

Broadcasting Stabilization for Dynamical Multi-Agent Systems

Myung-Gon Yoon, Jung-Ho Moon, Tae Kwon Ha

Abstract—This paper deals with a stabilization problem for multi-agent systems, when all agents in a multi-agent system receive the same broadcasting control signal and the controller can measure not each agent output but the sum of all agent outputs. It is analytically shown that when the sum of all agent outputs is bounded with a certain broadcasting controller for a given reference, each agent output is separately bounded: stabilization of the sum of agent outputs always results in the stability of every agent output. A numerical example is presented to illustrate our theoretic findings in this paper.

Keywords—Broadcasting Control, Multi-agent System, Transfer Function

I. INTRODUCTION

BOTH theoretically and practically, control problems of multi-agent systems have attracted much attention from control community. This is quite understandable because many important physical systems appearing in diverse fields can be described as multi-agent systems and therefore developments of appropriate control methodologies for those systems can have significant impacts.

As many multi-agents systems are composed of huge number of agents such as a flock of birds and a school of fish, for examples, in most multi-agent systems studied in literature, only a small number of agents are directly controlled in order to avoid the formidable cost and complexity in handling all agents. Desirable changes in the collective behaviors of a multi-agent system are induced by inter-agent communication of multi-agent systems.

The *single agent control* (SAC) approach is extremal in that, as the naming says, only a single agent of a given multi-agent system is directly controlled [1], [2]. The fundamental idea of the *pinning control* is similar but it is more focused on nonlinear multi-agent systems in time domain [3]–[7], whereas the SAC is a frequency domain approach. Another well known approach is the *leader-follower* scheme in which some *leader* agents serve as controllers effectively [8]–[11]. In the aforementioned control approaches for multi-agent systems, the problem of agent selection is critically important but rigorous justification for a certain way of selection is very hard in general.

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In this work we note that there are many dynamical multi-agent systems in which all agents are inherently capable of receiving some kind of external signals. For instance, all birds in a flock can simultaneously hear the same sound from a distance, not some of them. In this sort of multi-agent systems, one does not need to limit the number of directly controllable agents. There remains only the question of how to generate the signal monitored by all agents, call it a *broadcasting control signal*, for a desirable collective behaviors of multi-agent systems. In fact, several control methodologies in literature involve a broadcasting control input similar to ours, including [12] for mobile robots and [13]–[15] for biological applications.

For a better illustration of our focus in this work, let us consider a multi-agent system in Fig. 1, composed of eight identical agents labelled $1, \dots, 8$. Suppose we wish to design a broadcasting controller whose output, that is, the control signal, is transmitted to all agents at the same time. As the task of measuring all agent outputs simultaneously is impractical particularly for a large multi-agent system, we assume that the broadcasting controller can measure only the sum of outputs $y_1 + \dots + y_8$.

For a given *linear time-invariant* agent dynamics and a fixed configuration of inter-agent communication under our consideration in this paper, the task of controller design can be done by making use of any existing controller synthesis method. Leaving such a controller synthesis as a routine task, in this work, we wish to investigate whether or not the stabilization of output sum $y_1 + \dots + y_8$ with a broadcasting controller will result in a stable (bounded) agent output y_i for every i , given a bounded reference signal r in Fig. 1.

II. PROBLEM STATEMENT

This paper deals with multi-agent dynamic systems composed of identical agents, denoted by $\{1, 2, \dots, n\}$, whose dynamics is given by a possibly unstable, strictly proper rational transfer function

$$\frac{y_i(s)}{u_i(s)} = \frac{b(s)}{a(s)} \quad (1)$$

where $y_i(s)$ and $u_i(s)$ denote the Laplace transform of agent output $y_i(t)$ and input $u_i(t)$ of agent i . In the representation (1) the polynomials $b(s), a(s)$ are chosen to be coprime and $a(s)$ is *monic*, i.e., its leading coefficient is one. Here we have used the same names for functions in time-domain and their Laplace transforms in frequency domain for notational simplicity.

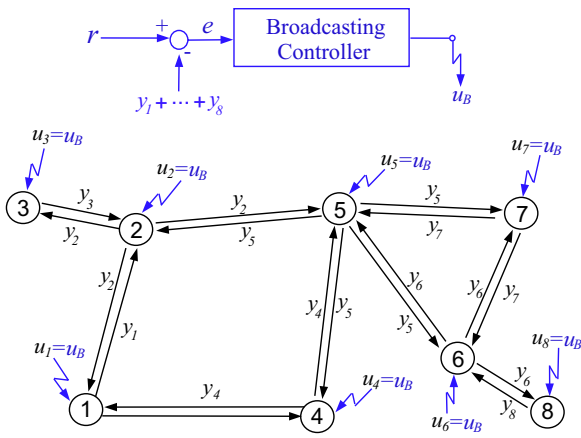


Fig. 1. Broadcasting Control of a Multi-agent System

We suppose that each agent receive an input

$$u_i(s) = u_i^c(s) + u_i^e(s) \quad (2)$$

where $u_i^c(s)$ is inter-agent communication signal and $u_i^e(s)$ denotes an external input. In many multi-agent system $u_i^c(s)$ is given as a linear combination of the outputs of other agents, i.e.,

$$u_i^c(s) = -Hy(s), \quad y := [y_1 \quad \cdots \quad y_n]^T \quad (3)$$

for some matrix H .

For the external input $u_i^e(s)$, we assume that all agents in a multi-agent system receive the same *broadcasting signal* u_B generated by an exogenous *broadcasting controller*, that is,

$$u_i^e = u_B. \quad (4)$$

Moreover, we assume that the controller can measure only the output sum of all agents

$$y_B = y_1 + \cdots + y_n, \quad (5)$$

instead of agent outputs $\{y_i\}$, because of measurement cost.

For a given reference signal $r(s)$, the broadcasting controller generates a control input of the form

$$u_B = c(s)(r - y_B) \quad (6)$$

for some controller $c(s)$ and reference signal r .

Remark 1: For presentational simplicity, we assume that the controller measures output sum y_B . Our results however can be straightforwardly modified for the case of measuring the *average output* y_B/n , instead of y_B .

The overall dynamics of a multi-agent system subject to a broadcasting controller, can be written as

$$\frac{a(s)}{b(s)}y = -Hy + \mathbf{1}u_B, \quad \mathbf{1} := \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad (7)$$

$$y_B = \mathbf{1}^T y, \quad (8)$$

where $\mathbf{1}$ is commonly called as the *all-one* vector.

Now let us suppose a controller $c(s)$ for the broadcasting controller is chosen to stabilize the transfer function from the reference $r(s)$ to the output $y_B(s)$. Then, the following problem will be investigated in this paper:

Problem 1: Is it sufficient for a broadcasting controller to stabilize the output sum $y_B = y_1 + \cdots + y_n$, measured by the controller, in order to stabilize each agent output?

In other words, *can it be possible that several agent outputs diverge while their sum remains bounded as time goes?*

A complete answer to this question will be developed in this paper.

III. MAIN RESULTS

A. Broadcasting Control

We assume that every communication between agents in a multi-agent system is always bi-directional:

Assumption 1: The matrix H describing inter-agent communication is symmetric, i.e., $H^T = H$.

From this assumption we have a spectral decomposition

$$H = \mu_1 P_1 + \cdots + \mu_m P_m \quad (9)$$

where μ_k denotes the distinct (real) eigenvalues of H and P_k is the orthogonal projection onto μ_k -eigenspace.

Making use of (9), it can be easily shown that the transfer function of the multi-agent system (7) is given by

$$g_i(s) := \frac{y_i}{u_B} = \sum_{k=1}^m \left(\frac{a(s)}{b(s)} + \mu_k \right)^{-1} \mathbf{e}_i^T P_k \mathbf{1} \quad (10)$$

$$= \sum_{k=1}^m \frac{b(s)}{a(s) + \mu_k b(s)} \mathbf{e}_i^T P_k \mathbf{1} \quad (11)$$

where $\{\mathbf{e}_i; i = 1, \dots, n\}$ denotes the standard orthonormal basis of n -dimensional vector space. Moreover, as $y_B = \mathbf{1}^T y$ and $\mathbf{1} = \sum_i \mathbf{e}_i$, we have

$$g_B(s) := \frac{y_B}{u_B} = \sum_{k=1}^m \frac{b(s)}{a(s) + \mu_k b(s)} \mathbf{1}^T P_k \mathbf{1}. \quad (12)$$

Let us rewrite those transfer functions as

$$g_i(s) := \frac{N_i(s)}{D_i(s)} \quad \text{and} \quad g_B(s) := \frac{N_B(s)}{D_B(s)} \quad (13)$$

with coprime polynomial pairs $\{N_i, D_i\}$ and $\{N_B, D_B\}$ and monic polynomials D_i, D_B . Similarly, let us represent the controller as $c(s) = X(s)/Y(s)$ with coprime polynomials $\{X, Y\}$ with monic Y .

Then the *closed loop* transfer function from the reference $r(s)$ to outputs can be written as

$$\frac{y_B(s)}{r(s)} = \frac{cg_B}{1 + cg_B} = \frac{XN_B}{N_B X + D_B Y}, \quad (14)$$

$$\frac{y_i(s)}{r(s)} = \frac{cg_i}{1 + cg_B} = \frac{D_B}{D_i} \cdot \frac{XN_i}{N_B X + D_B Y}. \quad (15)$$

As we have chosen a broadcasting controller $c(s)$ stabilizing the transfer function $g_B = N_B/D_B$, from (14), it is obvious that the polynomial $N_B X + D_B Y$ is stable, that is, its all roots are in the left half plane of the complex plane.

Therefore, in order to investigate the closed loop stability of transfer function $y_i(s)/r(s)$ considered in Problem 1, we have only to check if the polynomial D_i might have unstable roots which are not cancelled out by the roots of polynomial $D_B X N_i$.

Lemma 1: D_i is a factor of D_B for every $i = 1, \dots, n$.

Proof: (Sketch) The coprimeness of $a(s), b(s)$ gives that D_i, D_B are products of the term $\prod_k (a + \mu_k b)$ for k 's such that either $e_i^T P_k \mathbf{1} \neq 0$ or $\mathbf{1}^T P_k \mathbf{1} \neq 0$. Being a projection, $P_k^2 = P_k$ holds and therefore $\mathbf{1}^T P_k \mathbf{1} = \mathbf{1}^T P_k^2 \mathbf{1} = \langle P_k \mathbf{1}, P_k \mathbf{1} \rangle = \|P_k \mathbf{1}\|^2$. As a result $\mathbf{1}^T P_k \mathbf{1} = 0$ implies $e_i^T P_k \mathbf{1} = 0$, meaning that every factor $a + \mu_k b$ of D_i is also a factor of D_B . ■

This lemma reveals that D_B/D_i is merely a polynomial for every i and thus the closed loop stability of $y_i(s)/r(s)$ follows from that of $y_B(s)/r(s)$. It is important to notice that our developments up to now were independent of a network topology (configuration of agent connections) of a given multi-agent system.

To sum up, we obtain the following main result as a complete answer to our question stated in Problem 1:

Theorem 1: A broadcasting controller can stabilize every agent output separately, irrespectively of network topology, for any multi-agent system with purely bidirectional inter-agent communication.

In many control applications of multi-agent systems, the matrix H is an adjacency matrix or the Laplacian matrix of a mathematical *graph* describing the configuration of inter-agent communication channel and a network protocol, see e.g., [1], [16]. In a study of the broadcasting controllability of [17], for example, H was an adjacency matrix.

This paper makes no restrictions on H except that it is symmetric. Thus, for example, our results can be applied to a multi-agent system in which only some agents are equipped with a consensus protocol while others are not, as is the case of the following example.

IV. A NUMERICAL EXAMPLE

Consider the multi-agent system in Fig. 1, assuming that the agent dynamics is given by a second order system

$$g(s) = \frac{1}{s^2 + 10s + 5}. \quad (16)$$

Let us suppose that agents $\{1, 2, 3, 4\}$ employ a linear consensus protocol while agents $\{5, 6, 7, 8\}$ are not. Then, from the inter-agent communication topology in Fig. 1, we have the following matrix

$$H = \begin{bmatrix} \mathbf{-2} & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & \mathbf{-3} & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & \mathbf{-1} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & \mathbf{-2} & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (17)$$

in which the non-zero diagonal elements (bold) show that only agents $\{1, 2, 3, 4\}$ are subject to a linear consensus protocol.

Numerical calculations give that H has eight distinct eigenvalues $\{\mu_k\}$ and the parameters $\mathbf{1}^T P_k \mathbf{1}$ and $e_i^T P_k \mathbf{1}$ in the transfer function representations (11) and (12), as in Table I. For example, the transfer functions $g_B(s)$ and $g_1(s)$ are given in (18) and (19), respectively, where the bold numbers are matched with the corresponding bold numbers in Table I.

Note from Table I that $D_B = D_i$ holds for every $i = 1, \dots, 8$ in this particular example.

With the following proportional-integral type controller

$$c(s) = \frac{1}{2} \left(1 + \frac{1}{s} \right), \quad (20)$$

stabilizing the transfer function $g_B(s)$ and zero initial outputs, numerical simulations give the step responses ($r(s) = 1/s$) of $y_B = y_1 + \dots + y_n$ and all $\{y_i\}$ in Fig. 2. As theoretically expected, all agent outputs converge to constant numbers.

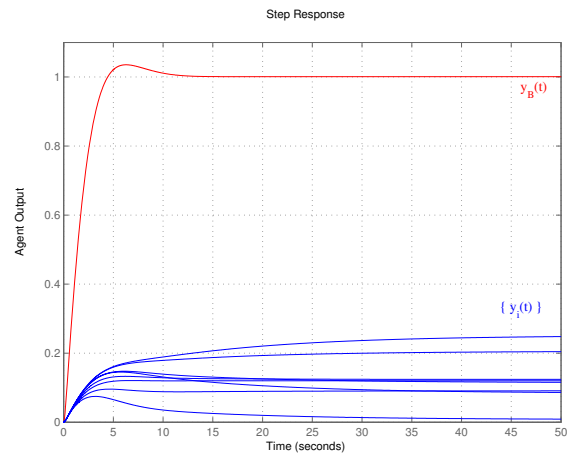


Fig. 2. Step Response of Multi-agent System with a Broadcasting Controller

It is impressive in Fig. 2 that most outputs $\{y_i\}$ vary slowly but their sum y_B changes rather quickly. This interesting result however is not surprising as the broadcasting controller (20) was design exactly for that purpose.

V. CONCLUSION

We have found the transfer functions of a multi-agent system combined with an exogenous broadcasting controller. Making use of special structures in the representation of the transfer functions, we was able to show that when the sum of all agent outputs is bounded with a certain broadcasting controller for a given bounded reference input, each agent output is also separately bounded. In other words, the broadcasting controller can achieve the stability of every agent, irrespectively of network topology, provided that every inter-agent communication in the multi-agent system is bidirectional.

TABLE I
TRANSFER FUNCTION PARAMETERS

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$
μ_k	2.3180	0.5011	-0.1855	-0.7707	-1.3438	-1.5755	-2.6495	-4.2941
$\mathbf{1}^T P_k \mathbf{1}$	5.1020	0.0547	1.6795	0.3323	0.6054	0.1757	0.0125	0.0379
$P_k \mathbf{1}$	0.1443	0.0424	0.4394	-0.0643	0.3371	0.2314	-0.0400	-0.0903
	0.2882	0.0497	0.4254	0.1076	0.0278	0.0116	-0.0550	0.1447
	0.0869	0.0331	0.5222	0.4694	-0.0809	-0.0202	0.0334	-0.0439
	0.3348	0.0563	0.3720	-0.1867	0.1934	0.0866	0.0810	0.0625
	1.3015	0.0985	0.2355	-0.1652	-0.2102	-0.1946	-0.0126	-0.0530
	1.2802	-0.0846	-0.0976	0.0269	-0.3721	0.2323	0.0041	0.0106
	1.1138	0.0279	-0.7434	0.1794	0.4333	-0.0239	0.0032	0.0099
	0.5523	-0.1688	0.5260	-0.0350	0.2769	-0.1474	-0.0016	-0.0025

$$g_B(s) = \frac{y_B}{u_B} = \frac{\mathbf{5.1020}}{(s^2 + 10s + 5) + \mathbf{2.3180}} + \frac{\mathbf{0.0547}}{(s^2 + 10s + 5) + \mathbf{0.5011}} + \dots + \frac{\mathbf{0.0379}}{(s^2 + 10s + 5) - \mathbf{4.2941}} \quad (18)$$

$$g_1(s) = \frac{y_1}{u_B} = \frac{\mathbf{0.1443}}{(s^2 + 10s + 5) + \mathbf{2.3180}} + \frac{\mathbf{0.0424}}{(s^2 + 10s + 5) + \mathbf{0.5011}} + \dots + \frac{\mathbf{-0.0903}}{(s^2 + 10s + 5) - \mathbf{4.2941}} \quad (19)$$

REFERENCES

- [1] M.-G. Yoon, "Single agent control for multi-agent dynamical consensus systems," *IET Control Theory & Applications*, vol. 6, no. 10, p. 14781485, 2012.
- [2] —, "Single agent control for cyclic consensus systems," *International Journal of Control, Automation, and Systems*, vol. 11, no. 2, pp. 243–249, 2013.
- [3] X. Li, X. Wang, and G. Chen, "Pinning a complex dynamical network to its equilibrium," *IEEE Transaction on Circuits and Systems I*, vol. 51, no. 10, pp. 2074–2087, 2004.
- [4] F. Chen, Z. Cen, L. Xiang, Z. Liu, and Z. Yuan, "Reaching a consensus via pinning control," *Automatica*, vol. 45, pp. 1215–1220, 2009.
- [5] T. Chen, X. Liu, and W. Lu, "Pinning complex networks by a single controller," *IEEE Transaction on Circuits and Systems I*, vol. 54, pp. 1317–1326, 2007.
- [6] W. Yua, G. Chena, and J. Lü, "On pinning synchronization of complex dynamical networks," *Automatica*, vol. 45, pp. 429–435, 2009.
- [7] J. Zhou, Q. Wu, and L. Xiang, "Pinning complex delayed dynamical networks by a single impulsive controller," *IEEE Transaction on Circuits and Systems I*, vol. 58, no. 12, 2011.
- [8] H. G. Tanner, "On the controllability of nearest neighbor interconnections," in *Proceeding of the 43rd Conf. Decision Control, Atlantis, Bahamas, December, 2004*, pp. 2467–2472.
- [9] A. Rahmani, M. Ji, M. Mesbahi, and M. Egerstedt, "Controllability of multi-agent systems from a graph-theoretic perspective," *SIAM J. Control Optim.*, vol. 48, no. 1, pp. 162–186, 2009.
- [10] S. Martini, M. Egerstedt, and A. Bicchi, "Controllability analysis of multi-agent systems using relaxed equitable partitions," *Int. J. Systems, Control and Communications*, vol. 2, pp. 100–121, 2010.
- [11] M. Ji, A. Muhammad, and M. Egerstedt, "Leader-based multi-agent coordination: Controllability and optimal control," in *American Control Conference, Minneapolis, June 14-16, 2006*.
- [12] S. Azuma, R. Yoshimura, and T. Sugie, "Broadcast control of multi-agent systems," in *in: 50th IEEE Cnf. on Decision and Control and European Control Conference, Orlando, FL, Dec. 12-15, 2011*.
- [13] J. Ueda, L. Odhner, and H. Asada, "Broadcast feedback of stochastic cellular actuators inspired by biological muscle control," *International Journal of Robotics Research*, vol. 26, no. 11-12, pp. 1251–1265, 2007.
- [14] A. Julius, A. Halasz, M. Sakar, H. Rubin, V. Kumar, and G. Pappas, "Stochastic modeling and control of biological systems: the lactose regulation system of escherichia coli," *IEEE Transactions on Automatic Control*, vol. 53, no. 1, pp. 51–65, 2008.
- [15] J. Ueda, L. Odhner, and H. Asada, "Broadcast feedback of large-scale, distributed stochastic control systems inspired by biological muscle control," in *American Control Conference, 2007*, pp. 1317–1322.
- [16] D. M. Cvetković, M. Doob, and H. Sachs, *Spectra of Graphs Theory and Applications*, ser. Pure and Applied Mathematics. New York: Academic Press, 1980.
- [17] M.-G. Yoon, P. Rowlinson, D. Cvetkoviacutec, and Z. Stanić, "Controllability of multi-agent dynamical systems with a broadcasting control signal," *Asian Journal of Control*, 2013, doi: 10.1002/asjc.793.