International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:7, No:7, 2013

Axisymmetric Vibrations of Layered Cylindrical Shells with Cracks

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Abstract**—**Vibrations of circular cylindrical shells made of layered composite materials are considered. The shells are weakened by circumferential cracks. The influence of circumferential cracks with constant depth on the vibration of the shell is prescribed with the aid of a matrix of local flexibility coupled with the coefficient of the stress intensity known in the linear elastic fracture mechanics. Numerical results are presented for the case of the shell with one circular crack.

Keywords**—**Layered shell, axisymmetric vibration, crack.

I. Introduction

UNTIL the fracture takes place the large class of composites behave as elastic bodies, following the Hooke's law. Usually the facture of bodies from such material is brittle, for example, destruction of bodies from fiber-glass. Therefore for investigations of small deformations in bodies from such materials, methods of the classical theory of elasticity of an anisotropic body can be used.

Circular cylindrical shells, made of composite materials, are widely used in many fields of engineering, especially in civil, mechanical, aerospace, marine and chemical industry. Vibration of circular cylindrical shells from composite materials is of interest in a number of different fields.

Since the crack- like defects are practically unavoidable during the manufacturing and operation of structural elements there exists the need for the information about the sensitivity of vibrational parameters of the shell with respect to defects. Vibration and stability of notched beams was investigated by Dimarogonas [1], Chondros and Dimarogonas [2], [3], Rizos et al. [4], Liang et al. [5], Kisa et al. [6], Lellep and Sakkov [7], Krawczuk, Ostachowich [8], [9] making use of the weightless rotational spring model. In [12] Lellep and Roots investigated axisymmetric vibrations of cylindrical shells with circumferential cracks.

According to this concept a beam with a crack can be treated as a structure consisting of two segments. These segments are connected each other with a rotational spring which stiffness is coupled with the stress intensity coefficient of the structure with the crack.

This idea was extended to composite structures and to buckling of composite columns by Nikpour and Dimarogonas [10], [11]. In this paper we will study free axisymmetric vibrations of layered cylindrical shells with cracks.

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II. Formulation of the Problem for Layered Shells

Consider a layered, circular cylindrical shell with length l (see Fig. 1). The shell can be divided into n ring segments. The symbol n denotes the number of total ring segments separated from the rest cylindrical shell by the sections where the thickness variations take place. Every jth ring segment of shell has q layers. Each layer is isotropic with thickness h_{ij}, Young's modulus E_{ij}, Poisson's ratio v_{ij}, and mass density ρ_{ij} as show in Fig. 1.

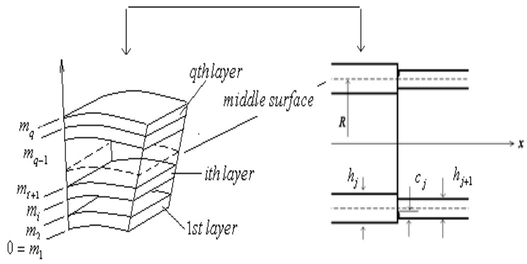

Fig. 1 Geometry of a layered shell

Let's denote

$$\rho_{ij}= \rho_{1j}d_{ij} \tag{1}$$

where d_{ij} is a constant of proportionality, d_{1j}=1and similarly Young's modulus for each layer

$$E_{ij}=E_{1j}e_{ij} \tag{2}$$

where e_{ij} is a constant of proportionality, e_{1j}=1.

We will denote the thickness of each layer h_{ij} by

$$h_{ij}=(z_{i+1j}-z_{ij})h_{1j} \tag{3}$$

where z_{ij} is a local coordinate of a layer with the thickness $h_j$andz_{1j}=0.

The mass of j th ring segment will be equal

$$\rho_{1j}h_{1j}\sum_{i=1}^{q} d_{ij}(z_{i+1j}-z_{ij}) \tag{4}$$

For the jth ring segment, the free axisymmetric vibration motion can be described by the equations [12].

$$\frac{\partial N_{1j}}{\partial x}=0$$
$$\frac{\partial^2 M_j}{\partial x^2}-\frac{N_j}{R}-\rho_j h_j\frac{\partial^2 w}{\partial t^2}=0 \qquad (5)$$

where for calculation of thin shells often use following formula [13]:

$$N_{1j}=\int_0^{h_j}\sigma_{1j}dz,\quad N_j=\int_0^{h_j}\sigma_j dz,\quad M_j=\int_0^{h_j}\sigma_{1j}zdz, \qquad (6)$$

where

$$\sigma_{1j}=\left[E_j/(1-\nu_j^{\,2})\right]\left[(\varepsilon_1+z\chi)+\nu_j\varepsilon\right],$$
$$\sigma_j=\left[E_j/(1-\nu_j^{\,2})\right]\left[\varepsilon+\nu_j(\varepsilon_1+z\chi)\right].$$

If for all ring segments $\nu_{ij}=\nu$,by using (1) - (3) and [13]

$$\varepsilon_1=\frac{\partial u}{\partial x},\ \varepsilon=\frac{w}{R},\ \chi=-\frac{\partial^2 w}{\partial x^2}.$$

Theforce N_j and bending moment M_j (6) can be written as

$$N_j=\int_0^{h_{1j}}\sigma_j dz+\int_{h_{1j}}^{h_{2j}}\sigma_j dz+...+\int_{h_{ij}}^{h_{i+1j}}\sigma_j dz+...$$
$$...+\int_{h_{q-1j}}^{h_{qj}}\sigma_j dz=b_jE_{1j}h_j\frac{w}{R},$$
$$M_j=-\frac{E_{1j}h_j^{\,3}}{12(1-\nu^2)}\bar{a}_j\frac{\partial^2 w}{\partial x^2}, \qquad (7)$$

where

$$\bar{a}_j=4\Big(\sum_{i=1}^{q}e_{ij}(z_{i+1j}^3-z_{ij}^3)\Big)-\frac{-3\Big(\sum_{i=1}^{q}e_{ij}(z_{i+1j}^2-z_{ij}^2)\Big)^2}{\sum_{i=1}^{q}e_{ij}(z_{i+1j}-z_{ij})}, \qquad (8)$$

and

$$b_j=\sum_{i=1}^{q}e_{ij}(z_{i+1j}-z_{ij}). \qquad (9)$$

By using (4) the equation of motion (5) of jth ring segment can be described by the equation

$$\bar{D}_j\bar{a}_j\frac{\partial^4 w}{\partial x^4}+\frac{E_{1j}h_j}{R^2}b_jw=-\rho_{1j}h_jc_j\ddot{w}, \qquad (10)$$

where $\bar{D}_j=E_{1j}h_j^{\,3}/12(1-\nu^2)$, ν=const and

$$c_j=\sum_{i=1}^{q}d_{ij}(z_{i+1j}-z_{ij}). \qquad (11)$$

Evidently, it is reasonable to look for the general solution of (10) in the form

$$w(x,t)=X_j(x)T(t) \qquad (12)$$

It follows from (10) with (12) that

$$X_j^{IY}-r_j^4X_j=0 \qquad (13)$$

where

$$r_j^4=\omega^2\cdot\frac{12\rho_1(1-\nu^2)}{E_1h_j^2}\frac{c_j}{\bar{a}_j}-\frac{12(1-\nu^2)}{R^2h_j^2}\frac{b_j}{\bar{a}_j}, \qquad (14)$$

Frequency of free vibrations of a layered shell will be equal

$$\omega=\sqrt{\frac{E_1}{\rho_1}}\frac{1}{R}\sqrt{\frac{k^4R^2}{12(1-\nu^2)}\frac{\bar{a}_j}{c_j}+\frac{b_j}{c_j}},$$

where $r_j=k/\sqrt{h_j}$.

The general solution of the linear fourth order equation (13) can be presented as

$$X_j(x)=A_j\sin(r_jx)+B_j\cos(r_jx)+$$
$$+C_j\sinh(r_jx)+D_j\cosh(r_jx) \qquad (15)$$

Assume that the ends of the shell are simply supported. We arrive at the boundary conditions at the points x=0 and $x=l$

w=0, M_x=0.

The continuity and jump conditions at $x=a$ are (see [12])

$$w(a+0)-w(a-0)=0,$$
$$w'(a+0)-w'(a-0)=\frac{72\pi}{E'h_1^2}f(s_1)M_x(a-0),$$
$$M_x(a-0)=M_x(a+0),$$
$$M_x'(a-0)=M_x'(a+0).$$

International Journal of Mechanical, Industrial and Aerospace Sciences
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Let us consider now the case when n=2. By using equation (7) we can rewrite the equation for definition of characteristic number k (see [12]) as

$$\begin{aligned}&((1-g_1)\cos kl_1\cosh kl_4+((g_2-g_4)\sin kl_1+\\&+g_3\cos kl_1)\sinh kl_4)((1-g_1)\cosh kl_1\cos kl_4-\\&-((g_2-g_4)\sinh kl_1+g_3\cosh kl_1)\sin kl_4)-\\&-((1+g_1)\cos kl_1\cos kl_4+((g_2+g_4)\sin kl_1+\\&+g_3\cos kl_1)\sin kl_4)((1+g_1)\cosh kl_1\cosh kl_4-\\&-((g_2+g_4)\sinh kl_1+g_3\cosh kl_1)\sinh kl_4)=0,\end{aligned}$$

where

$$\begin{aligned}&g_1=(h_0\bar{a}_0/h_1\bar{a}_1)^2,\\&g_2=(h_1/h_0)^{1/2},\\&g_4=g_1g_2,\\&g_3=6\pi f(s_1)\sqrt{h_1}g_1k\end{aligned}$$

and

$$l_1=a/\sqrt{h_0},\qquad l_4=(l-a)/\sqrt{h_1}.$$

Here $\bar{a}_0$ and $\bar{a}_1$ –are the values of $\bar{a}_j$ for j=0 and j=1, respectively.

III. Numerical Results

For an illustration of the method offered in that article the simply supported shell has been considered (See Fig. 2).

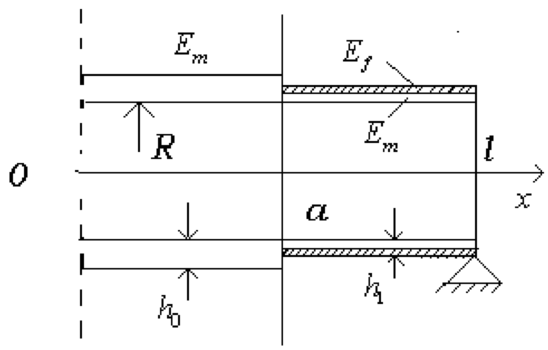

Fig. 2 Cylindrical shell with fiber-glass layer

The shell under consideration has a uniform shell wall with Young's modulus E_m for $x\in(0,a)$ whereas it consists of two layers with Young's moduli E_m and E_s respectively, and thickness h_1 for $x\in(a,l)$, as shown in Fig. 2. Geometrical parameters for the one-stepped shell are: l=0,6; h_0=0,006; $h_1=\gamma h_0$; γ=0,7. It is assumed herein that the material of the shell segment with $h_0$is a homogeneous elastic material - aluminum-lithium alloy with E_m=76GPa. The shell segment with h_1 has two layers. The inner layer is made of the same material as the other segment and the material of the top layer is a fiber-glass. In the segment with h_1, ν is the volume fraction of fibres. We will consider four kinds of fiber-glasses with E_s= 20GPa, 35GPa, 50GPa, 152GPa ($s= E_s/ E_m$), respectively. In this case the equation for definition of characteristic number k is

$$\begin{aligned}&((1-g_1)\cos kl_1\cosh kl_4+((g_2-g_4)\sin kl_1+\\&+g_3\cos kl_1)\sinh kl_4)((1-g_1)\cosh kl_1\cos kl_4-\\&-((g_2-g_4)\sinh kl_1+g_3\cosh kl_1)\sin kl_4)-\\&-((1+g_1)\cos kl_1\cos kl_4+((g_2+g_4)\sin kl_1+\\&+g_3\cos kl_1)\sin kl_4)((1+g_1)\cosh kl_1\cosh kl_4-\\&-((g_2+g_4)\sinh kl_1+g_3\cosh kl_1)\sinh kl_4)=0,\end{aligned}$$

where

$$\begin{aligned}&g_1=(h_0/h_1(f\nu\ +(1-\nu\ ))^2,\\&g_2=(h_1/h_0)^{1/2},\\&g_4=g_1g_2,\\&g_3=6\pi f(s_1)\sqrt{h_1}g_1k\end{aligned}$$

and

$$l_1=a/\sqrt{h_0},\qquad l_4=(l-a)/\sqrt{h_1}.$$

The results of calculations regarding to the shell with simply supported ends are presented in Figs. 3, 4. The influence of the crack c/h on the characteristic number k for the fixed values ν=0,2; β=0,2; γ=0,7 and different values of s is depicted in Fig. 3. In Fig. 4 different curves corresponding to different values of ν are presented in the case s=2; β=0,2; γ=0,7. Here $\beta=a_1/l$,$\gamma=h_1/h_0$, as in previous sections of the study.

Calculations carried out showed that the characteristic number k of the shell decreases when the crack depth increases as might be expected.

Calculations were made by means of the package Mathcad.

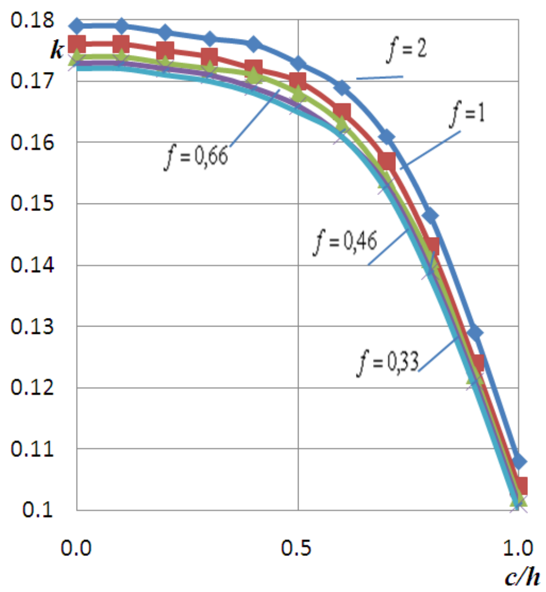

Fig. 3 Frequency parameters *k* for simply supported shells with one-step thickness variation and crack, the case ν=0,2; β=0,2; γ=0,7

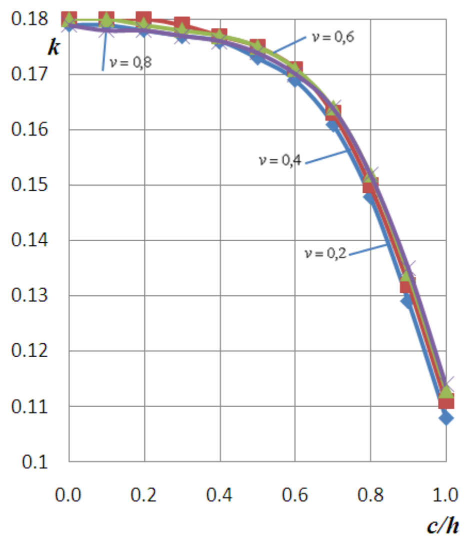

Fig. 4 Frequency parameters *k* for simply supported shells with one-step thickness variation and crack, the case *f*=2; β=0,2; γ=0,7

IV. Concluding Remarks

The natural frequency of vibrations is determined for various non-homogeneous materials.

Calculations carried out showed that the crack location and its dimensions have strong influence on the natural frequency of vibrations. Results of calculations showed that when the crack depth increases then the frequency of natural vibrations decreases. This theoretical results needs in the good computer engineering.

Acknowledgments

The partial support from the target financed project SF 0180081S08 “Models of applied mathematics and mechanics” and from theGrant MJD433 “Multiscale Methods for Fracture” is gratefully acknowledged.

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