Blind Source Separation Using Modified Gaussian FastICA

V. K. Ananthashayana, and Jyothirmayi M.

Abstract—This paper addresses the problem of source separation in images. We propose a FastICA algorithm employing a modified Gaussian contrast function for the Blind Source Separation. Experimental result shows that the proposed Modified Gaussian FastICA is effectively used for Blind Source Separation to obtain better quality images. In this paper, a comparative study has been made with other popular existing algorithms. The peak signal to noise ratio (PSNR) and improved signal to noise ratio (ISNR) are used as metrics for evaluating the quality of images. The ICA metric Amari error is also used to measure the quality of separation.

Keywords—Amari error, Blind Source Separation, Contrast function, Gaussian function, Independent Component Analysis.

I. INTRODUCTION

BLIND source separation (BSS) problem has become an important application in the research domain of statistical signal processing. In a basic blind source separation problem, the goal is to separate mutually statistically independent sources, without knowing the mixing coefficients from their instantaneous linear mixtures. But verification of independent condition of these sources is very difficult. Hence some higher order statistics are used in practice for achieving separation. Independent Component Analysis (ICA) is one of the statistical tools that use higher order statistics to discover the hidden factors from the set of observed data. BSS technique[5,6,8] has a variety of applications in communication, speech processing, multimedia and medical image processing [1, 2].

Mathematically, the BSS problem can be formulated as

$$X = WS + n \qquad (1)$$

where x is the observed image (mixed), S is the original source images obtained which is transformed by the mixing matrix W whose elements are unknown and n is the additive noise often omitted. Therefore, a multivariate data analysis, such as ICA [3, 4, 11,12,13], might be used to solve the source separation and restoration problem. In ICA, the separability of the mixed sources depends on the principal of redundancy reduction and the sources must be as independent as possible.

In this paper, the popular fixed point algorithm with a new objective function called Modified Gaussian function to extract the independent components is used. The objective function is based on the approximation of negentropy [4] that needs to be maximized. The statistical property of this function is demonstrated for its robustness in case of image mixtures. The images considered are here can be compared with the work in [9]. Further, we measure the quality of the separated images with Amari metric and assess the performance of separated images using ISNR and PSNR.

The remainder of this paper is organized as follows. In section II, the methodology for source separation using ICA is discussed; section III gives results and conclusion in section IV.

II. METHODOLOGY

A. ICA Model

ICA is formulated as a problem by which independent original sources are extracted based on the observations of mixture sources. The model can be expressed as a linear combination (Fig. 1.)

$$\begin{bmatrix} s_1(t) \\ \vdots \\ \vdots \\ s_N(t) \end{bmatrix} \rightarrow \begin{bmatrix} a_{11}(l) & \cdots & a_{1N}(l) \\ \vdots & \ddots & \vdots \\ a_{M1}(l) & \cdots & a_{MN}(l) \end{bmatrix} \rightarrow \begin{bmatrix} x_1(t) \\ \vdots \\ \vdots \\ x_M(t) \end{bmatrix} \rightarrow \begin{bmatrix} w_{11}(l) & \cdots & w_{1M}(l) \\ \vdots & \ddots & \vdots \\ w_{N1}(l) & \cdots & w_{NM}(l) \end{bmatrix} \rightarrow \begin{bmatrix} y_1(t) \\ \vdots \\ \vdots \\ y_N(t) \end{bmatrix}$$

Fig. 1 The schematic diagram of BSS

The linear equation for two source images can be expressed as:

$$\begin{aligned} X_1(t) &= a_{11} S_1 + a_{12} S_2 \\ X_2(t) &= a_{21} S_1 + a_{22} S_2 \end{aligned} \qquad (2)$$

where S_1 and S_2 are original sources, X_1 and X_2 are observed (mixture) sources and a is the unknown matrix that needs to be estimated. By determining a, we can separate the two original signals S_1 and S_2 from their mixtures $X_1(t)$ and $X_2(t)$. The number of sources N (known) can be estimated, when the number of sensors M is equal to or greater than N, $M \geq N$. The mixed signal X is represented as in (3). This ICA model is a generative model that describes the computation of S by

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estimating the mixing matrix a from the known random vector denoted by $S = A^{-1}X$ and $W = A^{-1}$

$$\begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MN} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ \vdots \\ \vdots \\ s_N \end{bmatrix} \qquad (3)$$

To estimate W some general and fundamental assumptions are made.

a) The components s_N (i.e. the sources) are statistically independent.

b) The independent components must have nongaussian distribution or utmost one source may have gaussian distribution. (Fig. 8)

Depending on the application, the sources and the mixture may be real or complex. Source separation is carried out by estimating a separating matrix W such that the separated output Y=WX contains an estimate of the sources.

To satisfy the assumption (a), the mixed signal is first whitened [10] thereby restricting the search space for the mixing matrix to the space of orthogonal matrices. To measure the non gaussianity, kurtosis or differential entropy called negentropy can be used. Since kurtosis is not a robust measure of nongaussianity, we consider negentropy. The function [4] J_G, a maximizing entropy equation can be used as a gradient approximation of negentropy

$$J_G(w) = \left[E\left\{ G(w^T x) \right\} - E\left\{ G(v) \right\} \right]^2 \qquad (4)$$

where G is a non quadratic function, v is a Gaussian variable of zero mean and unit variance and w can be found under the constraint $E\{(w^T x)^2\} = 1$

B. Contrast Functions

The First thorough mathematical framework for BSS and ICA was established by Comon in [6]. In order to achieve separation, it is important to choose a non-linearity function g, a derivative of G [4, 7, 8, 14]. The objective function used in FastICA is

$$g(y) = y \exp\left(-y^2/2 \right) \qquad (5)$$

Fig. 2a gives the distribution spread for this equation. As g(.) is dependent on the estimated sources y the distribution obtained is sigmoid.
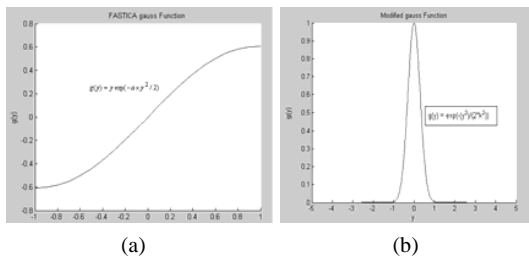

(a) (b)

Fig. 2 Plot showing a) the Gaussian function for the FastICA
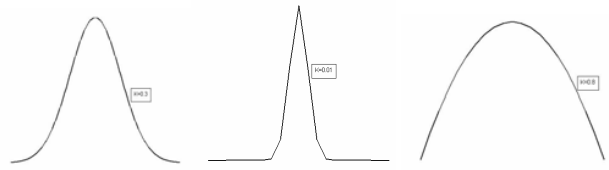b) The Gaussian function for MGFICA

Fig. 3 Probability distribution functions showing the variation of K value (K=0.3, k=0.01, K=0.9)

Estimation of W is a measure of minimization of mutual information and maximization of negentropy which can be interpreted as estimation of single Independent component.

In this section, we describe our approach based on the objective function 'The Modified Gaussian function', abbreviated as MGFICA. The unmixing matrix W is estimated by maximizing the contrast function with respect to W given by:

$$W = \sum_{i=1}^{n} \sum_{j=1}^{p} g\ (w_j^T x_i) \qquad (6)$$

where $g(y)$ is the modified Gaussian function, defined by

$$g(y) = \exp(-(y)^2/(2K^2)) \qquad (7)$$

where, K is a parameter that gives a measure of distribution, (Fig. 3). By varying the parameter K, it is possible to describe the function as Gaussian, platykurtic, and lepokurtic distributions that control the spread of the distribution. The plot of a contrast function defined by A.Hyvarinen [4] is shown as a comparison with the modified Gaussian plot Fig.2 for K=0.1. Optimal value of K can be found from experimentation. The constant parameter gives the algorithm a kind of 'Self adaptation' quality. From the distributions it is clear that, for certain values, the probability distribution function (pdf) is spiky and hence the entropy will be small. Hence it is possible to choose an efficient contrast function based on the probability spread function that has maximum entropy, converges faster and the sources are separated with less distortion.

C. Modified Gaussian FastICA Algorithm (MGFICA)

To simplify the algorithm, the data X is centered and whitened by computing covariance $E\{xx^T\}$. FastICA[10] uses convergence criteria in which the old and the new values of W estimated must point in the same direction. To extract all the independent components one by one, we need to run the algorithm several times. To prevent the vectors $w_1 \ldots w_N$ converge to same maxima we orthogonalize the vectors after every iteration. For the function g, we use a robust approximation of negentropy as in (7) and for the problem of BSS, we demonstrate that quality of separation is improved with our modified Gaussian function. Hence choice of the right function is also very important as a robust estimator. Table I provides the algorithmic description for the MGFICA.

TABLE I
ALGORITHM FOR MGFICA

1. Center the data to make its mean zero.
2. Whiten the data to give Z.
3. Choose an initial random vector W of unit norm.
4. $W \leftarrow E\{Zg(W^{T}Z)\} - E\{g'(W^{T}Z)\}W$
5. Normalize $W \leftarrow W/\|W\|$
6. If not converged, go to step4.

III. EXPERIMENTAL RESULTS

We demonstrate the capability of the proposed algorithm to successfully separate four images and compare its performance to competing algorithms. Four source images[1] of size 256x512 are shown in Fig. 4. The pixel intensities are normalized in the [0,255] interval. The observed image obtained by mixing original images with a 4x4 random matrix resulting in a mixture of size 4x131072 Fig. 5. The separated image obtained after applying MGFICA algorithm is shown in the Fig. 6 and with FastICA is shown in Fig. 7. A comparison of Fig. 6 and Fig. 7 for source separation can be visualized. The variation of *K* for different values is also experimented and optimal value is found to be 0.3 for quality separation. For the evaluation of the quality of demixing matrix W, the Amari's error *d(W,A)*, an Amari metric (Amari et al.(1996)) is used. The Amari error (8) is used as a measure to compare different ICA algorithms and the matrices should be scaled in the same way. The value so obtained is such that it is scaled to a value between 0 and 1 and 0 corresponds to optimal separation. Table II shows Amari error for comparison with other algorithms.

$$d(W,A) = \frac{1}{2N(N-1)}\left\{\sum_{i=1}^{N}\left(\frac{\sum_{j=1}^{N}|b_{ij}|}{\max_{j}|b_{ij}|} - 1\right) + \sum_{j=1}^{N}\left(\frac{\sum_{i=1}^{N}|b_{ij}|}{\max_{i}|b_{ij}|} - 1\right)\right\} \quad (8)$$

where $b_{ij} = (W^{-1}A)_{ij}$, W and A are NxN matrices.

For the purpose of objectively testing the performance of separation problem, the Peak Signal to Noise Ratio (PSNR) is often used (9). We also evaluate the performance based on Improved Signal to Noise Ratio (ISNR) as in (10). Results are tabulated in Table III and Table IV.

$$psnr = 20\log_{10}\left(\frac{255}{\sum_{m=1}^{M}\sum_{n=1}^{N}s(m,n) - y(m,n)}\right) dB \quad (9)$$

$$Isnr = 10\log_{10}\left(\frac{s(m,n) - x(m,n)}{s(m,n) - y(m,n)}\right) dB \quad (10)$$

[1] http://www.cis.hut.fi/projects/ica/data/images/
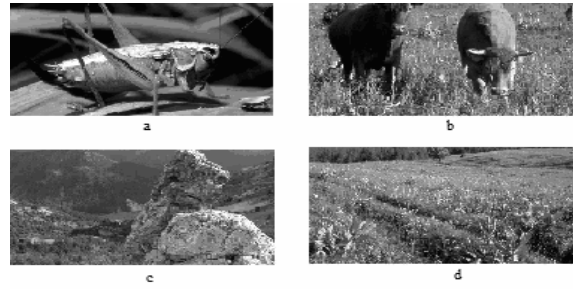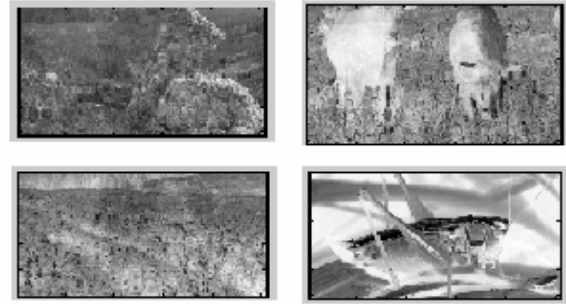

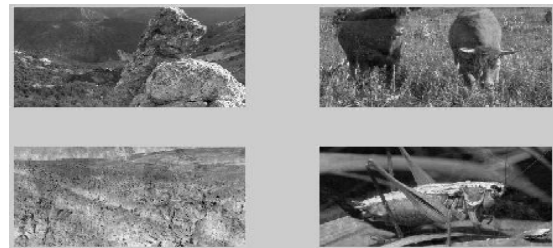Fig. 4 Original Images

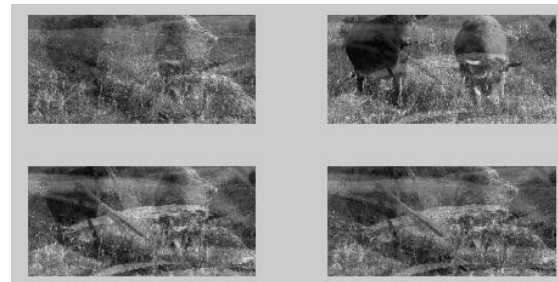
Fig. 5 Mixed images

Fig. 6 Images Separated from MGFICA
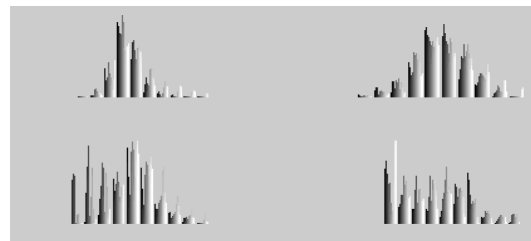

Fig. 7 Images separated from FastICA

Fig. 8 Histogram representation of mixed images

TABLE II
AMARI ERRORS

Algorithms	Amari Error between mixing and demixing matrix
MGFICA	0.3877985
FASTICA with Gauss Function	0.4530318
FASTICA with Tanh Function	0.4689753
Radical ICA	0.5191616

TABLE III
COMPARISON WITH DIFFERENT METRICS

Fig No	PSNR in dB		
	Fast ICA	Radical ICA	Modified ICA
a	12.13	16.4401	26.33
b	11.06	17.7517	30.43
c	16.33	17.8795	10.492
d	10.61	17.3477	23.83

TABLE IV
COMPARISON WITH DIFFERENT METRICS

Fig No	ISNR in dB		
	Fast ICA	Radical ICA	MGFICA
a	-0.1	-1.0	-8.14
b	-3.0	1.09e03	13.14
c	3.6	1.05e03	12.7
d	1.5	1.1976e03	11.72

We estimate the demixing matrix W from which the sources are separated. This result is compared with the different algorithms like FastICA and Radical [9]. Table III, IV shows the quality assessment for all the separated Images which gives an improvement in PSNR and ISNR.

IV. CONCLUSION

The proposed Modified Gaussian ICA is used to solve the problem of Source Separation. The effectiveness of the algorithm is the selection of the negentropy based function that can give optimal solution. The simplicity of using a modified gaussian function is the main criteria in this algorithm. Experimental result shows that there is a better quality of separation of the mixed images (Amari error) and the PSNR has improved moderately than in the existing ICA algorithms. The visual quality as well as the objective quality in terms of ISNR of the reconstructed images can also be compared.

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