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Breaking of Charge Independence of Nucleon-Nucleon Interaction Using Phase Shift Calculations

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Abstract**—**Using calculated phase- shift values, for pp, nn, and np elastic scattering in the energy range 1MeV to 350MeV, the charge independence breaking of nucleon-nucleon interaction is investigated. We have used Darboux transformation to calculate phase-shift for the first three values of ℓ. It is seen that the charge independency holds in most of the energy range but it is broken in the particular energy range of 120MeV to 200MeV. It is indicated that a charge dependent term be added to the nucleon-nucleon potential in this energy range.

Keywords**—**Phase-shift, charge independence breaking, Darboux transformation.

I. Introduction

THE interaction between two nucleons is one of the central questions in nuclear physics. Traditionally, nucleon-nucleon $(\mathrm{N}-\mathrm{N})$ potentials are constructed by fitting np data for $\mathrm{T}=0$ states and either np or pp data for $\mathrm{T}=1$ states. Unfortunately, potential models which have been fit only to the np data often give a poor description of the pp data, even after applying the necessary corrections for the coulomb interaction. Fundamentally, this problem is due to charge independence breaking in the strong interaction. Recently, several realistic potentials are constructed which accurately fit the proton- proton and neutron-proton scattering phase -shifts. These potentials contain terms which break charge independence [1].

Charge independence breaking of the nucleon-nucleon interaction means that, in the isospin $\mathrm{T}=1$ state, the proton-proton $(\mathrm{Tz}=+1)$, neutron-proton $(\mathrm{Tz}=0)$, or neutron-neutron $(\mathrm{Tz}=1)$ interaction are different, after electromagnetic effects have been removed. The results are analyzed in the $1s_0$ state for the scattering lengths (a) and effective ranges (r) [1].

These are

$app = -17.3 \pm 0.4\, fm$	$rpp = 2.85 \pm 0.04\, fm$
$ann = -18.8 \pm 0.3\, fm$	$rnn = 2.75 \pm 0.11\, fm$
$anp = -23.75 \pm 0.01\, fm$	$rnp = 2.75 \pm 0.05\, fm$

The differences between these scattering lengths represent CIB and CSB effects. The major cause of charge independence breaking in the nucleon- nucleon interaction is the mass difference between the charged and the neutral pions. The charge dependence of nucleon-nucleon interaction due to the pion mass difference has been calculated [2], [3]. Whit in QCD, the charge independence breaking of nucleon-nucleon interaction is of course due to the difference in the up and down quarks masses and charges.

Our aim in this paper is the investigation of the charge independence breaking of nucleon-nucleon interaction by evaluating phase- shift.

II. Theory

Two nucleons scattering process consists of pp, np, and nn scattering. In non relativistic limit the scattering process in described by Schrodinger equation in center of mass system.

For zero potential, obviously no scattering occurs and phase -shifts $\delta_\ell = 0$ when interaction potential exists, for the region with $r > R$ (R is the potential width) the shape of the wave function is unchanged but $\delta \neq 0$. If the potential is attractive, then $\delta > 0$ and the nodes of the wave function are pulled toward the origin. Whereas for repulsive potential, $\delta < 0$ and the nodes are pushed away from the origin [4].

Depending upon the angular momentum of the incoming wave relative to the target, it is assumed that the incoming wave consists of many partial waves, each having different ℓ and consequently different phase changes. In terms of phase change, the scattering amplitude is given by [5]

$$f(\theta) = \frac{\sqrt{4\pi}}{k}\sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\exp(i\delta_\ell)\sin\delta_\ell\, Y_{\ell,0}(\theta) \tag{1}$$

and the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{4\pi}{k^2} \mid \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\exp(i\delta_\ell)\sin\delta_\ell\, Y_{\ell,0} \mid^2 \tag{2}$$

also the scattering cross section is

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)sin^2\,\delta_\ell \tag{3}$$

In order to obtain the nucleon-nucleon potential, at first we obtain the phase shift by using Darboux transformation. Then by using calculated phase shifts, the potentials determine for the first three values of ℓ for pp, nn, and np interaction in the energy range of 1 MeV to 350 MeV. The basic idea of this study is to associate angular momentum the long-distance asymptotic behavior of the potential, irrespective of its singularity at the origin. This is in the spirit as [6] where these

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asymptotic limits are independent of each other. This starting point will provide a new possibility for getting a correct effective range expansion of the phase shift which is the following Taylor expansion in the vicinity of $k = 0$

$$k^{2\ell+1} cot\delta_\ell(x) = -\frac{1}{a_{0\ell}} + \frac{1}{2} r_{0\ell} k^2 + \cdots \quad (4)$$

Here $a_{0\ell}$ is the scattering length and r_0 the effective range [7]. The expression (4) implies that for a given in the series expansion of tan $\delta_\ell(k)$ the coefficients of the terms containing powers of k below $2\ell + 1$ must vanish. In the frame of super-symmetric (SUSY) quantum mechanics, we solve this problem by introducing adequate long-distance Darboux transformation.

III. Darboux Transformation

The Darboux transformation method consists in getting solution φ of one Schrodinger equation [8]

$$h_1\varphi = E\varphi,\ h_1 = -\frac{d^2}{dx^2} + V_1(x) \quad (5)$$

when solution ψ of another equation

$$h_{0\psi} = E\psi \quad , \quad h_0 = -\frac{d^2}{dx^2} + V_0(x) \quad (6)$$

are known.

This is achieved by acting on $\boldsymbol{\psi}$ with a differential operator L of the form

$$\varphi = L\psi \ , \quad L = -\frac{d}{dx} + \omega(x) \quad (7)$$

where the real function $\omega(x)$ called super-potential, is defined as the logarithmic derivative of a known solution of (6) denoted by u in following. One has

$$\omega = \frac{\acute{u}(x)}{u(x)},\ \ h_0 u = \propto u \quad (8)$$

with $\propto \leq E_0$, where E_0 is the ground state energy of h_0. The function u is called transformation or factorization function and $\propto$ its factorization constant or factorization energy. The potential $V1$ is defined in terms of the super potential ω as

$$V_1(x) = V_0(x) - 2\acute{\omega}(x) \quad (9)$$

Equation (7) defines a first- order Darboux transformation. Now, let us start by first considering ℓ -fixed transformations as in [8]. This means that we use a special chain of $N = 2n - v$ first- order Darboux transformations with $v \geq 0$ generated by the following system of transformation functions:

$$v_1(x) \dots, v_v(x), u_{v+1}(x), v_{v+1}(x), \dots\dots, u_n(x), v_n(x) \quad (10)$$

$$h_0 u_j(x) = -a_j^2 u_j(x) \ , \ h_0 v_j(x) = -b_j^2 v_j(x) \quad (11)$$

where v_j are regular $[v_j = 0]$ and u_j irregular $[v_j \neq 0]$ ones, the latter being expressed in terms of the Jost solutions as

$$u_j(x) = A_j f(x, -ia_j) + B_j f(x, ia_j). \quad (12)$$

They have arbitrary eigenvalues $-a_j^2$ and $-b_j^2$, respectively, but always below E_0. If we are interested in the final action of the chain only, the solution $\psi_N(x,k)$of the transformed equation with the Hamiltonian

$$h_N = -\frac{d^2}{dx^2} + V_N \quad (13)$$

corresponding to the energy $E = k^2$

$$\psi_N(x,k) = W[u_1, \dots u_N, \psi_0(x,k)] W^{-1}(u_1, \dots u_N) \quad (14)$$

where W are Wronskians expressed in terms of u_j , denoting symbolically any function of (10) and of $\psi_0(x,k)$ which is a solution of original Schrodinger equation corresponding to the same energy E. In the Hamiltonian (13), the transformed potential is

$$V_N = V_0 - 2\frac{d^2}{dx^2} lnW(u_1, \dots u_N) \quad (15)$$

The corresponding phase- shift $\delta_\ell^N(k)$ can be written as

$$\delta_\ell^N(k) = \delta_\ell^0(k) + \Delta_\ell^N(k) \quad (16)$$

where $\delta_\ell^0(k)$ is the initial phase- shift due to the potential V_0 and $\Delta_\ell^N(k)$ (k) is the phase-shift produced by the chain of N Darboux transformations,

$$\Delta_\ell^N(k) = -\sum_{j=v+1}^{n} \arctan\left(\frac{k}{a_j}\right) - \sum_{j=1}^{n} \arctan\left(\frac{k}{b_j}\right) \quad (17)$$

Here a_j and $b_j$are arbitrary eigenvalues. In the next section, we will use these results to calculate phase-shift in np, pp, and nn interactions for $\ell = 0, 1, 2$ states.

IV. Calculations

A. Phase- Shift

1. Interaction (np) for $\ell = 0$

The asymptotic behavior of scattering wave function of the potential V_N with $\ell = 0$ and singularity strength v at long distance is

$$\psi_N(x,k) \sim \sin[kx - \frac{1}{2} v\pi + \delta_N(k)] \ , \ x \to \infty \quad (18)$$

In order to calculate $\delta_\ell^0(k)$, the equation bellow is used.

$$\delta_N(k) = \delta_0(k) - \sum_{j=v+1}^{n} \arctan\left(\frac{k}{a_j}\right) - \sum_{j=1}^{n} \arctan\left(\frac{k}{b_j}\right) \quad (19)$$

From (19) we take $\delta_0(k) = 0$ and also a_j and b_j are found by using the poles of the S matrix [8]. We set $v = 0$ therefor we have $N = 2n = 6$, and the poles are

$$a_1 = -0.0401 \ , \quad a_2 = -0.7540 \ , \quad a_3 = 4.1650 \ , \quad b_1 = 0.6152 \ , \quad b_2 = 2.0424 \ , \quad b_3 = 4.6000 \qquad (20)$$

in fm^{-1} units. For $\ell = 0$ the effective range expansion is

$$\lim_{k\to 0} k\, cot\delta(k) = -\frac{1}{a} + \frac{1}{2} r_0 k^2 - p r_0^3 k^4 + \cdots \qquad (21)$$

Expanding the phase shift (19) in a power series, one obtains

$$\frac{1}{a^N} = \frac{1}{a^0} + \left[\sum_{j=v+1}^{n} a_j^{-1} + \sum_{j=1}^{n} b_j^{-1}\right]^{-1} \qquad (22)$$

and

$$r^{(N)} = r^{(0)} + \frac{2a^{(N)}}{3}\left[1 - \left(\frac{1}{a^{(N)}}\right)^3 \left(\sum_{j=v+1}^{n} a_j^{-3} + \sum_{j=1}^{n} b_j^{-3}\right]\right. \qquad (23)$$

These formulas can be used to calculate phase- shift values in the (np) scattering. The obtained results are given in Table I.

2. Interaction (np) for $\ell = 1$

In order to study this situation we make the following assumption

$$L_1 = -\frac{d}{dx} + \frac{1}{x + x_0} \qquad (24)$$

The transformed potential is

$$V_1 = \frac{2}{(x + x_0)^2} \ , \ x_0 \geq 0 \qquad (25)$$

If one new applies operator (24) on an oscillating solution of the free particle equation, one obtains

$$\varphi_1(x,k) = -k\cos(kx + \delta_1^1) + \frac{\sin(kx + \delta_1^1)}{x + x_0} \qquad (26)$$

$$\delta_1^1 = \arctan k x_0 \qquad (27)$$

where

$$x_0 = \sum_{j=v+1}^{n} a_j^{-1} + \sum_{j=1}^{n} b_j^{-1} \qquad (28)$$

Note nevertheless that in resulting phase- shift, given by (16), δ_ℓ^ℓ has to be replaced by δ_1^1 of (27) since the initial potential for the sub chain of ℓ-fixed transformations is new $V1$ of (25). For the Nth order transformation the potential $V1$ and the phase- shift δ_1^1 play and auxiliary role.

As an application we look for neutron -proton which reproduces the pruned phase-shift of [9] for the $1P^1$ partial wave. This phase- shift obtained from (16) and exhibited in Table I. a_j and b_j are the six appropriate poles of the S matrix[8].

$$a_1 = -0.7290 \ , \quad a_2 = -0.7295 \ , \quad a_3 = 1.0368 \ , \quad b_1 = 0.4403 \ , \quad b_2 = 2.04408 \ , \quad b_3 = 3.3818 \qquad (29)$$

in fm^{-1} units. With use form (28) one gets $x0 = 3.0578 fm$. Now we can expand all arc tangent functions appearing in (16) in power series. This leads to a correct effective range expansion given by

$$k^3 cot\, \delta_1^7(k) = -0.3182 - 3.1511k^2 + \cdots \qquad (30)$$

from which one can extract the scattering length a_{01} and the effective range r_{01} defined according to (4).

3. Interaction (np) for ℓ = 2

Now we want to apply two subsequent transformations associated with function corresponding to $E = 0$ and then, as above, a sub-chain of ℓ fixed transformations. After the first transformation whit the function $u_{0,1} = x + x0$, the potential V_1 of (25) has $u = \frac{1}{(x+x_0)}$ and $u = (x + x0)^2$ as linearly independent solutions at combination $E = 0$. Their linear combination

$$u_{0,2} = Cu + \tilde{u} \qquad (31)$$

which is the transformation function for the second transformation step defined by the operator

$$L_2 = -\frac{d}{dx} + \frac{u_{0,2}(x)}{u_{0,2}(x)} \qquad (32)$$

contains two free parameters C and x_0. These can be chosen such as the series expansion for tan starts at K^5. The intermediate (or background) potential

$$V_2 = -2\left[\ln u_{0,2}(x) u_{0,1}(x)\right] = \frac{6(x+x_0)[(x+x_0)^3 - 2c]}{[c + (x+x_0)^3]^2} \qquad (33)$$

obtained from the $L = L1L2$ Darboux transformation (for more details see [8]) plays new the role of the initial potential for an ℓ-fixed sub chain of transformations. The background phase- shift corresponding to V_2 is

$$\delta_2^2 = arctan \frac{3kx_0^3}{3x_0 - k^2(3x_0^3 + c)} \qquad (34)$$

Note that the function $\varphi_2(x,k) = L2L1\, sin(kx + \delta_2^2)$ is (16) we have to identify $\delta_\ell^\ell(k)$ with $\delta_2^2(k)$ of (34). After expanding in power series all arc tangent functions we find that the coefficient of the term linear in k vanishes for x_0 given by (28) and

$$C = \frac{1}{3}\left[\sum_{j=1}^{n}(a_j^{-3} + b_j^{-3}\right] \qquad (35)$$

series expansion of $\tan \delta_2^N$. With a fit of a similar quality to that performed for the $1D_2$ partial wave phase shift of [8] with the following four poles of the S matrix

$$a_1 = -0.2047 \quad , \quad a_2 = -1.9800$$
$$b_1 = 1.2305 \quad , \quad b_2 = 4.9631 \tag{36}$$

in fm^{-1} units. From (28) and (35) we get $x0 = -4{:}375fm,\ c = 116{:}08fm^3$. This leads to the following effective range expansion

$$k^5 cot\ \delta_2^6(k) = 0.4496 + 10.9878k^2 + \cdots \tag{37}$$

where the superscript $N = 6$ represents 4 transformation functions associated with the poles (36) plus two zero-eigenvalue functions $u_{0,1}$ and $u_{0,2}$ defined above.This is consistent with the formula $(N = 2n - v + \ell)$ with $n = 2$, $v = 0$, $\ell = 2$. These values phase -shift is shown in Table I. From (37) one can extract the scattering length a_{02} and the effective range r_{02} defined in (4).

4. Interaction (pp,nn) for $\ell = 0$

For analyzing the scattering proton-proton we used the "shape -independent" effective range expansion

$$x(k^2) = -\frac{1}{a} + \frac{1}{2}rk^2 \tag{38}$$

The result of this method is a modified effective range expansion

$$x(k^2) = -\frac{1}{a} + \frac{1}{2}rk^2 - \frac{pk^4}{1+qk^2} \tag{39}$$

where $p = 0.554$ and $q = 3.055$ are given by parameters of the OPE (the pion mass and pion –nucleon coupling constant) [10], [11]. Data for scattering length a and the effective range r of the $1S_0$ partial wave, and in the Table IV for the $1S_0$ proton-proton case are summarized. We have calculated phase shift for the S state for proton-proton scattering. These values are shown in Table II.

We have calculated phase- shift for the S state for neutron-neutron scattering whit using scattering length $a = 23.75 \pm 0.01fm$ and the effective range $r = 2.75 \pm 0.05fm$ these values are shown in Table III.

5. Interaction (pp, nn) for $\ell = 1$

In discussing the low energy behavior of the P waves, the proton-proton the "shape independent" effective range expansion (38) is applied to the function $x(k^2)$[11].

$$x(k^2) = k^3 cot\delta_1 \tag{40}$$

In Table V scattering length a and the effective range r for the P wave are summarized. We have calculated phase- shift for the P states in proton-proton and neutron-neutron scattering. These values are shown in Tables II and III.

B. Scattering Amplitude

According to (1) for S-state we have:

$$f_0(\theta) = \frac{1}{k}\exp(i\delta_0)\, sin\delta_0 \tag{41}$$

while for P-state scattering amplitude is given by:

$$f_1(\theta) = f_0(\theta) + \frac{1}{k}\exp(i\delta_1)\, sin\delta_1 cos\theta \tag{42}$$

and for D-state

$$f_2(\theta) = f_1(\theta) + \frac{1}{2k}\exp(i\delta_2)\, sin\delta_2(3cos^2\theta - 1) \tag{43}$$

Using the phase-shift calculated values of Tables I-III the scattering amplitude have been calculated.

C. Nucleon- Nucleon Potential

As a Fourier transformation of the potential, the scattering amplitude depends upon the momentum transfer. For a spherical potential we have

$$V = -\frac{h^2}{4\mu\pi^2}2\pi\int_0^\pi\int_0^k q^2 f(q)\exp(i.\bar{q}\bar{r})\, dq\, sin\theta\, d\theta \tag{44}$$

For each value of ℓ, the amplitude is written as

$$f_0(\theta) = a \tag{45}$$

$$f_1(\theta) = a + b\cos(\theta) \tag{46}$$

$$f_2(\theta) = a + b\cos(\theta) + c(3cos^2(\theta) - 1) \tag{47}$$

Using $(q = 2ksin\frac{\theta}{2})$ the following equation are obtained

$$f_0(q) = a \tag{48}$$

$$f_1(q) = a + b(1 - \frac{q^2}{2k^2}) \tag{49}$$

$$f_2(q) = a + b[1 - \frac{q^2}{2k^2}] + c[3(1 - \frac{q^2}{2k^2})^2 - 1] \tag{50}$$

Substituting for the amplitude for each value of ℓ, we arrive at

$$V_0(r) = -\frac{h^2}{2\mu\pi}a\int_0^\pi\int_0^{2k} q^2\exp(-iqr\, cos\theta)\, sin\theta\, d\theta\, dq \tag{51}$$

$$V_1(r) = V_0(r) - \frac{h^2}{2\mu\pi}b\int_0^\pi\int_0^{2k} q^2\,(1 - \frac{q^2}{2k^2}) \exp(-iqr\, cos\theta)\, sin\theta\, d\theta\, dq \tag{52}$$

$$V_2(r) = V_1(r) - \frac{h^2}{2\mu\pi}c\int_0^\pi\int_0^{2k} q^2\,(3\left(1 - \frac{q^2}{2k^2}\right)^2 \text{-1}) \exp(-iqr\, cos\theta)\, sin\theta\, d\theta\, dq \tag{53}$$

Using the previous calculated values of Tables I-III, the potentials have been calculated for the first three values of ℓ for pp, nn and np interactions in the energy range of $1MeV$ to $350MeV$. The results of calculations are given in Tables VI-VIII.

V. Summary and Conclusion

Breaking of charge independence of nucleon-nucleon interaction is of special interest in the nuclear theory. In this article, the nucleon-nucleon elastic scattering $(\mathrm{np}, \mathrm{pp}, \mathrm{nn})$ in the energy rangy of $1MeV$ to $350MeV$ for the first three values of ℓ, is considered. The phase -shifts for mentioned interactions are calculated by using Darboux transformation. Then by using calculated phase -shifts, the nucleon-nucleon potential are determined.

The results of the calculated inter nucleon potentials indicate that the strong nuclear force acting between nucleons is indeed charge-independent in most of the energy range observer here. There are small portions of energy range in which charge-independence breaking is noted, but within the experimental errors such violation may not be ignored. It is interesting to note that such charge-independence breaking occurs only in the range of $120MeV$ to $200MeV$ for all states, suggesting that a charge dependent term be added to the nucleon-nucleon potential previously introduced in the literature in which such dependency is absent in this energy range.

TABLE I
THE CALCULATED PHASE SHIFT IN (NP) INTERACTION

$E_{lab}(MeV)$	$^1S_0(deg)$	$^1P_1(deg)$	$^1D_2(deg)$
1	62.289	-0.212	0.0066≈0
5	63.872	-1.55	-0.057
10	59.023	-3.22	0.158
25	50.775	-6.904	0.6546
50	40.298	-9.875	1.695
100	26.211	-14.681	3.812
150	16.012	-18.396	5.674
200	7.854	-22.472	7.239
250	1.006	-24.104	8.545
300	-4.909	-26.401	9.637
350	-10.123	-28.436	10.509

TABLE II
THE CALCULATED PHASE SHIFT IN (PP) INTERACTION

$E_{lab}(MeV)$	$^1S_0(deg)$	$^3P_0(deg)$	$^3P_1(deg)$	$^3P_2(deg)$	$^1D_2(deg)$
1	32.86	0.1380	-0.084	0.0118	0.00
5	55.4	1.68	-0.11	0.24	0.04
10	56.1	3.78	-2.089	0.69	0.017
25	48.93	8.89	-5.03	2.84	0.70
50	39.08	11.76	-8.65	5.92	1.71
100	25.06	10.02	-13.67	11.09	3.77
150	14.96	4.96	-17.84	14.21	5.67
200	7.21	-0.53	-21.95	15.94	7.26
250	-0.34	-5.83	-25.3	16.79	8.55
300	-6.67	-10.61	-28.94	17.62	9.54
350	-11.43	-15.06	-31.51	17.41	10.27

TABLE III
THE CALCULATED PHASE SHIFT IN (NN) INTERACTION

$E_{lab}(MeV)$	$^1S_0(deg)$	$^3P_0(deg)$	$^3P_1(deg)$	$^3P_2(deg)$	$^1D_2(deg)$
1	57.69	0.26	-0.125	0.023	0.00
5	61.5	1.96	-1.19	0.31	0.05
10	58.65	4.33	-2.35	0.79	0.18
25	50.06	9.02	-5.23	2.89	0.074
50	38.92	11.86	-8.62	6.63	1.77
100	24.61	9.62	-13.68	11.51	3.88
150	14.56	4.82	-17.97	14.43	5.80
200	6.09	-0.83	-22.01	16.09	7.42
250	-0.96	-6.03	-25.39	16.99	8.72
300	-6.98	-10.88	-28.69	17.66	9.72
350	-12.36	-15.54	-31.91	17.82	10.46

TABLE IV
EFFECTIVE RANGE PARAMETERS FOR THE (PP) $^1S_0$

Equation for expansion	a(fm)	r(fm)
(38)	-7.76 ± 0.0098	2.687 ± 0.0146
(39)	-7.826 ± 0.001	2.803 ± 0.015
(39)	-7.828 ± 0.008	2.80 ± 0.02
(38)	-7.822 ± 0.003	2.775 ± 0.006
(39)	-7.844 ± 0.003	2.859 ± 0.006

TABLE V
EFFECTIVE RANGE PARAMETERS FOR THE P WAVES

state	a(fm)	r(fm)
$^1P_1$	2.4 ± 1.3	-12.6 ± 2.2
$^3P_0$	-2.84 ± 0.02	4.45 ± 0.05
$^3P_1$	1.90 ± 0.01	-7.56 ± 0.05
$^3P_2$	-0.31 ± 0.01	7.59 ± 0.28

TABLE VI
INTERNUCLEON POTENTIALS FOR PP, NN AND NP INTERACTIONS IN $S-$ STATE

$E_{lab}(MeV)$	$V_{pp}(MeV)$	$V_{nn}(MeV)$	$V_{np}(MeV)$
1	-0.6521	-0.6521	-0.6521
5	-2.927	-2.927	-2.927
10	-4.816	-4.801	-4.801
25	-6.702	-6.6825	-6.6825
50	-4.025	-4.015	-4.015
100	0.8723	0.697	0.697
150	0.826	0.731	0.731
200	0.1577	0.157	0.157
250	1.52E-4	1.5E-4	1.5E-4
300	-0.0479	-0.048	-0.048
350	-0.3268	-0.322	-0.322

TABLE VII
INTERNUCLEON POTENTIALS FOR PP; NN AND NP INTERACTIONS IN P- STATE.

$E_{lab}(MeV)$	$V_{pp}(MeV)$	$V_{nn}(MeV)$	$V_{np}(MeV)$
1	-06521	-06521	-06521
5	-2.929	-2.929	-2.929
10	-4.895	-4.895	-4.895
25	-6.693	-6.693	-6.693
50	-4.4203	-4.4203	-4.4203
100	-1.368	-1.368	-1.368
150	3.745	3.745	3.745
200	1.334	1.334	1.334
250	2.432	2.432	2.432
300	6.81	6.81	6.81
350	7.035	6.201	6.201

TABLE VIII
INTERNUCLEON POTENTIALS FOR *PP, NN* AND *NP* INTERACTIONS IN D - STATE.

$E_{lab}(MeV)$	$V_{pp}(MeV)$	$V_{nn}(MeV)$	$V_{np}(MeV)$
1	-0.6523	-0.6523	-6-0.6523
5	-2.929	-2.9929	-2.9929
10	-4.895	-4.895	-4.895
25	-6.692	-5.968	-5.968
50	-4.408	-4.206	-4.206
100	-1.3926	-1.393	-1.393
150	3.2153	2.717	2.717
200	-0.2071	-02071	-0.2071
250	0.8754	-4.83	-4.502
300	3.19	-2.23	-2.12
350	5.397	-0.21	-0.86

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