

# BPNN Based Processing for End Effects of HHT

Chun-Yao Lee and Yao-chen Lee

**Abstract**—This paper describes a method of signal process applied on an end effects of Hilbert-Huang transform (HHT) to provide an improvement in the reality of spectrum. The method is based on back-propagation network (BPN). To improve the effect, the end extension of the original signal is obtained by back-propagation network. A full waveform including origin and its extension is decomposed by using empirical mode decomposition (EMD) to obtain intrinsic mode functions (IMFs) of the waveform. Then, the Hilbert transform (HT) is applied to the IMFs to obtain the Hilbert spectrum of the waveform. As a result, the method is superiority of the processing of end effect of HHT to obtain the real frequency spectrum of signals.

**Keywords**—Neural network, back-propagation network, Hilbert-Huang transform

## I. INTRODUCTION

HILBERT-Huang transform (HHT) is a new method to nonlinear and non-stationary analysis. HHT consists of two part of empirical mode decomposition (EMD) and Hilbert Transform (HT); herein EMD is the key for HHT. A number of literatures discuss the methodology of EMD. However, some unresolved arguments still exist, such as the choice of envelope, a criterion for the sifting process to stop, intermittency problems, and end effects. This study tries to find a solution of end effects problem in vibration signals analysis.

The signal waveform of the extreme value of the upper and lower envelope is obtained using EMD. But both sides of the boundary points are non-extreme point. Therefore, the envelope on the boundary there will not be depicted properly, resulting in distortion of the boundary on both sides of the decomposition results. In order to resolve the end effects, this study presents BPN to extend the boundary waveform to improve the problem that the envelope is be not depicted properly on both sides of the boundary. Finally, the comparison of the results with/without the proposed procedure is discussed.

## II. HILBERT-HUANG TRANSFORM

HHT is a new signal analysis method, which is proposed by [1]. Through the adaptive base, it could analyze signals of non-stationary and nonlinear. HHT can be divided into two parts, EMD and Hilbert transform. The original signal is decomposed into several IMFs and a monotonic signal trend by

HHT. Then we obtain the instantaneous amplitude and the instantaneous frequency by Hilbert transform to each of IMFs. Finally, we get the corresponding distribution of time, frequency and amplitude. The procedures are explicitly explained as below.

### A. Empirical Mode Decomposition

HT is superior in stationary signals but not in non-stationary and DC signal. Therefore, Huang in [2] proposes a method of EMD which the signal of the similar frequency components and a trend can be decomposed where IMFs must satisfy the following conditions.

- 1) The number of extremes and the number of zero-crossings must equal or differ by one.
- 2) The signal at any point, the mean value of upper and lower envelope must be zero.

If IMFs satisfy the above conditions, we can ensure that IMFs oscillate to zero as the center. The signal repeats to be decomposed into IMFs and a residual, where the residual is the trend of signal. The steps of EMD are explicitly explained as below, and the procedures are shown in Fig. 1.

- step 1.* Find local maximum points of the signal, and connect the points to produce the upper envelope by cubic spline. Repeat the procedure for local minimum points to produce the lower envelope.
- step 2.* Compute the mean value of the upper and lower envelopes,  $m_{1,k}$ .
- step 3.* Compute  $h_{1,k} = x(t) - m_{1,k}$ .
- step 4.* Repeat step1 to step4, judging  $h_{1,k}$ . If  $h_{1,k}$  satisfies the definition of IMFs, save as  $c_n = h_{n,k}$ .
- step 5.* Compute the residual value  $r_n = h_{n,0} - c_n$ .
- step 6.* Determine the residual value is a monotonic trend, and if so, to complete decomposition; if not, repeat steps one to four, made the rest of the intrinsic mode function.

The original signal is decomposed by EMD and several IMFs and a monotonic trend can obtained, as shown in (1).

$$x(t) = \sum_{j=1}^n c_j + r_n \quad (1)$$

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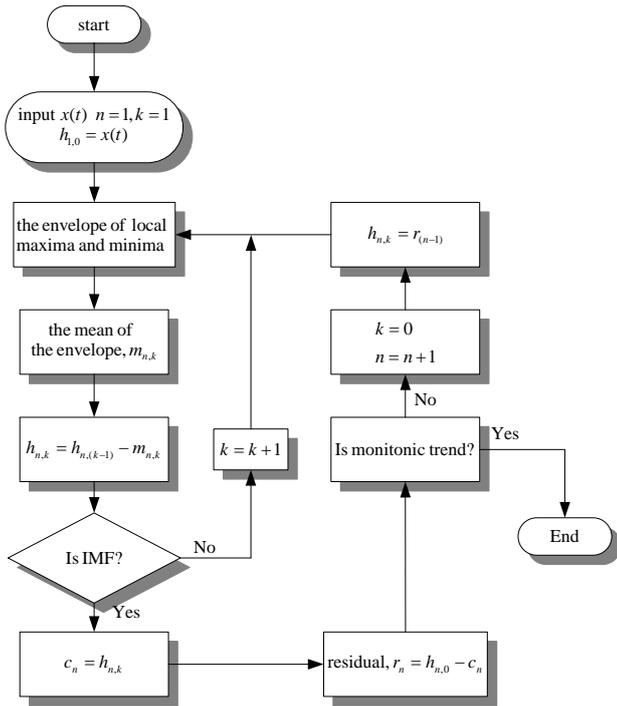


Fig. 1 The flowchart of EMD.

### B. Hilbert Transform

The IMFs  $c_j(t)$  are transformed by Hilbert transform as shown in (2). Where  $PV$  is the Cauchy principal value.

$$y_j(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{c_j(\tau)}{t - \tau} d\tau \quad (2)$$

$c_j(t)$  and  $y_j(t)$  can be composed the conjugate form, as shown in (3).

$$z(t) = c_j(t) + iy_j(t) = a_j(t)e^{i\theta_j(t)} \quad (3)$$

We can obtain the instantaneous amplitude  $a_j(t)$  and the instantaneous phase  $\theta_j(t)$ , respectively, as shown in (4) and (5).

$$a_j(t) = \sqrt{c_j^2(t) + y_j^2(t)} \quad (4)$$

$$\theta_j(t) = \tan^{-1} \frac{y_j(t)}{c_j(t)} \quad (5)$$

From (5), the instantaneous frequency can be obtained, as shown in (6).

$$\omega_j(t) = \frac{d\theta_j(t)}{dt} \quad (6)$$

Hilbert spectrum can be obtained from the relationship of instantaneous amplitude, instantaneous frequency and time.

### III. END EFFECT OF HHT

There are two dash-line curves represented the upper and lower envelopes of the signal, respectively. However, the envelopes on both sides of the end of origin wave are depicted abnormally, which causes the waveform is distortion on the ends. The waveform obtained by original signal minus IMF is regarded the next iteration original signal. Therefore, the distortion of IMF occurs and the distortion trend is from ends to the center of the curve, even the whole curve completely distorts finally [2]. It is called end effect in this paper. We employs neural network to solve the problem.

#### A. Back-propagation Network

Pual Werbos in 1974 described multi-network training algorithm based on back-propagation network (BPN). David Rumelhart et al. in 1985 proposed BPN [3], [4] take advantage in the research.

BPN is a widespread method for categorization and prediction. It is belong to supervised learning network. The structure mainly contains input layer, output layer, and hidden layer. The principle of BPN is gradient steepest descent method, by repeated conveying the error gradient forward to update weights and bias, so that the output achieving target value. The procedure of BPN can be divided into the feed-forward phase and the back propagation phase. The network computes the output value in the feed-forward phase, and updates weights and bias in the back propagation phase.

The input and output of neural network must be dependent. In the process of the neural network, data have to be divided into two parts, training and test set. The training set is used for the network training to construct the network model, and the test set is used for testing the accuracy of the network. When accuracy does not achieve the target, the network trains again or increases the number of training set, to achieve the target accuracy. The procedures are explicitly explained as below.

#### The feed-forward phase

- 1) Input  $P$  and calculate the output value of the hidden layer,  $a_1$ , as shown in (7). Where  $f_1$  refers to the activation function of the hidden layer,  $w_1$  refers to the weights of the hidden layer, and  $b_1$  refers to the biases of the hidden layer.

$$a_1 = f_1(w_1 p + b_1) \quad (7)$$

- 2) The output value of the hidden layer,  $a_1$ , is regarded the input value, and the output value of the output layer,  $a_2$ , is computed, shown in (8). Where  $f_2$  refers to the activation function of the output layer,  $w_2$  refers to the weights of the output layer, and  $b_2$  refers to the biases of the output layer.

$$a_2 = f_2(w_2 a_1 + b_2) \quad (8)$$

### The propagation phase

- 1) The mean square error is considered a indicator of weight and bias. Then, we compute the mean square error of output  $a_2$  and target  $d$ , as shown in (9).

$$E(t) = [d(t) - a_2(t)]^2 \quad (9)$$

- 2) Compute error gradients for each weight and each bias for revising the measured size and direction, as shown in (10) to (13).

$$\frac{\partial E(t)}{\partial w_2(t)} = -[e(t)f_2'(w_2(t)a_1(t) + b_2(t))]a_1(t) \quad (10)$$

$$\frac{\partial E(t)}{\partial b_2(t)} = -e(t)f_2'[w_2(t)a_1(t) + b_2(t)] \quad (11)$$

$$\frac{\partial E(t)}{\partial w_1(t)} = -\left[ \sum_k e(t)f_2'(w_2(t)a_1(t) + b_2(t)w_2(t)) \right] \quad (12)$$

$$f_1'(w_1(t)p(t) + b_1(t))p(t)$$

$$\frac{\partial E(t)}{\partial b_1(t)} = -\left[ \sum_k e(t)f_2'(w_2(t)a_1(t) + b_2(t)w_2(t)) \right] \quad (13)$$

$$f_1'(w_1(t)p(t) + b_1(t))$$

- 3) Suppose the learning rate is  $\alpha$ . Adjust each weight and each bias, as shown in (14) to (17).

$$w_2(t+1) = w_2(t) - \alpha \frac{\partial E(t)}{\partial w_2(t)} \quad (14)$$

$$b_2(t+1) = b_2(t) - \alpha \frac{\partial E(t)}{\partial b_2(t)} \quad (15)$$

$$w_1(t+1) = w_1(t) - \alpha \frac{\partial E(t)}{\partial w_1(t)} \quad (16)$$

$$b_1(t+1) = b_1(t) - \alpha \frac{\partial E(t)}{\partial b_1(t)} \quad (17)$$

### B. The Processing of End Effect

The origin signal is extended on both end sides by BPN. We can obtain IMFs from the full waveform by EMD, and the Hilbert spectrum can be obtained from IMFs by Hilbert transform. The extension of IMFs and Hilbert spectrum is removed.

### IV. TEST RESULT

The IMF, c1 diverges slightly on both sides of ends. The IMF, c2 has more serious divergence than c1, and the upper bound and lower bound reach +10 and -10. It is unreasonable for the values of c2 on both sides are too large relative on middle. The condition of c3 and c4 is similar to c2. Therefore, the values of IMFs on both sides are not correct. Observed c1 - c4, we can find a phenomenon that the divergence occurs more near the middle of signal. For signal analysis, the layers of IMF, c2-c4,

are more uncorrected. The IMFs with the proposed procedure has no divergence on both end sides. The result has shown that can improve the end effect problem by BPN.

The divergent of Hilbert spectrum occurs. In the case, the Hilbert spectrum on both sides without the proposed procedure is incorrect. The problem can be solved by the proposed procedure, in which there is no divergence on both sides correctly and the correct Hilbert spectrum can be obtained. As a result, the method is superiority of the processing of end effect of HHT to obtain the real Hilbert spectrum of signals.

### V. CONCLUSION

In this paper, we can observe the results with and without the proposed procedure to the both sides of the origin signal. These results show that using the proposed procedure for the end effect problem of HHT is feasible. The processed signal has no divergence by BPN, and can correctly show its Hilbert spectrum. The above results indicate that BPN for end effect of HHT is superiority.

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