

# Blind Non-Minimum Phase Channel Identification Using 3<sup>rd</sup> and 4<sup>th</sup> Order Cumulants

S. Safi, and A. Zeroual

**Abstract**—In this paper we propose a family of algorithms based on 3<sup>rd</sup> and 4<sup>th</sup> order cumulants for blind single-input single-output (SISO) Non-Minimum Phase (NMP) Finite Impulse Response (FIR) channel estimation driven by non-Gaussian signal. The input signal represents the signal used in 10GBASE-T (or IEEE 802.3an-2006) as a Tomlinson-Harashima Precoded (THP) version of random Pulse-Amplitude Modulation with 16 discrete levels (PAM-16). The proposed algorithms are tested using three non-minimum phase channel for different Signal-to-Noise Ratios ( $SNR$ ) and for different data input length. Numerical simulation results are presented to illustrate the performance of the proposed algorithms.

**Keywords**—Higher Order Cumulants, Channel identification, Ethernet communication.

## I. INTRODUCTION

**F**INITE Impulse Response (FIR) system identification from output measurements only is a well-defined problem in several science and engineering areas such as communications, speech signal processing, adaptive filtering, spectral estimation, radar Doppler, sonar, geophysical, biomedicine, blind equalization, plasma physics, seismic data processing, image reconstruction, harmonic retrieval, time-delay estimation and array processing ([1], [2], [3], [4], [6], [7], [8], [9], [10], [11], [12], [13], [14], [16]). The interest in Higher Order Cumulants (HOC) (or Higher Order Statistics (HOS)) has been permanently growing in the last years. Mainly finite impulse response (FIR) system identification based on HOC of system output has received more attention. Tools that deal with problems related to either non-linearity, non-Gaussianity, or non-minimum phase (NMP) systems are available, because they contain the phase information of the underlying linear system in contrast to second order statistics, and they are of great value in applications, such as radar, sonar, array processing, blind equalization, time delay estimation, data communication, image and speech processing and seismology. Many algorithms have been proposed in the literature for the identification of FIR system using cumulants. These algorithms can be classified into three broad classes of solutions: Closed form solutions [13], [17] Optimization-based solutions [11], [15] and Linear algebra solutions [13], [14], [21]. Recently, the latter have received great attention because

they have “simpler” computation and are free of the problems of local extreme values that often occur in the optimization solution. Although the closed-form solutions have similar features, they usually do not smooth out the noises caused from the observation and computation [16], [18], [19] and [20]. Therefore, while these solutions are more interesting from the theoretical point of view, they are not recommended for practical applications [5], [21]. In this paper, we propose a family of algorithms based on linear algebra solution, exploit third, and fourth order cumulants. In the part of simulation, we have tested the proposed algorithms for channel identification, principally the Ethernet channel. The Ethernet has evolved into the most widely implemented physical and link layer protocol today. Fast Ethernet increased speed from 10 to 100 megabits per second (Mbit/s). Gigabit Ethernet was the next iteration, increasing the speed to 1000 Mbit/s. The initial standard for gigabit Ethernet was standardized by the IEEE in June 1998 as IEEE 802.3z. 10 gigabit Ethernet or 10GbE is the most recent (as of 2006) and fastest of the Ethernet standards. It defines a version of Ethernet with a nominal data rate of 10 Gbit/s, ten times as fast as gigabit Ethernet. 10GbE over fiber and InfiniBand “like” copper cabling are specified by the IEEE 802.3-2005 standard. 10GbE over twisted pair has been released under the IEEE 802.3an amendment. The 802.3an standard defines the wire-level modulation for 10GBASE-T as a Tomlinson-Harashima Precoded (THP) version of pulse-amplitude modulation with 16 discrete levels (PAM-16), encoded in a two-dimensional checkerboard pattern known as DSQ128. Several proposals were considered for wire-level modulation, including PAM with 12 discrete levels (PAM-12), 10 levels (PAM-10), or 8 levels (PAM-8), both with and without Tomlinson-Harashima Precoding (THP). PAM-5 was used in the older 1000BASE-T gigabit Ethernet standard [22], [23] [24] and [25].

In this paper, we will consider a NMP channel excited by a random PAM-16, for different signal to noise ratio ( $SNR$ ) and for different size data input. The results show the performance of the proposed algorithms for small data input in noisy environment.

## II. PROBLEM STATEMENT

### A. Channel modeling

Consider the transmission of  $N$  i.i.d zero-mean symbols with unit energy, belonging to some alphabet  $A$ , across a frequency selective channel with memory  $p$  and Additive white Gaussian Noise (AWGN) (Fig. 1).

The NMP channel output is modeled as the output of a FIR system that is excited by an unobservable input and is

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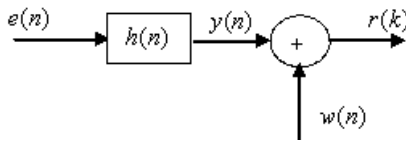


Fig. 1. Discrete time Non-Minimum phase channel.

corrupted at its output by an additive white Gaussian noise. The output time series is described by

$$y(n) = \sum_{i=0}^q h(n)e(n-i) \quad (1)$$

Where  $\{e(n)\}$  is the input sequence,  $\{h(n)\}$  the impulse response coefficients, is the order of FIR system and  $\{y(n)\}$  is the non-measurable sequences. the observed "measurable" output  $\{r(n)\}$  is given by

$$r(n) = y(n) + w(n) \quad (2)$$

where  $\{w(n)\}$  is the noise sequence.

In order to simplify the construction of the algorithm we assume that: The input sequence,  $\{e(n)\}$ , is independent and identically distributed (i.i.d) zero mean, and non-Gaussian. The system is causal, i.e.  $h(n) = 0$  for  $i \leq 0$  and  $i \geq q$ , where  $h(0) = 1$  and  $h(q) \neq 0$ . The system order  $q$  is known or may be computed by a priori knowledge [5], [6]. The measurement noise sequence  $\{w(n)\}$  is assumed zero mean, i.i.d, Gaussian and independent of  $\{e(n)\}$  with unknown variance.

### B. Basic Relationships

In this section, we present the general relationships for impulse response coefficients that constitute the basic relations in development of most linear HOC based methods proposed in the literature.

The  $m^{th}$  order cumulants of the  $\{y(n)\}$  can be expressed as a function of the impulse response coefficients  $\{h(i)\}$  as follows [20]:

$$C_{my}(t_1, \dots, t_{m-1}) = \gamma_{me} \sum_{i=0}^q h(i)h(i+t_1)\dots h(i+t_{m-1}) \quad (3)$$

Where  $\gamma_{me}$  is the cumulant at origin of the input sequences. From this relation (3), the methods [8], [1], [9], [11], [10], [13], [14], [21] are derived. The third and fourth order cumulants of  $\{y(n)\}$ , are expressed respectively by

$$C_{3y}(t_1, t_2) = \gamma_{3e} \sum_{i=0}^q h(i)h(i+t_1)h(i+t_2) \quad (4)$$

$$C_{4y}(t_1, t_2, t_3) = \gamma_{4e} \sum_{i=0}^q h(i)h(i+t_1)h(i+t_2)h(i+t_3) \quad (5)$$

Our interest is to search the relationships between the third and fourth order cumulants. Therefore, the Fourier transform of the third order cumulant (4) is given by

$$S_{3y}(\omega_1, \omega_2) = \gamma_{3e} H(\omega_1)H(\omega_2)H(-\omega_1 - \omega_2) \quad (6)$$

The same, The Fourier transform of fourth order cumulant (5) is given as well by

$$S_{4y}(\omega_1, \omega_2, \omega_3) = \gamma_{4e} H(\omega_1)H(\omega_2)H(\omega_3) \times H(-\omega_1 - \omega_2 - \omega_3) \quad (7)$$

with  $H(\omega) = \sum_{i=0}^{\infty} h(i) \exp^{-ji\omega}$  ( $j^2 = -1$ )

If we replace  $\omega_2$  in equation (6) by  $\omega_2 + \omega_3$ , the equation (6) becomes

$$S_{3y}(\omega_1, \omega_2 + \omega_3) = \gamma_{3e} H(\omega_1)H(\omega_2 + \omega_3) \times H(-\omega_1 - \omega_2 - \omega_3) \quad (8)$$

from the equation (7) and (8) we obtain the following equation

$$H(\omega_2)H(\omega_3)S_{3y}(\omega_1, \omega_2 + \omega_3) = \epsilon H(\omega_2 + \omega_3) \times S_{4y}(\omega_1, \omega_2, \omega_3) \quad (9)$$

where  $\epsilon = \left(\frac{\gamma_{3x}}{\gamma_{4x}}\right)$ . So, if we take the inverse Fourier transform of the equation (9), we obtain the fundamental relationships, which link the third and fourth order cumulants to the system parameters (proof see appendix A)

$$\sum_{i=0}^q h(i)h(i+t_3-t_2)C_{3y}(t_1, t_2-i) = \sum_{i=0}^q \epsilon h(i)C_{4y}(t_1, t_2-i, t_3-i) \quad (10)$$

for different values of  $t_1, t_2$  and  $t_3$  we can obtain different algorithms.

### C. Proposed algorithms

In order to identify the parameter  $\{h(i)\} : i = 1, \dots, q$ , of the communication channel impulse response, we use a Least Squares (LS) solution of the system of equation (10) as follow:

If we take  $t_1 = \lambda$  and  $t = t_2 = t_3$  in equation (10), for each  $\lambda$  we can estimate the parameters  $h(i)$  by a Least Squares (LS) solution of the following systems

$$\sum_{i=0}^q \epsilon h(i)C_{4y}(\lambda, t-i, t-i) = \sum_{i=0}^q h^2(i)C_{3y}(\lambda, t-i) \quad (11)$$

the equation (11) can be written into the following system

$$\left( \sum_{i=0}^q \epsilon h(i)C_{4y}(\lambda, t-i, t-i) - \sum_{i=1}^q h^2(i)C_{3y}(\lambda, t-i) \right) = C_{3y}(\lambda, t) \quad (12)$$

else if we suppose that the system is causal, i.e. that  $h(i) = 0$  if  $i < 0$ . So, for  $t = -q, \dots, 0, \dots, q$  the system of equations (12) can be written in matrix form as follow

$$\begin{pmatrix} C_{4y}(\lambda, -q, -q) & \cdots & -C_{3y}(\lambda, -2q) \\ \vdots & & \vdots \\ C_{4y}(\lambda, 0, 0) & \cdots & -C_{3y}(\lambda, -2q) \\ \vdots & & \vdots \\ C_{4y}(\lambda, q, q) & \cdots & -C_{3y}(\lambda, 0) \end{pmatrix} \begin{pmatrix} \epsilon \\ \epsilon h(1) \\ \vdots \\ \epsilon h(q) \\ h^2(1) \\ \vdots \\ h^2(i) \\ \vdots \\ h^2(q) \end{pmatrix} = \begin{pmatrix} C_{3y}(\lambda, -q) \\ \vdots \\ C_{3y}(\lambda, 0) \\ \vdots \\ C_{3y}(\lambda, q) \end{pmatrix} \quad (13)$$

the above equation (13) can be written in compact form as

$$Mh_\lambda = d \quad (14)$$

with  $M$  the matrix of size  $(2q + 1) \times (2q)$  element,  $h_\lambda$  a column vector is constituted by the values  $\epsilon, \epsilon h(1), \dots, \epsilon h(q), h^2(1), \dots, h^2(q)$  and  $d$  is a column vector of size  $(2q + 1) \times (1)$  that is composed by third order cumulants  $C_{3y}(\lambda, -q), \dots, C_{3y}(\lambda, 0), \dots, C_{3y}(\lambda, q)$  as indicated in the equation (18). The Least Squares solution ( $LS$ ) of the system of equation (18), permit an identification of the parameters  $h_\lambda(n)$  blindly and without any 'information' of the input channel. The solution will be written under the following form

$$h_p = (M^T M)^{-1} M^T d \quad (15)$$

1) *First algorithm ALG1*: if  $\lambda = -q$  and for  $t = -q, \dots, q$  into the equation (11), we obtain the algorithm (ALG1) proposed in [21].

The Least Squares solution of the system provides an estimation of the parameters  $\epsilon, h(i)$  and  $h^2(i)$ . In order to exploit this redundancy we propose the following solution for estimating the parameters of the communication channel impulse response  $\{h(i) : i = 1, \dots, q$

$$\hat{h}_{\lambda=-q}(i) = \frac{1}{2} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} + \text{sign} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} \right) (\hat{h}^2(i))^{1/2} \right) \quad (16)$$

where  $\text{sign}(x) = 1$  if  $x > 0$ ,  $-1$  if  $x < 0$  and  $0$  if  $x = 0$ .

2) *Second algorithm ALG2*: now if  $\lambda = -q + 1$  and for  $t = -q, \dots, q$  into the equation (13), we obtain the second  $LS$  algorithm ( $ALG2$ ), the parameters of the communication channel impulse response are given by

$$\hat{h}_{\lambda=-q+1}(i) = \frac{1}{2} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} + \text{sign} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} \right) (\hat{h}^2(i))^{1/2} \right) \quad (17)$$

3) *Third algorithm ALG3*: if we take  $\lambda=0$  and for  $t = -q, \dots, q$  into the equation (13), we obtain the thirth  $LS$  algorithm ( $ALG3$ ) and the parameters of the communication channel impulse response are given by

$$\hat{h}_{\lambda=0}(i) = \frac{1}{2} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} + \text{sign} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} \right) (\hat{h}^2(i))^{1/2} \right) \quad (18)$$

4) *Fourth algorithm ALG4*: if we take  $\lambda = q - 1$  and for  $t = -q, \dots, q$  into the equation (13), we obtain the fourth  $LS$  algorithm ( $ALG4$ ) and the in the same way the parameters of the communication channel impulse response are given by

$$\hat{h}_{\lambda=q-1}(i) = \frac{1}{2} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} + \text{sign} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} \right) (\hat{h}^2(i))^{1/2} \right) \quad (19)$$

5) *Fifth algorithm ALG5*: if we take  $\lambda = q$  and for  $t = -q, \dots, q$  into the equation (13), we obtain the fifth  $LS$  algorithm ( $ALG5$ ) and in the same manner the parameters of the communication channel impulse response are given by

$$\hat{h}_{\lambda=q}(i) = \frac{1}{2} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} + \text{sign} \left( \frac{\hat{\epsilon} h(i)}{\hat{\epsilon}} \right) (\hat{h}^2(i))^{1/2} \right) \quad (20)$$

6) *Mean algorithm ALGm*: so, in general, if the system order is  $q$  we can construct  $2q + 1$  algorithms for each  $\lambda \in [-q, q]$ , from this algorithms we can obtain the means of the estimated parameters as follow

$$\hat{h}_{mean}(i) = \frac{1}{2q + 1} \sum_{\lambda=-q}^q h_\lambda(i) \quad (21)$$

### III. SIMULATIONS AND DISCUSSIONS

In order to evaluate the performance of the proposed algorithms, we consider a non-minimum phase channel in which the order is known. The channel output was corrupted by an Additive White Gaussian Noise (AWGN) for different sample sizes and for 100 Monte Carlo runs.

#### A. First channel

We consider the channel described by the model FIR-NMP (2) with the zeros located at 0.75 and 1.333, given by the following equations

$$y(n) = e(n) - 2.083e(n-1) + 1.0e(n-2), \text{ in noise free case.}$$

$$S(n) = y(n) + w(n), \text{ in presence of Gaussian noise.}$$

Where the signal to-noise-ratio ( $SNR$ ) is defined [14] by

$$SNR = 10 \log \left( \frac{E(y^2(n))}{E(w^2(n))} \right) \quad (22)$$

To measure the accuracy of parameter estimation with respect to the real values, we define the mean square error ( $MSE$ ) for each run as

$$MSE = \sum_{i=0}^q \left( \frac{h(i) - \hat{h}(i)}{h(i)} \right)^2 \quad (23)$$

where,  $h(i)$ ,  $\hat{h}(i)$  for  $i = 0, \dots, q$ , represent the true and estimated parameters respectively. In the following figure (Fig. 2) we have represented the magnitude, phase and zeros of the impulse response of the first channel. The magnitude response is more flat and the phase response is linear.

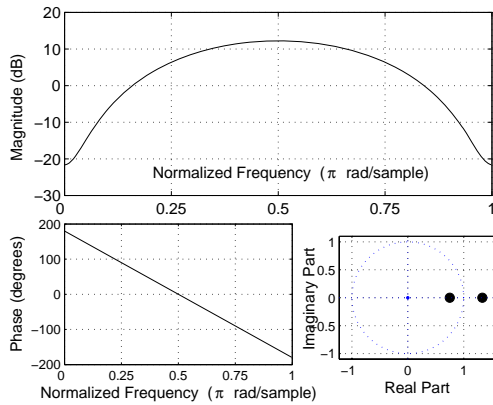


Fig. 2. First channel impulse response characteristics.

TABLE I

TRUE AND ESTIMATED PARAMETERS OF THE FIRST CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 1024 SAMPLES (100 ITERATIONS)

Algorithms	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	MSE
ALG1	$-1.729 \pm 1.158$	$1.221 \pm 0.811$	0.077
ALG2	$-1.736 \pm 0.748$	$1.249 \pm 0.791$	0.089
ALG3	$-1.511 \pm 0.791$	$1.036 \pm 0.724$	0.076
ALG4	$-1.669 \pm 0.989$	$0.984 \pm 0.797$	0.039
ALG5	$-1.344 \pm 0.698$	$0.980 \pm 0.534$	0.126
ALGm	$-1.764 \pm 0.721$	$1.257 \pm 0.763$	0.094

1) performance in noise free case: In the following Table (I) we have summarized the simulation results when the length data input is 1024.

From the Table (TABLE I) we can conclude that:

- All proposed algorithms gives approximately the same results on the estimation of the impulse response. These estimations are closed to the true ones.
- The variances of the estimated parameters are acceptable.
- The mean squares error is small for all algorithms, which is demonstrates that the estimated channel impulse response parameters are more closed to the true ones.

A above results -TABLE I- are obtained in noise free case, but in the following Tables TABLE II and III we estimate the non-minimum phase channel impulse response parameters described by the system 1 for a very low SNR (8dB and 0dB) and for the input data length  $N = 1024$ . In the following figure (Fig. 3) we represent the estimated magnitude and phase of the first channel impulse response in noise free case and for 1024 input data length. In the figure (Fig. 3) we remark that the estimated magnitude response, using all proposed algorithms, are not very different from the true ones. Concerning the estimation of the phase response, we observe a constant gap between the estimated phases and the true ones. This observation concerns principally the ALG1, ALG2, ALG3 and ALGm algorithms. But the estimated phase response using the ALG4 and ALG5 have approximately the same phase in comparison with the true ones.

In the figure (Fig. 4) we have plotted the estimation of the magnitude and phase of first, the case of the SNR = 8dB and for data length of  $N = 1024$ . This is a test of the influence

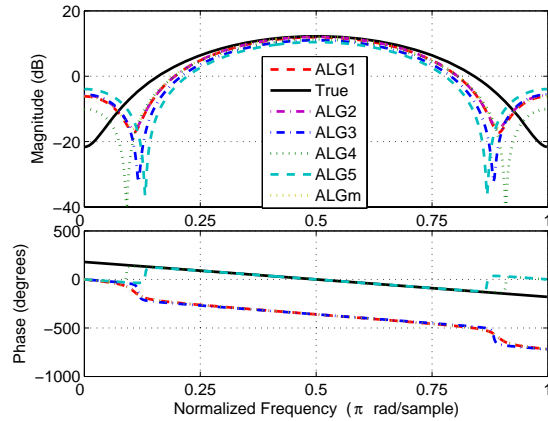


Fig. 3. Estimated magnitude and phase of the channel 1 impulse response in noise free case, using the proposed algorithms, when the data input is  $N = 1024$ .

TABLE II

TRUE AND ESTIMATED PARAMETERS OF FIRST CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 1024 SAMPLES (100 ITERATIONS, SNR = 8dB)

Algorithms	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	MSE
ALG1	$-1.683 \pm 0.864$	$1.336 \pm 0.748$	0.149
ALG2	$-1.895 \pm 0.878$	$1.399 \pm 0.922$	0.167
ALG3	$-1.525 \pm 1.047$	$1.184 \pm 0.828$	0.105
ALG4	$-1.368 \pm 0.659$	$0.818 \pm 0.647$	0.150
ALG5	$-1.203 \pm 0.942$	$1.002 \pm 0.732$	0.178
ALGm	$-1.535 \pm 0.842$	$1.147 \pm 0.794$	0.145

of the noise on the estimation.

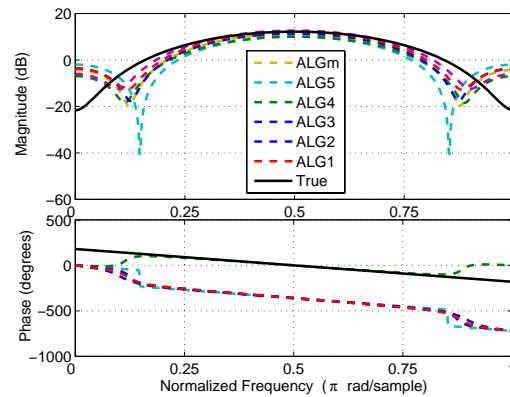


Fig. 4. Estimated magnitude and phase of the channel 1 impulse response, using the proposed algorithms, when the data input is  $N = 1024$  and SNR = 8dB.

In the case of an SNR = 8dB and for a data length  $N = 1024$ , we remark from the figure (Fig.4) that the presence of noise has “nearly” no influence on the estimation of the magnitude and phase response using all the proposed algorithms.

From the Tables (TABLE II and III), we notice that the estimated parameters are slowly affected by the noise presence, this is more apparent if we observe the variance

TABLE III

TRUE AND ESTIMATED PARAMETERS OF FIRST CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 1024 SAMPLES (100 ITERATIONS,  $SNR = 0dB$ )

Algorithms	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	MSE
ALG1	$-1.558 \pm 1.033$	$1.611 \pm 1.174$	0.436
ALG2	$-1.922 \pm 1.171$	$1.403 \pm 0.839$	0.168
ALG3	$-1.586 \pm 0.427$	$1.386 \pm 0.827$	0.205
ALG4	$-1.903 \pm 0.656$	$1.457 \pm 0.578$	0.216
ALG5	$-2.036 \pm 1.027$	$1.577 \pm 0.827$	0.333
ALGm	$-1.801 \pm 0.756$	$1.486 \pm 0.865$	0.271

of the estimated parameters. The MSE is not very different from the noiseless case (TABLE I).

Now what will happen if the variance of noise is equal to the output channel variance (i.e.  $SNR = 0dB$ ). The following figure (Fig. 5) will explain this situation. If the

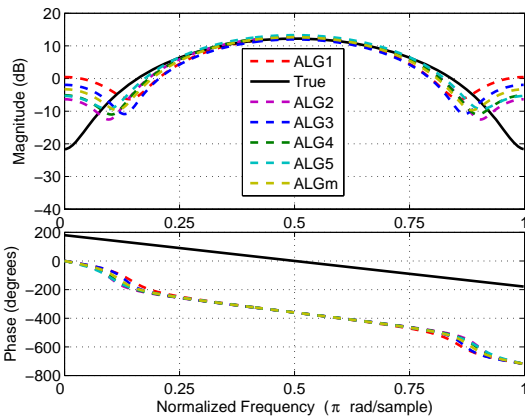


Fig. 5. Estimated magnitude and phase of the first channel impulse response, using the proposed algorithms, when the data input is  $N = 1024$  and  $SNR = 0dB$ .

noise variance is equal to the output channel variance (i.e.  $SNR = 0dB$ ) and for the same data length, we conclude that all algorithms give the “same” estimations of the phase which have a constant difference comparing to the true ones. The estimated magnitude response using the proposed algorithms are not different from the original ones excepted those parts of magnitude  $\leq 0dB$ .

As a result, we can say that:

- The proposed algorithms yield approximately the same estimation of system parameters.
- We observe some increases of the variance estimations, this is due to the presence of noise and short data input (due to a bias estimation of the cumulants).
- If  $\lambda$  approaches zero, i.e  $\lambda = -1, 0$  and  $1$ , the algorithms (ALG2, ALG3 and ALG4) give good estimation for all  $SNR$  and for different input data length (low MSE (Fig. 6), because in this case we use more cumulants information if the lags are centered round to zero.

In Tables IV and V we increase the sample data to 2048 and for two  $SNR$  (0 and 16 dB). From the results shown in the TABLE IV and V, we can deduce that the estimated

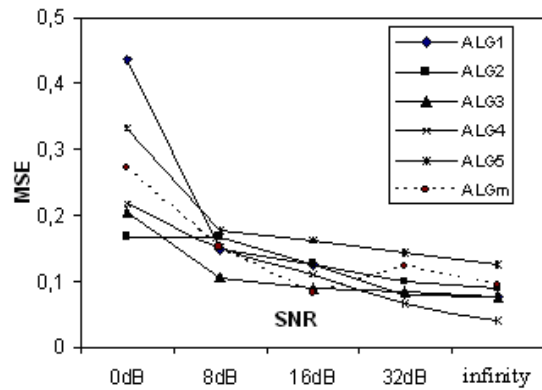


Fig. 6. MSE (first channel) for each algorithm and for different  $SNR$  and for a data length  $N = 1024$ .

parameters are not very different from those shown in TABLE II. This implies that the data length does not influence the estimation of third fourth order cumulants. The MSE are approximately of the same values.

TABLE IV

TRUE AND ESTIMATED PARAMETERS OF FIRST CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 2048 SAMPLES (100 ITERATIONS,  $SNR = 0dB$ )

Algorithms	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	MSE
ALG1	$-1.329 \pm 0.778$	$1.208 \pm 0.824$	0.174
ALG2	$-1.725 \pm 1.109$	$1.342 \pm 0.852$	0.146
ALG3	$-1.330 \pm 0.676$	$1.142 \pm 0.898$	0.150
ALG4	$-1.355 \pm 1.152$	$0.733 \pm 0.687$	0.193
ALG5	$-1.279 \pm 1.132$	$1.039 \pm 1.051$	0.150
ALGm	$-1.404 \pm 0.962$	$1.093 \pm 0.932$	0.162

TABLE V

TRUE AND ESTIMATED PARAMETERS OF FIRST CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 2048 SAMPLES (100 ITERATIONS,  $SNR = 16dB$ )

Algorithms	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	MSE
ALG1	$-1.619 \pm 0.719$	$1.296 \pm 0.807$	0.137
ALG2	$-2.093 \pm 1.214$	$1.362 \pm 1.013$	0.131
ALG3	$-1.474 \pm 0.851$	$1.066 \pm 0.801$	0.089
ALG4	$-1.461 \pm 0.692$	$0.783 \pm 0.569$	0.136
ALG5	$-1.384 \pm 0.556$	$0.857 \pm 0.499$	0.133
ALGm	$-1.606 \pm 0.726$	$1.073 \pm 0.659$	0.125

We remark from the above results that the noise have a small influence on the magnitude and phase estimation, but now we will know the influence of the data length on the estimation of the impulse response estimation. In the Fig.5 we have represent the results for an  $SNR = 16dB$  and the data length is  $N = 2048$ .

In Fig.7 we have increased the data length to have  $N = 2048$  and the  $SNR = 16dB$ , we remark that the estimated magnitude of the first channel impulse response are not very different from the true ones excepted those with a magnitude  $\leq 0dB$ . The estimated phase response using the algorithms ALG1, ALG2 and ALG3 have approximately the same allure

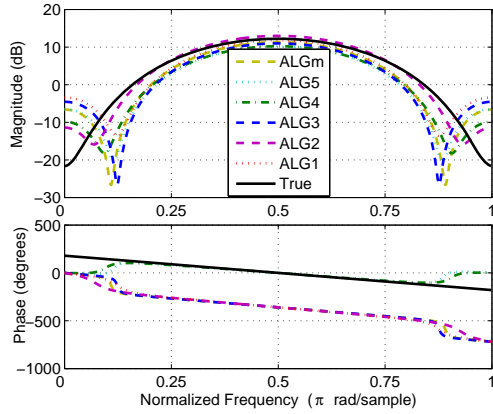


Fig. 7. Estimated magnitude and phase of the channel 1 impulse response, using the proposed algorithms, when the data input is  $N = 2048$  and  $SNR = 16dB$ .

comparing the true ones; but the algorithms  $ALG4$ ,  $ALG5$  and  $ALGm$  give a constant gap comparing to the true ones.

**B. Second channel**

In this section, we increase the channel order (in order to know the influence of the increasing system order on the parameters estimation). Let us consider the channel impulse response described by the system FIR-NMP(3), with the zeros that are located at  $-0.955$ ,  $0.812$  and  $1.226$  given by the equation:

$$y(n) = e(n) - 1.083e(n - 1) - 0.95e(n - 2) + 0.95e(n - 3),$$

in noise free case.

$$r(n) = y(n) + w(n),$$

in presence of an AWGN.

In the following figure (Fig. 8) we represent the magnitude, phase and zeros of the impulse response of second channel. The magnitude response represents more selectivity in frequency and the phase response is not linear.

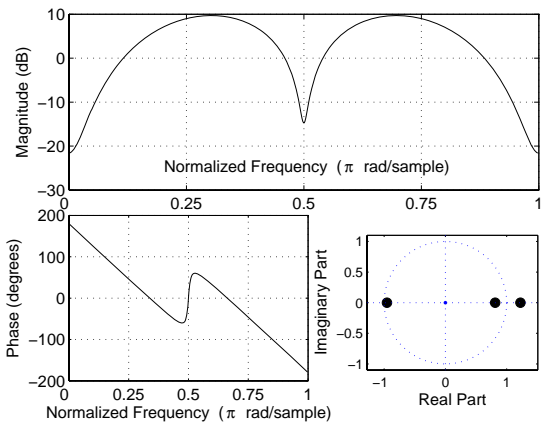


Fig. 8. Second channel impulse response characteristics

In the following Table (TABLE VI) we represent the simulation results where the data input is  $N = 1024$  and for an  $SNR = 16dB$ . From the Table VI we can conclude that:

TABLE VI  
TRUE AND ESTIMATED PARAMETERS OF SECOND CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 1024 SAMPLES (100 ITERATIONS,  $SNR = 16dB$ )

Algorithms	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	$\hat{h}(3) \pm \sigma$	MSE
ALG1	$-0.956 \pm 0.782$	$-1.027 \pm 0.396$	$1.073 \pm 0.862$	0.037
ALG2	$-1.207 \pm 0.598$	$-1.081 \pm 0.663$	$1.276 \pm 0.720$	0.149
ALG3	$-0.996 \pm 0.829$	$-1.083 \pm 0.491$	$1.243 \pm 0.639$	0.121
ALG4	$-1.351 \pm 0.986$	$-0.940 \pm 0.525$	$1.415 \pm 1.313$	0.301
ALG5	$-1.147 \pm 0.526$	$-1.330 \pm 0.802$	$1.383 \pm 0.825$	0.371
ALGm	$-1.131 \pm 0.697$	$-1.092 \pm 0.586$	$1.278 \pm 0.776$	0.144

- The estimated parameters are good for all algorithms.
- The ALG1 gives a better estimation compared to the other algorithms, if we observe the values of variance  $\sigma$  and  $MSE$ .
- The ALG4 and ALG5 are not very efficient compared to other algorithms, because they have the greatest  $MSE$  compared to the rest of algorithms.

In the following figure (Fig. 9) we represent the  $MSE$  for the proposed algorithms, when the length of the data input is  $N = 1024$  and the  $SNR = 16dB$ . The results in TABLE

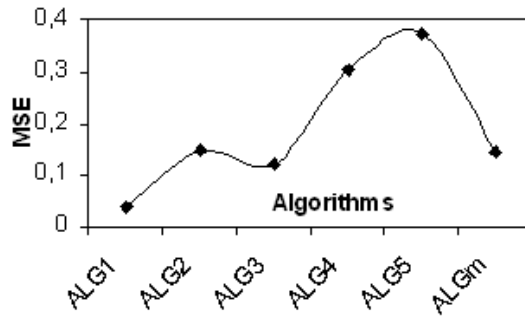


Fig. 9.  $MSE$  for each algorithm and for different  $SNR$  and for a data length  $N = 1024$ .

VI shows that the noise presence has a small influence on the parameters estimation (overestimation of the parameter  $h(3)$ ), but not a great influence on the parameters estimation using all algorithms. This can be observed in the values of the  $MSE$ . From Fig. 9 we can observe the robustness of the ALG1 compared to the other algorithms. The imprecision of the algorithms  $ALG4$  and  $ALG5$  is minimized by using the mean algorithm  $ALGm$ .

In the figure (Fig. 10) we represent the estimated magnitude and phase response of the second channel using the proposed algorithms, when the  $SNR = 16$  and the data length  $N = 1024$ . The estimations of magnitude impulse response of the second channel, using all proposed algorithms, are not more close to the real ones when the data length  $N = 1024$  and the  $SNR = 16dB$  (Fig. 10). but the phase estimations are different from the true ones by "constant" value.

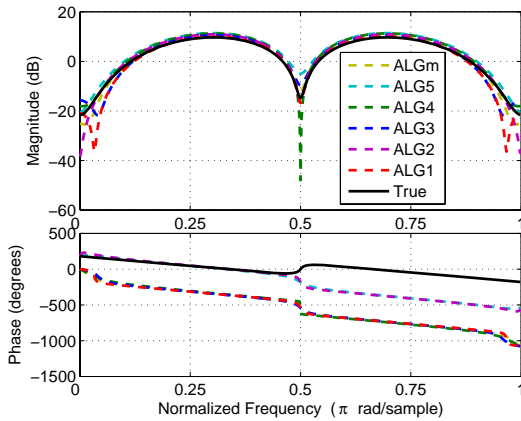


Fig. 10. Estimated magnitude and phase of the second channel impulse response, using the proposed algorithms, when the data input is  $N = 1024$  and  $SNR = 16dB$ .

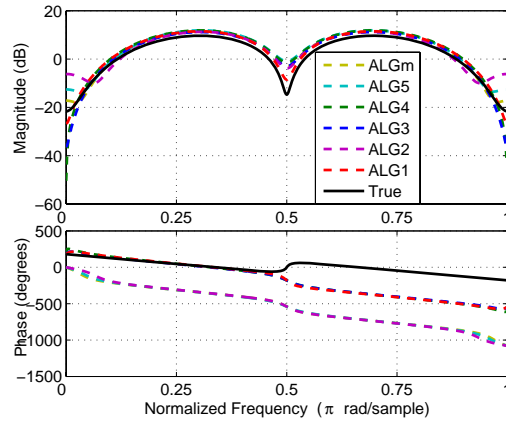


Fig. 11. Estimated magnitude and phase of the second channel impulse response, using the proposed algorithms, when the data input is  $N = 512$  and  $SNR = 0dB$ .

In the following Table (Table VII) we represent the parameters estimation in the case of a small data input ( $N = 512$ ) and for a very noisy environment ( $SNR = 0dB$ ), in order to test the robustness of the proposed algorithms to noise and small data input.

TABLE VII

TRUE AND ESTIMATED PARAMETERS OF SECOND CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 512 SAMPLES (100 ITERATIONS,  $SNR = 0dB$ )

Algorithms	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	$\hat{h}(3) \pm \sigma$	MSE
ALG1	$-1.260 \pm 0.730$	$-1.211 \pm 0.675$	$1.425 \pm 0.796$	0.352
ALG2	$-1.001 \pm 0.502$	$-1.076 \pm 0.652$	$1.568 \pm 0.755$	0.447
ALG3	$-0.968 \pm 0.428$	$-1.363 \pm 0.763$	$1.344 \pm 0.689$	0.372
ALG4	$-1.130 \pm 0.560$	$-1.425 \pm 0.943$	$1.552 \pm 0.871$	0.654
ALG5	$-1.081 \pm 0.433$	$-1.335 \pm 0.443$	$1.652 \pm 0.775$	0.711
ALGm	$-1.088 \pm 0.530$	$-1.282 \pm 0.695$	$1.508 \pm 0.777$	0.468

The results shown in the Table VII demonstrate that:

- There is a little influence of the noise, because the proposed algorithms are based on cumulants of order great than 2 ( $3^{rd}$  and  $4^{th}$  order cumulants)
- There is some influence of short data input ( $N = 512$ ), this implies a bias of the cumulants estimation.
- In conclusion, if we use high data length we will minimize the influence of the bias caused when we estimate the cumulants. In addition we can minimize this influence by dividing the input data ( $N$ ) into  $M$  interval in which we estimate the cumulants and then we take its means.

In the following figure (Fig. 11) we have considered the bad situation, i.e small data length ( $N = 512$ ) and for a high noise variance ( $SNR = 0dB$ ), of the impulse response estimation using the proposed algorithms. When the output channel is more affected by the noise, i.e.  $SNR = 0$ , we estimate the magnitude impulse response (using all algorithms) with more

precision so that the data length is only  $N = 512$  (Fig.11). But, the estimation phases impulse response have the same constant gap comparing to the true ones.

### C. Third channel

In this example, we consider a non-minimum phase impulse response channel, given by the following equation:

$$y(n) = e(n) + 0.327e(n-1) - 0.815e(n-2) + 0.470e(n-3),$$

in noise free case.

$$r(n) = y(n) + w(n),$$

in presence of an AWGN.

The channel characteristics are illustrated in the following figure (Fig. 14). The magnitude of the channel impulse response is more selective in frequency and the phase response are not linear. We have selected the third algorithm ( $ALG3$ ), i.e. for

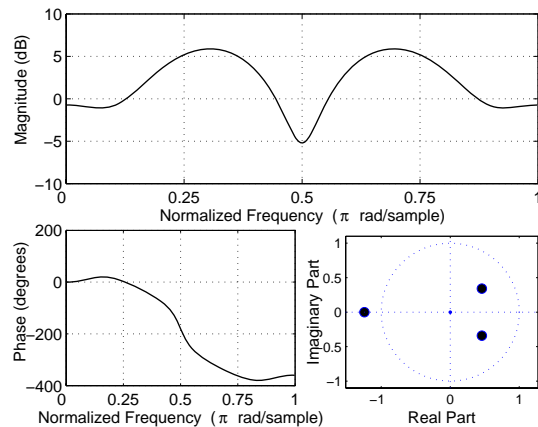


Fig. 12. Third channel impulse response characteristics.

$\lambda = 0$  in which we use the maximum information of the estimated signal using higher order cumulants. In the TABLE VIII we represent the estimated impulse response parameters. From the TABLE VIII we conclude that the algorithm ( $ALG3$ ) gives a good impulse response parameters estimation, for



TABLE VIII

TRUE AND ESTIMATED PARAMETERS, USING ALG3, OF THIRD CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 1024 SAMPLES (100 ITERATIONS AND FOR DIFFERENT DATA  $SNR$ )

$SNR(dB)$	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	$\hat{h}(3) \pm \sigma$	$MSE$
$\infty$	0.375 $\pm$ 0.659	-0.923 $\pm$ 0.561	0.488 $\pm$ 0.371	0.041
40	0.232 $\pm$ 0.458	-0.775 $\pm$ 0.336	0.525 $\pm$ 0.299	0.101
32	0.253 $\pm$ 0.499	-0.861 $\pm$ 0.332	0.584 $\pm$ 0.206	0.113
24	0.209 $\pm$ 0.482	-0.815 $\pm$ 0.357	0.566 $\pm$ 0.238	0.170
16	0.167 $\pm$ 0.574	-0.7341 $\pm$ 0.520	0.478 $\pm$ 0.368	0.249
8	0.136 $\pm$ 0.991	-0.828 $\pm$ 0.916	0.442 $\pm$ 0.555	0.344

different  $SNR$  and for a data input  $N = 1024$ . The influence of noise is not very important, this influence is due, principally, to the use of cumulants biased estimator.

In the following figure (Fig. 13), we represent the estimation of the magnitude and phase of the channel impulse response for different  $SNR$  and for a data input  $N = 1024$ .

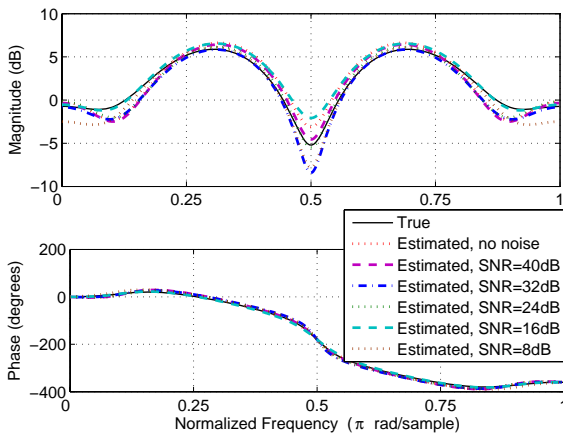


Fig. 13. Estimated magnitude and phase of the second channel impulse response, using the ALG3 algorithm, when the data input is  $N = 1024$  and for different  $SNR$ .

In order to minimize the influence of the bias cumulants estimator, we consider in the following Table (TABLE IX) the data input  $N = 2048$ . The simulation results are illustrated for different  $SNR$ .

The TABLE IX demonstrate a light improvement of channel impulse response parameters estimation, especially on the variance estimation -no more fluctuation comparing to the TABLE VIII- and a small  $MSE$ .

In Fig.14 so below, we represent the estimation of the magnitude and phase of the third channel in the case of input data length  $N = 2048$ .

The Fig. 14 proof that the ALG3 gives a very good estimation for phase response, the estimated phase are closed to the true ones, and an important estimation on the magnitude estimation. To conclude, the proposed algorithms are able to estimate the phase and magnitude of the non minimum phase channel impulse response in very noisy environments, this is

TABLE IX

TRUE AND ESTIMATED PARAMETERS, USING ALG3, OF THE THIRD CHANNEL EXCITED BY A PAM(16) INPUT SEQUENCE OF 2048 SAMPLES (100 ITERATIONS AND FOR DIFFERENT  $SNR$ )

$SNR(dB)$	$\hat{h}(1) \pm \sigma$	$\hat{h}(2) \pm \sigma$	$\hat{h}(3) \pm \sigma$	$MSE$
$\infty$	0.227 $\pm$ 0.341	-0.762 $\pm$ 0.195	0.550 $\pm$ 0.134	0.066
40	0.245 $\pm$ 0.312	-0.770 $\pm$ 0.109	0.560 $\pm$ 0.074	0.102
32	0.220 $\pm$ 0.364	-0.760 $\pm$ 0.132	0.552 $\pm$ 0.163	0.142
24	0.192 $\pm$ 0.394	-0.761 $\pm$ 0.253	0.550 $\pm$ 0.146	0.203
16	0.182 $\pm$ 0.373	-0.741 $\pm$ 0.237	0.541 $\pm$ 0.191	0.227
8	0.171 $\pm$ 0.373	-0.691 $\pm$ 0.258	0.529 $\pm$ 0.197	0.266

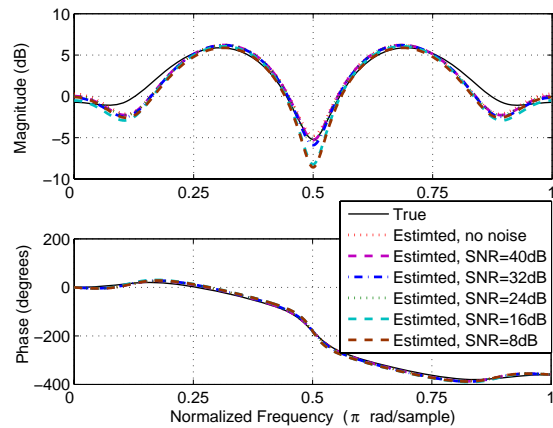


Fig. 14. Estimated magnitude and phase of the second channel impulse response, using the ALG3 algorithm, when the data input is  $N = 2048$  and for different  $SNR$ .

due to the cumulants properties such as: the cumulants of the Gaussian noise are zeros.

#### IV. CONCLUSION

In this paper, we have proposed a family of algorithms based on third and fourth order cumulants. These algorithms are used for the estimation of parameters of non-minimum phase channel. The results show the performance of all the proposed algorithms mainly for (ALG3). The proposed algorithms can be used for all channel (minimum phase or non-minimum phase). Therefore, these algorithms will be used for modeling the internet traffic. It is important to know that we can use all family of random PAM input (i.e.: PAM (2), PAM (4), PAM (8)). The current work is aimed to show the performance of the proposed algorithms to a practical channel such as BRAN A and BRAN E used in mobile communication.

#### APPENDIX A

##### PROOF OF THE EQUATION (10)

From the equation (9)

$$\begin{aligned} & \epsilon H(\omega_2 + \omega_3) S_{4y}(\omega_1, \omega_2, \omega_3) \\ &= H(\omega_2) H(\omega_3) S_{3y}(\omega_1, \omega_2 + \omega_3) \end{aligned} \quad (24)$$



If we take the inverse Fourier Transform of the above equation (24) we obtain the following equation form

$$\sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} C_{3y}(n, l) e^{-jn\omega_1} e^{-jl(\omega_2+\omega_3)} \times \left( h(m) e^{-(jm\omega_2)} h(k) e^{-(jk\omega_3)} \right) = \epsilon \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \left( \sum_{k=-\infty}^{+\infty} h(k) C_{4y}(n, m, l) e^{-(jn\omega_1)} e^{-(jm\omega_2)} \right) \times \left( e^{-(jl\omega_3)} e^{-(jk(\omega_2+\omega_3))} \right) \quad (25)$$

$$\sum_{m=0}^q h(m) h(t_2 - t_1 + m) C_{3y}(n, t_1 - m) = \epsilon \sum_{k=0}^q h(k) C_{4y}(n, t_1 - k, t_2 - k) \quad \forall (t_1, t_2, n) \in [-q, q] \quad (29)$$

so, we have obtained the equation (10)

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Or in another form as follow

$$\sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} h(m) h(k) C_{3y}(n, l) e^{-(jn\omega_1)} \times \left( e^{-(j(l+m)\omega_2)} e^{-(j(k+l)\omega_3)} \right) = \epsilon \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \left( \sum_{k=-\infty}^{+\infty} h(k) C_{4y}(n, m, l) e^{-(jn\omega_1)} e^{-(j(m+k)\omega_2)} \right) \times \left( e^{-(j(l+k)\omega_3)} \right) \quad (26)$$

if we take if we take (in the left hand) of the above equation that

$$l + m = t_1 \text{ and } k + l = t_2$$

and in right hand that

$$k + m = t_1 \text{ and } k + l = t_2$$

we obtain the following equation

$$\sum_{t_1, t_2, n=-\infty}^{+\infty} \left( \sum_{m=-\infty}^{+\infty} h(m) h(t_2 - t_1 + m) C_{3y}(n, t_1 - m) \right) \times \left( e^{-(jn\omega_1)} e^{-(jt_1\omega_2)} e^{-(jt_2\omega_3)} \right) = \epsilon \sum_{t_1, t_2, n=-\infty}^{+\infty} \left( \sum_{k=-\infty}^{+\infty} h(k) C_{4y}(n, t_1 - k, t_2 - k) e^{-(jn\omega_1)} \right) \times \left( e^{-(jt_1\omega_2)} e^{-(jt_2\omega_3)} \right) \quad (27)$$

from the equation (27) we obtain the following equality

$$\sum_{m=-\infty}^{+\infty} h(m) h(t_2 - t_1 + m) C_{3y}(n, t_1 - m) = \epsilon \sum_{k=-\infty}^{+\infty} h(k) C_{4y}(n, t_1 - k, t_2 - k) \quad \forall (t_1, t_2, n) \in [-q, q] \quad (28)$$

where  $q$  is the impulse response channel order. If the system is causal (our case of study) the equation (28) takes the following

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