Axisymmetric Vibration of Pyrocomposite Hollow Cylinder

V. K. Nelson and S. Karthikeyan

Abstract—Axisymmetric vibration of an infinite Pyrocomposite circular hollow cylinder made of inner and outer pyroelectric layer of 6mm-class bonded together by a Linear Elastic Material with Voids (LEMV) layer is studied. The exact frequency equation is obtained for the traction free surfaces with continuity condition at the interfaces. Numerical results in the form of data and dispersion curves for the first and second mode of the axisymmetric vibration of the cylinder BaTio$_3$ / Adhesive / BaTio$_3$ by taking the Adhesive layer as an existing Carbon Fibre Reinforced Polymer (CFRP) are compared with a hypothetical LEMV layer with and without voids and as well with a pyroelectric hollow cylinder. The damping is analyzed through the imaginary parts of the complex frequencies.

Keywords—Axisymmetric vibration, CFRP, hollow cylinders, LEMV, pyrocomposite

I. INTRODUCTION

CERAMIC materials and single crystals showing pyroelectric behavior are being used in many applications in electronics and optics. A huge leap in the research on smart materials came in the 1950's, leading to the widespread use of barium titanate (BaTiO$_3$) based ceramics in capacitor applications and pyroelectric transducer devices. Measurement of X-ray intensity in the medical diagnostic range by pyroelectric detector is analyzed by Carvalho and Alter [1]. The pyroelectric vidicon camera can be used as a medical thermograph [2]. Black et al [3] have studied that the pyroelectric thermal imaging system for use in medical diagnosis. In the Past, the propagation of wave in a pyroelectric cylinder of arbitrary cross section with a circular cylindrical cavity, a pyroelectric circular cylinder of crystal class 6 and a pyroelectric cylinder of inner and outer arbitrary shape are studied [4]-[6]. Paul and Nelson [7] have extended the study of Vasudeva and Govinda Rao [8]-[9] on the influence of distributed voids in the interfacial LEMV adhesive zones of the isotropic Sandwich plate to the axisymmetric vibration of Piezo composite hollow cylinder. A continuum theory of LEMV with distinct properties has been developed by Cowin and Nunziato [10]. In layered composites pores or voids are found in the interface region and it is known to affect the estimation of physical and mechanical properties of the composites [11]. Voorhees and

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Green [12] have studied the mechanical behavior of sandwich composites made of thin porous core and denser face materials. Damage detection and vibration control of a new smart board designed by mounting piezoelectric fibers with metal cores on the surface of a CFRP composite was studied by Takagikiyoshi [13].

In the present analysis axisymmetric free vibration of pyrocomposite circular hollow cylinder of crystal class 6mm with LEMV as a bonding layer is considered as in figure (1). The frequency equation for axisymmetric vibration has been derived for traction free shorted inner and outer surfaces with interface continuity conditions on both sides of the LEMV layer. Numerical work is carried out for the axisymmetric vibration of the cylinder with equal thickness of the material BaTio$_3$ combined by a thin LEMV/CFRP layer. Dispersion curves have been drawn for propagation of waves along the axis of the composite cylinder with LEMV (with and without voids) is compared on replacing LEMV by CFRP and a pyroelectric hollow cylinder.
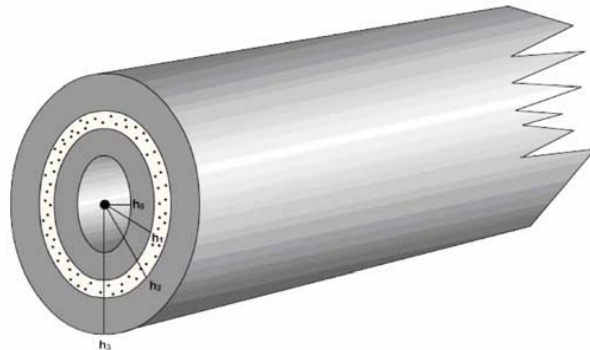

Fig. 1 Pyrocomposite hollow cylinder of thickness h = h$_3$ – h$_0$ with inner and outer Pyroelectric layers (h$_0$, h$_3$ are inner and outer radius of the cylinders) connected together by LEMV as a bonding layer

II. GOVERNING EQUATIONS

The equations governing elastic, electric and thermal behavior are given by Mindlin [14] - [15]

$$T^l_{ij,j} = \rho^l u^l_{i,tt},$$

$$D^l_{i,i} = 0,$$

$$k^l_{ij}\theta^l_{1,ij} = \theta^l_0 \sigma^l_{,t}$$

$$T^l_{ij} = C^l_{ijkl} S^l_{kl} - e^l_{kij} E^l_k - \lambda^l_{ij}\theta^l_1,$$

and

$$D^l_j = e^l_{jkl} S^l_{kl} + \varepsilon^l_{jk} E^l_k + p^l_j \theta^l_1,$$

$$\sigma^l = \lambda^l_{kl} S^l_{kl} + p^l_k E^l_k + \alpha^l \theta^l_1 \quad and \quad \alpha^l = \rho^l C^l_v \theta_0^{-1l}$$

(1)

where, $T^l_{ij}, S^l_{kl}, D^l_j, E^l_k, \sigma^l$ and θ^l_1 are stresses, strains, electric displacements, electric fields, entropy and temperature. Temperature field is assumed to be uniform throughout the cylinder. Here, C^l_v is the specific heat capacity, θ_0 is the reference temperature, and ρ^l is the density. Here, $C^l_{ij}, e^l_{mn}, \varepsilon^l_{kl}, \beta^l_j$ and p^l_m are elastic, piezoelectric, dielectric, thermal stress coefficients and pyroelectric constants respectively. The comma followed by an independent variable denotes partial differentiation of that coefficient with respect to that independent variable. And $E^l_i = -\phi^l_{,i}$. The superscript $(l=1,2)$ is to denote the constants and variables of inner and outer pyroelectric materials of hexagonal (class 6 mm).

For crystal class 6 mm, the material constants are

$$C^l = \begin{bmatrix} C^l_{11} & C^l_{12} & C^l_{13} & 0 & 0 & 0 \\ C^l_{12} & C^l_{11} & C^l_{23} & 0 & 0 & 0 \\ C^l_{13} & C^l_{23} & C^l_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C^l_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C^l_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C^l_{66} \end{bmatrix}$$

$$, \beta^l = \begin{bmatrix} \beta^l_1 \\ \beta^l_2 \\ \beta^l_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, p^{lT} = \begin{bmatrix} 0 \\ 0 \\ p^l_3 \end{bmatrix}$$

$$e^l = \begin{bmatrix} 0 & 0 & 0 & 0 & e^l_{15} & 0 \\ 0 & 0 & 0 & e^l_{15} & 0 & 0 \\ e^l_{31} & e^l_{31} & e^l_{33} & 0 & 0 & 0 \end{bmatrix},$$

$$\varepsilon^l = \begin{bmatrix} \varepsilon^l_{11} & 0 & 0 \\ 0 & \varepsilon^l_{11} & 0 \\ 0 & 0 & \varepsilon^l_{33} \end{bmatrix}$$

where, $C^l_{66} = \left(\dfrac{C^l_{11} - C^l_{12}}{2} \right)$ The stress components T_{ij}, electric displacements D_m and the entropy σ satisfy the following equations for axisymmetric vibration of hexagonal symmetry when the uniform temperature field throughout the cylinder,

$$T^l_{rr,r} + T^l_{rz,z} + r^{-1}(T^l_{rr}) = \rho^l u^l_{,tt},$$

$$T^l_{zz,z} + T^l_{rz,r} + r^{-1}T^l_{rz} = \rho^l w^l_{,tt},$$

$$r^{-1}[r D^l_{r,r} + D^l_r] + D^l_{z,z} = 0,$$

(2)

$$k^l_{11} \overline{\nabla}^2 T^l + k^l_{33} T^l_{,zz} = T^l_0 \sigma^l_{,t},$$

where,

$$\overline{\nabla}^2 = \frac{\partial^2}{\partial r^2} + r^{-1}\frac{\partial}{\partial r}.$$

The equations of axisymmetric motion, Gauss's equation and the entropy equation in cylindrical polar coordinates r,θ, z for class 6 are

$$C^l_{11}[u^l_{,rr} + r^{-1}u^l_{,r} + r^{-2}u^l] + C^l_{44} u^l_{,zz} + (C^l_{44} + C^l_{13})w^l_{,rz} + (e^l_{31} + e^l_{15})\phi^l_{,rz} - \beta^l_1 T^l_{,r} = \rho^l u^l_{,tt},$$

$$(C^l_{44} + C^l_{13})[u^l_{,rz} + r^{-1}u^l_{,z}] + C^l_{44}(w^l_{,rr} + r^{-1}w^l_{,r}) + C^l_{33} w^l_{,zz} + C^l_{15}(\phi^l_{,rr} + r^{-1}\phi^l_{,r}) + e_{33}\phi^l_{,zz} - \beta^l_3 T^l_{,z} = \rho^l w^l_{,tt},$$

$$\varepsilon^l_{11}(\phi^l_{,rr} + r^{-1}\phi^l_{,r} +) + \varepsilon^l_{33}\phi^l_{,zz} - (e^l_{31} + e^l_{15})[u^l_{,rz} + r^{-1}u^l_{,z}] - e^l_{15}(w^l_{,rr} + r^{-1}w^l_{,r}) - e_{33} w^l_{,zz} - p^l_3 T^l_{,z} = 0,$$

$$k^l_{11}[T^l_{,rr} + r^{-1}T^l_{,r}] + k^l_{33} T^l_{,zz} - T^l_0 d^l T^l_{,t}$$
$$T^l_0[\beta^l_1(u^l_{,rt} + r^{-1}u^l_{,t}) + \beta^l_3 w^l_{,zt} - p^l_3 \phi^l_{,zt}],$$

(3)

where k_{ii} is heat conduction coefficient, $d = \dfrac{\rho C_v}{T_0}$, u and w are the displacements along r, z direction, ϕ is the electric potential, ρ is the mass density and t is the time. The solutions of Eqn. (3) is considered in the form

$$u^l(r,\theta,z,t) = (U^l_{,r})\exp\{i(kz+pt)\},$$

$$w^l(r,\theta,z,t) = i\left(\frac{W^l}{h}\right)\exp\{i(kz+pt)\},$$

(4)

$$\phi^l(r,\theta,z,t) = i\left(\frac{C^l_{44}}{e^l_{33}}\right)\left(\frac{\Phi^l}{h}\right)\exp\{i(kz+pt)\},$$

$$T^l(r,\theta,z,t) = \left(\frac{C^l_{44}}{\beta^l_3}\right)\left(\frac{T^l}{h^2}\right)\exp\{i(kz+pt)\}$$

where, U^l, W^l, Φ^l, T^l are functions of r, k is the wave number, p is the angular frequency and $i = \sqrt{-1}$. We introduce the non dimensional quantities x and ε such that $x = \left(\dfrac{r}{h}\right)$, $\varepsilon = kh$ and h = h₃ − h₀ (h₀, h₃ are inner and outer radius of the cylinders) thickness of the composite hollow cylinder.

Using the above solution in Eqn (3) can be rewritten as

$$\begin{vmatrix} \overline{C}_{11}^{\,l}\nabla^2 + A_1 & -A_2 & A_3 & -A_4 \\ A_2\nabla^2 & \overline{C}_{44}^{\,l}\nabla^2 + A_5 & \left(\overline{e}_{15}^{\,l}\nabla^2 + A_6\right)\overline{C}_{44}^{\,l} & A_7 \\ A_3\nabla^2 & \overline{e}_{15}^{\,l}\nabla^2 + A_6 & -\left(K_{33}^{-2\,l}\nabla^2 + A_8\right) & -A_9 \\ A_4\nabla^2 & A_7 & -A_9 & i\overline{K}_1^{\,l}\nabla^2 + A_{10} \end{vmatrix}$$
$$\times\ (U^l, W^l, \Phi^l, T^l) = 0$$
(5)

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \left(\frac{1}{x}\right)\frac{\partial}{\partial x}$$
$$A_1 = \varepsilon^2 - (ch)^2$$
$$A_2 = \varepsilon(1 + \overline{C}_{13}^{\,l})$$
$$A_3 = \varepsilon(\overline{e}_{31}^{\,l} + \overline{e}_{15}^{\,l})$$
$$A_4 = \overline{\beta}$$
$$A_5 = \varepsilon^2 \overline{C}_{33}^{\,l} - (ch)$$
$$A_6 = \varepsilon^2$$
$$A_7 = \varepsilon$$
$$A_8 = \overline{K}_{33}^{2\,l}\varepsilon^2$$
$$A_9 = p^l\varepsilon$$
$$A_{10} = i\overline{K}_3\varepsilon^2 - \overline{d}$$

and

$$\overline{C}_{ij}^{\,l} = \frac{C_{ij}^{\,l}}{C_{44}^{1}}, \quad \overline{e}_{ij} = \frac{e_{ij}^{\,l}}{e_{33}^{\,l}}, \quad \overline{\beta} = \frac{\beta^l}{\beta_3^l},$$
$$\overline{d}^{\,l} = \frac{\rho^l C_{44}^{\,l} C_v^{\,l}}{\beta_3^{l2}\theta_0}, \quad \overline{K}_{ij}^2 = \frac{\varepsilon_{ij}^{\,l} C_{44}^{\,l}}{e_{33}^{\,l2}},$$
$$\overline{K}_{ij} = \frac{\left(\rho^l C_{44}^{\,l}\right)^{1/2} K_{ij}^{\,l}}{\beta_3^{l2} h\theta_0^{\,l}(ch)}, \quad \overline{p} = \frac{p_3 C_{44}^{\,l}}{e_{33}\beta_3}, \quad \overline{k}_i = \frac{k_i}{ch}$$

The solutions of the Eqn (5) are taken as

$$U^l = \sum_{j=0}^{4}\left[A_j^l J_0(\alpha_j^l x) + B_j^l Y_0(\alpha_j^l x)\right]$$
$$W^l = \sum_{j=0}^{4}\left[A_j^l J_0(\alpha_j^l x) + B_j^l Y_0(\alpha_j^l x)\right]d_j^l$$
$$\Phi^l = \sum_{j=0}^{4}\left[A_j^l J_0(\alpha_j^l x) + B_j^l Y_0(\alpha_j^l x)\right]e_j^l$$
$$T^l = \sum_{j=0}^{4}\left[A_j^l J_0(\alpha_j^l x) + B_j^l Y_0(\alpha_j^l x)\right]h_j^l$$
(6)

And $\left(\alpha_j^l x\right)^2$ are the four roots of the Eqn (5) when replacing $\nabla^2 = -\left(\alpha_j^l x\right)^2$.

The constants d_j^l, e_j^l and h_j^l can be evaluated using the following relations:

$$\left(\overline{C}_{11}^{\,l}\nabla^2 + A_1\right) - A_2 d^l{}_j + A_3 e^l{}_j - A_4 h^l{}_j = 0,$$
$$A_2\nabla^2 + \left(\overline{C}_{44}^{\,l}\nabla^2 + A_5\right)d^l{}_j + \left(\overline{e}_{15}^{\,l}\nabla^2 + A_6\right)\overline{c}_{44}^{\,l} e^l{}_j - A_7 h^l{}_j = 0,$$
$$A_3\nabla^2 + \left(\overline{e}_{15}^{\,l}\nabla^2 + A_6\right)d^l{}_j - \left(K_{33}^{-2\,l}\nabla^2 + A_8\right)e^l{}_j - A_9 h^l{}_j = 0,$$
$$A_4\nabla^2 + A_7 d^l{}_j - A_9 e^l{}_j + \left(i\overline{K}_1^{\,l}\nabla^2 + A_{10}\right)h^l{}_j = 0.$$
(7)

In the context of the theory of LEMV, the equations of motion and balance of equilibrated force are given by [10]

$$\rho\ddot{u}_i = \mu u_{i,jj} + (\lambda + \mu)u_{j,ji} + \beta\psi_{,i},$$
$$\rho k\ddot{\psi} = \alpha\psi_{,ii} - \omega\dot{\psi} - \xi\psi - \beta u_{k,k}$$
(8)

where u, v, w are displacements in r, θ and z directions $\alpha, \beta, \xi, \omega$ and k (equilibrated inertia) are material constants characterizing the core of LEMV, ρ is the density and λ, μ are the Lame's constants and ψ is the new kinematical variable associated with a material with voids comes into contact with another material without voids. The displacement equations of motion and balance of equilibrated force for an isotropic LEMV as in [17] are

$$(\lambda + 2\mu)(u_{,rr} + r^{-1}u_{,r} - r^{-2}u) + \mu u_{,zz}$$
$$+ (\lambda + \mu)w_{,rz} + \beta\psi_{,r} = \rho u_{,tt}$$

$$(\lambda + \mu)(u_{,rz} + r^{-1}u_{,z}) + \mu(w_{,rr} + r^{-1}w_{,r})$$
$$+ (\lambda + 2\mu)w_{,zz} + \beta\psi_{,z} = \rho w_{,tt}$$
(9)

$$-\beta(u_{,r} + r^{-1}u) - \beta w_{,z} + \alpha(\psi_{,rr} + r^{-1}\psi_{,zz})$$
$$- \rho k\psi_{,tt} - \omega\psi_{,t} - \xi\psi = 0$$

The solution for Eqn. (9) is taken as

$$u(r,\theta,z,t) = (U_{,r})\exp\{i(kz + pt)\},$$
$$w(r,\theta,z,t) = i\left(\frac{W}{h}\right)\exp\{i(kz + pt)\},$$
$$\psi(r,\theta,z,t) = \left(\frac{\Phi}{h^2}\right)\exp\{i(kz + pt)\},$$
(10)

Substituting Eqn. (10) in Eqn (9) and using the dimensionless variables x and ε, the Eqn. (10) becomes

$$\begin{vmatrix} (\lambda + 2\mu)\nabla^2 + B_1 & -B_2 & B_3 \\ B_2\nabla^2 & \overline{\mu}\nabla^2 + B_4 & B_5 \\ -B_3\nabla^2 & B_5 & \alpha\nabla^2 + B_6 \end{vmatrix}$$
$$(U, W, \Phi) = 0$$
(11)

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \left(\frac{1}{x}\right)\frac{\partial}{\partial x}$$

$$B_1 = \left(\frac{\rho}{\rho^1}\right)(ch)^2 - \overline{\mu}\,\varepsilon^2,$$

$$B_2 = (\overline{\lambda} + \overline{\mu})\varepsilon,$$

$$B_3 = \overline{\beta},$$

$$B_4 = \left(\frac{\rho}{\rho^1}\right)(ch)^2 - (\overline{\lambda} + 2\overline{\mu})\,\varepsilon^2,$$

$$B_5 = \overline{\beta}\,\varepsilon,$$

$$B_6 = \left(\frac{\rho}{\rho^1}\right)(ch)^2\,\overline{k} - \overline{\alpha}\,\varepsilon^2 - i\,\overline{\omega}\,(ch) - \overline{\xi},$$

and

$$\overline{\lambda} = \frac{\lambda}{C^1_{44}}, \quad \overline{\mu} = \frac{\mu}{C^1_{44}}, \quad \overline{\alpha} = \frac{\alpha}{h^2\,C^1_{44}}, \quad \overline{\beta} = \frac{\beta}{C^1_{44}},$$

$$\overline{\xi} = \frac{\xi}{C^1_{44}}, \quad \overline{\omega} = \left(\frac{\omega}{h}\right)(C^1_{44}\,\rho)^{-\frac{1}{2}}, \quad \overline{K} = \frac{k}{h^2}$$

The solutions of the Eqn. (11) are taken as

$$U = \sum_{j=0}^{3}\left[A_j\,J_0(\alpha_j x) + B_j\,y_0(\alpha_j x)\right]$$

$$W = \sum_{j=0}^{3}\left[A_j\,J_0(\alpha_j x) + B_j\,y_0(\alpha_j x)\right]d_j$$

$$\Phi = \sum_{j=0}^{3}\left[A_j\,J_0(\alpha_j x) + B_j\,y_0(\alpha_j x)\right]e_j$$

$$(12)$$

$(\alpha_j x)^2$ are the three roots of the equation (11) when replacing $\nabla^2 = -(\alpha_j^1 x)^2$. The constants d_j^1 and e_j^1 can be evaluated using the following relations:

$$B_2\,\nabla^2 + (\overline{\mu}\,\nabla^2 + B_4)d_j + B_5\,e_j = 0,$$

$$-B_3\,\nabla^2 + (B_5)\,d_j + (\alpha\,\nabla^2 + B_6)e_j = 0$$

$$(13)$$

The governing equation for CFRP core material can be deduced from Eqn. (9) by taking the void volume fraction $\psi = 0$, and the Lame's constants as $\lambda = c_{12}$, $\mu = \dfrac{(c_{11} - c_{12})}{2}$

III. FREQUENCY EQUATIONS

The frequency equation has been derived by using the following boundary and interface conditions
(i) Since the inner and outer surfaces are traction free and coated with electrodes which are shorted, the boundary conditions becomes
$T^l_{rr} = T^l_{rz} = \phi^l = T^l_{,r} = 0$ at $x_0 = h_0/h$ and $x_3 = h_3/h$
(ii) On the interfaces (inner and middle, outer and middle), the continuity conditions are
$T^l_{rr} = T_{rr}$; $T^l_{rz} = T_{rz}$; $u^l = u$; $w^l = w$

$\phi^l = 0$, $T^l_{,r} = 0$ (At non-pyroelectric core material) and $\psi_{,r} = 0$ due to void volume fraction field.

The interface condition $\psi_{,r} = 0$ on the void volume fraction field ψ is suggested by Atkin et al [18]. (When a material with voids comes into contact with another material without voids). The frequency equation is obtained as 22 x 22 determinantal equation, on substituting the solutions in the boundary- interface conditions. It is written in symbolic form as

$$\left|E(i,j)\right| = 0, \ (i, j = 1, 2, 3, ..., 22) \qquad (14)$$

The non-zero elements at $x_0 = \dfrac{h_0}{h}$ by varying j from 1 to 4 are

$$E(1, j) = 2\overline{c}_{66}{}^1\left(\frac{\alpha_j^1}{x_0}\right)J_1\left(\alpha_j^1\,x_0\right)$$
$$-\left[\overline{c}_{11}\,(\alpha_j^1)^2 + \overline{c}_{13}^1\,\varepsilon\,d_j^1 + \overline{e}_{31}^1\,\varepsilon\,e_j^1 + \overline{\beta}\,h_j\right]J_0(\alpha_j^1 x_0)$$

$$E(2, j) = \left(\varepsilon + d_j^1 + \overline{e}_{15}^1\,e_j^1\right)\left(\alpha_j^1\right)J_1\left(\alpha_j^1\,x_0\right)$$

$$E(3, j) = e_j^1\,J_0\left(\alpha_j^1\,x_0\right)$$

$$E(4, j) = \frac{h_j^1}{x_0}J_0(\alpha_j^1\,x_0) - (\alpha_j^1)\,J_1(\alpha_j^1\,x_0)$$

And the other nonzero elements
$E(1, j+4), E(2, j+4), E(3, j+4)$ and $E(4, j+4)$ are obtained by replacing J_0 by J_1 and Y_0 by Y_1.

At $x_1 = \dfrac{h_1}{h}$

$$E(5, j) = 2\overline{c}_{66}{}^1\left(\frac{\alpha_j^1}{x_1}\right)J_1\left(\alpha_j^1\,x_1\right)$$
$$-\left[\overline{c}_{11}\,(\alpha_j^1)^2 + \overline{c}_{13}^1\,\varepsilon\,d_j^1 + \overline{e}_{31}^1\,\varepsilon\,e_j^1 + \overline{\beta}\,h_j\right]J_0(\alpha_j^1 x_1)$$

$$E(5, j+8) = -\begin{bmatrix} 2\overline{\mu}\left(\dfrac{\alpha_j}{x_1}\right)J_1\left(\alpha_j\,x_1\right) \\ +\left\{-(\overline{\lambda} + \overline{\mu})(\alpha_j)^2 + \overline{\beta}\,e_j - \overline{\lambda}\,\varepsilon\,d_j\right\} \\ J_0(\alpha_j x_1) \end{bmatrix}$$

$$E(6, j) = \left(\varepsilon + d_j^1 + \overline{e}_{15}^1\,e_j^1\right)\left(\alpha_j^1\right)J_1\left(\alpha_j^1\,x1\right)$$

$$E(6, j+8) = -\overline{\mu}\,(\varepsilon + d_j)(\alpha_j)\,J_1\left(\alpha_j\,x1\right)$$

$$E(7,j)=(\alpha_j^1)J_1\left(\alpha_j^1 x_1\right)$$

$$E(7,j+8)=-(\alpha_j)J_1\left(\alpha_j^1 x_1\right)$$

$$E(8,j)=d_j^1 J_0\left(\alpha_j^1 x_1\right)$$

$$E(8,j+8)=-d_j^1 J_0\left(\alpha_j^1 x_1\right)$$

$$E(9,j)=e_j^1 J_0\left(\alpha_j^1 x_0\right)$$

$$E(10,j)=e_j(\alpha_j)J_1(\alpha_j x_1)$$

$$E(11,j)=\frac{h_j^1}{x_1}J_0(\alpha_j^1 x_1)-(\alpha_j^1)J_1(\alpha_j^1 x_1)$$

and the other nonzero element at the interfaces $x = x_1$ can be obtained on replacing J_0 by J_1 and Y_0 by Y_1 in the above elements. They are

$$E(i,j+4),E(i,j+8),E(i,j+11),E(i,j+14),(i=5,6,7,8)$$

and

$$E(9,j+4),E(10,j+4),E(11,j+4).$$

At the interface $x = x_2$, nonzero elements along the following rows $E(i,j),(i=12,13,...,18 \text{ and } j=8,9,..,20)$ are obtained on replacing x_1 by x_2 and superscript 1 by 2 in order.

Similarly, at the outer surface $x = x_3$, the nonzero elements $E(i,j),(i=19,20,21,22 \text{ and } j=14,15,..,22)$ can be had from the nonzero elements of first four rows by assigning x_3 for x_0 and superscript 2 for 1.

In the case of without voids in the interface region, the frequency equation is obtained by taking $\psi = 0$ in Eqn. (9) which reduces to a 20 x 20 determinantal equation. The frequency equations derived above are valid for different inner and outer materials of 6mm class and arbitrary thickness of layers.

IV. NUMERICAL RESULTS

Zeros of the frequency equations are evaluated using Muller's method [19]. The elastic, piezoelectric, dielectric and pyroelectric constants for $BaTio_3$ are taken from Ref. [20]-[21]. The material constants of LEMV bonding layer are taken as the hypothetical material no.2, in Table III of Puri and Cowin [16]. The value of dimensionless number N, which is void volume measure factor, defined in eq. (3.4) of Ref [16], and the value of N is found to be $0 \le N \le 0.66$. The material constants of CFRP bonding layer are taken from [22]. The frequencies are calculated by fixing real wave numbers for the thin core of thickness 0.002m and the inner and outer shell of thickness 0.03m. The complex frequencies for the axisymmetric waves in the first and second modes are given in Tables (1) and (2). The imaginary parts of the frequencies of the axisymmetric vibration of pyrocomposite LEMV cylinder

are compared with pyrocomposite CFRP and pyroelectric cylinders. Due to the thermal effect of the pyroelectric shells when combined with porous nature of the interfacial layers LEMV/CFRP shows a poor variation in the attenuation of the complex frequencies when compared to Piezocomposite hollow cylinder [7]. The dispersion curves of the Pyrocomposite hollow cylinder in the first and second axial modes are plotted in figs. (2) and (3) respectively. In both the figs (2) and (3), the bold, dotted, continuous and discontinuous lines indicates the Pyrocomposite with interfacial layers LEMV with N=0, N=0.33, CFRP and a single layered pyroelectric cylinder respectively. The limitations of Higher-order Mindlin Plate Theory are discussed by Ji Wang [23].

TABLE I DIMENSIONLESS COMPLEX FREQUENCIES FOR DIFFERENT VALUES OF REAL WAVE NUMBERS IN THE FIRST AXIAL MODE

| Wave no. (ε)	Dimensionless Frequencies (ch)			
	Pyro. Material	Composite with core		
		LEMV (N=0)	LEMV (N=0.33)	CFRP
0.01	0.1935E-01 +i 0.1520E+00	0.1904E-01 +i 0.1101E-15	0.1904E-01 +i 0.1101E-15	0.2030E-01 +i 0.1923E-03
0.2	0.1999E+00 +i 0.1086E-03	0.1999E+00 +i 0.1174E-17	0.1999E+00 +i 0.1174E-17	0.2137E+00 +i 0.1426E-14
0.6	0.6000E+00 +i 0.9198E-19	0.6000E+00 +i 0.9198E-19	0.6000E+00 +i 0.5194E-11	0.6000E+00 +i 0.9198E-19
1.2	0.1200E+01 +i 0.1608E-22	0.1200E+01 +i 0.9706E-12	0.1200E+01 +i 0.5908E-10	0.1200E+01 +i 0.2679E-11
1.8	0.1800E+01 +i 0.7103E-23	0.1800E+01 +i 0.1093E-11	0.1800E+01 +i 0.4409E-09	0.1800E+01 +i 0.3225E-11
2.4	0.2400E+01 +i 0.9370E-24	0.2400E+01 +i 0.1147E-10	0.2400E+01 +i 0.2048E-10	0.2400E+01 +i 0.1147E-10
3.0	0.3000E+01 +i 0.4243E-23	0.3000E+01 +i 0.1770E-11	0.3000E+01 +i 0.6504E-11	0.3000E+01 +i 0.1770E-11
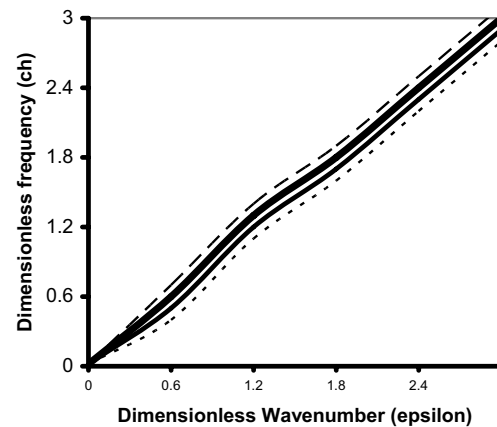
Fig. 2 Comparison of dispersion curves of composite hollow cylinder BaTio3 / CFRP / BaTio3 (Thin line), hollow cylinder BaTio3 / LEMV (N=0) / BaTio3 (bold line), hollow cylinder BaTio3 / LEMV (N=0.33) / BaTio3 (dotted line) and Pyroelectric cylinder (discontinuous line) in the first axial Mode

TABLE II DIMENSIONLESS COMPLEX FREQUENCIES FOR DIFFERENT VALUES OF REAL WAVE NUMBERS IN THE SECOND AXIAL MODE

Wave no. (ε)	Dimensionless Frequencies (ch)			
	Pyro. Material	Composite with core		
		LEMV (N=0)	LEMV (N=0.33)	CFRP
0.01	0.1052E+00 +i 0.2325E-01	0.3040E-01 +i 0.2289E-16	0.3040E-01 +i 0.2289E-16	0.3112E-01 +i 0.7774E-03
0.2	0.2000E+00 +i 0.0000E+00	0.2000E+00 +i 0.2693E-30	0.2000E+00 +i 0.2693E-30	0.2129E+00 +i 0.2129E-15
0.6	0.6000E+00 +i 0.4653E-31	0.6000E+00 +i 0.4653E-31	0.6000E+01 +i 0.9198E-19	0.6000E+01 +i 0.4653E-31
1.2	0.12000E+01 +i 0.1423E-29	0.1200E+01 +i 0.1423E-29	0.1200E+00 +i 0.9707E-12	0.1200E+00 +i 0.1423E-29
1.8	0.1800E+01 +i 0.7103E-23	0.1800E+01 +i 0.6348E-25	0.1800E+01 +i 0.1093E-11	0.1800E+01 +i 0.6348E-25
2.4	0.2400E+01 +i 0.1119E-29	0.2400E+01 +i 0.9370E-24	0.2400E+01 +i 0.4723E-11	0.2400E+01 +i 0.9370E-24
3.0	0.3000E+01 +i 0.2089E-22	0.3000E+01 +i 0.2089E-22	0.3000E+01 +i 0.6504E-11	0.3000E+01 +i 0.4244E-22
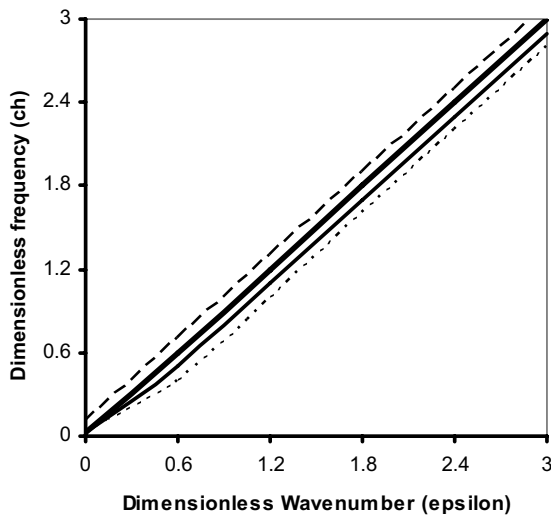
Fig. 3 Comparison of dispersion curves of composite hollow cylinder BaTio3 / CFRP / BaTio3 (Thin line), hollow cylinder BaTio3 / LEMV (N=0) / BaTio3 (bold line), hollow cylinder BaTio3 / LEMV (N=0.33) / BaTio3 (dotted line) and Pyroelectric cylinder (discontinuous line) in the second axial Mode

V. CONCLUSION

The frequency equation for free axisymmetric vibration of pyrocomposite hollow cylinder with LEMV as core material is derived. Dispersion curves in the first and second axial modes of the pyrocomposite hollow cylinders with a hypothetical LEMV core and an existing CFRP core are compared with a single layered pyroelectric cylinder. The damping observed is not significant due to the thermal effect of the pyroelectric layers and the presence of voids in the interfacial LEMV/CFRP layers. However an increase in damping often results in corresponding decrease in mechanical property. The present model with CFRP core may be modified suitably to have a similar practical application discussed in [24]-[25].

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