FINITE STRIP STR-NW CURVES   12x12  CLMPED-FREE
C BUCKLING OF PLATE CLAMPED -FREE UNDER  AXIAL LOAD
DIMENSION ND(6,2),B(4,4),KS(14,14),KO(14,14),
#KG(12,12),KE(12,12)
#,NB(2),KEI(12,12),KEIT(12,12),P(12,12),DP(12),
#EP(12),Z(12,12)
 #,STR(30),F9(30),F99(30)
 DOUBLEPRECISION PI,PHA,PHB,PHC,PHD,FLA,B,KO,
#KG,KE,KS,E,BB,EY,ER,TT,KEI,KEIT,#P,DP,EP,Z
INTEGER,I7,NOD,IND,I9,LL,NEL,QM,JM
OPEN(1,FILE='MAT12.OUT')
OPEN(2,FILE='MAT22.OUT')
303 RUN=1
F9(QM)=FLA
F99(JM)=FLABP
208PI=4*ATAN(1.0)
 I1=1
NN=IND*LL
I9=NN-I7
1000  DO 2 L=1,NN
     DO 2 M=1,NN
2    KS(L,M)=0.0
ASTIC STIFFNESS OF SINGLE STRIP

300   PHA=(PI**4.)*E*BB *(TT**3.)/(10080.*
#(1-UN*UN)*FLA**3.)+GK*FLA*BB/840.
     PHB=PI*PI*E*(TT**3.)/(360.*(1.-UN*UN)*BB*FLA)
PHC=E*FLA *(TT**3.)/(24.* (1.-UN*UN)*BB**3.)
     PHD=PI*PI*BB*TT/(840.* FLA)
 33  IF(RUN-2) 3,4,602
3   B(1,1)=PHD*156.
B(2,1)=PHD*22.*BB
B(3,1)=PHD*54.
B(4,1)=PHD*(-13.)*BB
B(2,2)=PHD*4.*BB*BB
B(3,2)=PHD*13.*BB
B(4,2)=PHD*(-3.)*BB*BB
B(3,3)=PHD*156.
B(4,3)=PHD*(-22.)*BB
B(4,4)=PHD*BB*BB*4.
     GO TO 11
4    B(1,1)=PHA*156.+PHB*36.+PHC*12.
B(2,1)=PHA*22.*BB+PHB*(3.+15.*UN)*BB+PHC*6.*BB
B(3,1)=PHA*54.+PHB*(-36.)+PHC*(-12.)
B(4,1)=PHA*(-13.)*BB+PHB*(3.)*BB+PHC*6.*BB
B(2,2)=4.*BB*BB*(PHA+PHB+PHC)
B(3,2)=PHA*13.*BB+PHB*(-3.)*BB+PHC*(-6.)*BB
B(4,2)=PHA*BB*BB*(-3.)+PHB*BB*BB*(- 1.)+PHC*BB*BB*2.
B(3,3)=PHA*156.+PHB*36.+PHC*12.
B(4,3)=PHA*(-22.)*BB+PHB*(-3.-15.*UN)*BB+PHC*(-
6.)*BB
B(4,4)=4.*BB*BB*(PHA+PHB+PHC)
11   DO 7  I=1,4
     DO 7  J=1,4
 7   B(I,J)=B(J,I)
C    #,I=1,4)
        N1=1
52   L1=1
50   DO 90 I=1,LL
  DO 90 J=1,LL
     L2=I+(N1-1)*LL
     M3=J+(L1-1)*LL
L=LL*ND(I1,N1)-LL+I
M=LL*ND(I1,L1)-LL+J
90  KS(L,M)=KS(L,M)+B(L2,M3)
     L1=L1+1
IF(L1-NOD) 50,50,51
51   N1=N1+1
IF(N1-NOD)52,52,53
53I1=I1+1
  IF(I1-NEL)300,300,320
320  DO 500  L=1,NN
     DO 500 M=1,NN
KO(L,M)=KS(L,M)
500  CONTINUE
     DO   600  II=1,I7
```

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```
K=NB(II)
K=K-II+1
ITERM=NN-1
     DO 510 L=K,ITERM
     IP1=L+1
     DO 510 M=1,NN
KO(L,M) =KS(IP1,M)
510  CONTINUE
  NM1=NN-1
     DO 540 L=1,NM1
     DO 540 M=1,NN
KS(L,M) =KO(L,M)
540  CONTINUE
     DO 520 M=K,ITERM
     JP1=M+1
     DO 520 L=1,NN
520  KO(L,M)=KS(L,JP1)
NN=NN-1
     DO 545 L=1,NN
     DO 545 M=1,NN
KS(L,M)=KO(L,M)
545  CONTINUE
600  CONTINUE
IF(RUN-2) 16,17,602
16   DO 161 I=1,I9
     DO 161 J=1,I9
161  KG(I,J)=KO(I,J)
      GO TO  207
17   DO 171 I=1,I9
     DO 171 J=1,I9
171  KE(I,J)=KO(I,J)
207  RUN=RUN+1
IF(RUN-2)208,208,602
02CALL CHOLDC(KE,N,12)
WRITE(2,992)((KE(I,J),J=1,N),I=1,N)
  CALL LMI(KE,KEI,KEIT,N) CALL PMAT(KEI,KG,KEIT,P,N)
              CALL TRIDIAG(P,N,DP,EP)
  Z=P
             CALL TQLI(DP,EP,N,12,Z)
  STR(JM)=1./DP(N)
  X7=STR(JM)/(E*((TT/BP)**2))
  IF(JM.GT.1.)GOTO 1078
  STRCR=STR(JM)
  QM=QM+1
  JM=JM+1
  IF(QM.GT.QMMAX) GOTO 0111
  GOTO 306
1078  DEL=STR(JM)-STRCR
IF(DEL.GT.0.0) GOTO 2933
  STRCR=STR(JM)
  QM=QM+1
  JM=JM+1
  GOTO 306
2933WRITE(2,33331)STRCR,X7,F99(JM),ER, GK,QM,JM
33331 FORMAT(1X,'STRCR(JM)= ',F10.1,1X,'X7=
',F6.2,1X,'FLA/BP= ',F5.2,
    #1X,'ER= ',F8.6,1X,'GK= ',F4.2,1X,'QM= ',I2,1X'JM=
',I2)
10111 GK=GK+DGK
  IF(GK.GT.GKMAX)GOTO 20111
  GOTO 309
20111 STOP
     END
```

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- Scientific Researcher (Second)
- Scientific Researcher (First)- Central Organization for Research and Manufacturing- 1998
- Head of Researchers - Central Organization for Research and Manufacturing –(2007).
- Consultant- Central Organization for Research and Manufacturing-(2008)
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