Average Turbulent Pipe Flow with Heat Transfer Using a Three-Equation Model

Khalid Alammar

***Abstract*—**Aim of this study is to evaluate a new three-equation turbulence model applied to flow and heat transfer through a pipe. Uncertainty is approximated by comparing with published direct numerical simulation results for fully-developed flow. Error in the mean axial velocity, temperature, friction, and heat transfer is found to be negligible.

***Keywords*—**Heat Transfer, Nusselt number, Skin friction, Turbulence.

I. Introduction

THE The problem of turbulence dates back to the efforts of Claude-Louis Navier and George Gabriel Stokes, as well as others in the early nineteenth century. Searching for its solution, it was a source of great despair for many notably great scientists, including Werner Heisenberg, Horace Lamb, and many others. The complete description of turbulence remains one of the unsolved problems in modern physics. A great deal of early work on turbulence can be found, for example, in Hinze [1].

Recently, direct numerical simulation (DNS) has emerged as an indispensible tool to tackle turbulence directly, albeit at relatively low Reynolds numbers. Several DNS studies on turbulent pipe flow have been performed recently, including Eggels et al. [2], Loulou et al. [3], and Wu and Moin [4]. The latter has carried out DNS on a turbulent pipe flow at Reynolds number of 44,000, which is the largest among the other three studies. Mean velocity, Reynolds stresses, and turbulent intensities are presented and discussed, along with visualization of flow structure. Good agreement was attained with the Princeton Superpipe data on mean flow statistics and Lawn [5] data on turbulence intensities. Large eddy simulation (LES) is another tool that somewhat bridges between DNS and Reynolds-averaged Navier-Stokes (RANS) methods. In LES, large turbulent structures in the flow field are resolved, while the effect of sub-grid scales (SGS) are modeled. LES investigation, for example, has been carried out by Rudman and Blackburn [6] on a turbulent pipe flow at Reynolds number of 38,000. Mean velocity and Reynolds stresses are presented and discussed, along with visualization of flow structure. Results were reported to compare favorably with measurements.

While much work on isothermal turbulence have been carried out using DNS and LES, less attempts have been carried out to investigate turbulent flow with heat transfer using these methods. For example, Redjem-Saad [7] have investigated fully-developed flow and heat transfer characteristics in pipes using DNS at the low Reynolds number of 5,500 based on bulk velocity and pipe diameter. Main emphasis is placed on Prandtl number effects on turbulent heat transfer. Temperature fluctuations and heat fluxes were found to increase when increasing Pr. The turbulent Prandtl number was found to be independent of Pr for Prandtl number > 0.2. They also confirm the intermittent behavior close to the wall, which is more pronounced with increasing Pr. The predicted Nusselt number is in good agreement with published data. Similar investigation was carried out by Satake et al. [8] who performed DNS analysis on a turbulent pipe flow with heat transfer for five Reynolds numbers, ranging from 5,300 to 40,000 based on the bulk velocity and pipe diameter. Their results of average friction and Nusselt number were in good agreement with published data.

While DNS and LES are fairly accurate for modeling turbulent flows with heat transfer, they remain limited to relatively low-range Reynolds numbers. This drawback explains the wide-spread of turbulence modeling in industrial applications where the use of DNS techniques remains formidable. Turbulence modeling includes eddy viscosity models which utilize the Boussinesq hypothesis [1] for relating the Reynolds stresses to the average flow field. In turn, the eddy viscosity is determined by using any of a variety of techniques, including zero-, one-, and two-equation models, most notably the k-ε model. While such models vary in complexity, they share several shortcomings, including isotropy of the eddy viscosity and the lack of generality in wall treatment. Such shortcomings lead to poor results in separated flows and other non-equilibrium turbulent boundary layers [9].

A second-order turbulence model, which also falls under RANS methods, is the Reynolds stress model. While the model relaxes the isotropic assumption, it remains more complicated and costly due to the need for solving six additional transport equations along with many unknown terms. For more on the subject of turbulence modeling, the reader is referred to, for example, Launder and Spalding [10].

In this paper, the accuracy of a three-equation turbulence model is assessed. Using the model, average turbulent flow and heat transfer through a pipe is simulated for Reynolds numbers of 40,000 and 44,000. Uncertainty is approximated by comparing with DNS results of Wu and Moin [11] and Satake et al. [8]. Results for fully-developed mean axial velocity and temperature are presented and discussed.

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II. Theory

Starting with the incompressible Navier-Stokes equations in Cartesian index notation, and with Reynolds decomposition, averaging, and following Boussinesq hypothesis, we have;

$$\frac{\partial(\overline{u}_i)}{\partial x_i} = 0 \tag{1}$$

$$\rho\left[\frac{\partial(\overline{u}_i)}{\partial t} + (\overline{u}_j)\frac{\partial(\overline{u}_i)}{\partial x_j}\right] + \frac{\partial(\overline{p})}{\partial x_i} - \frac{\partial}{\partial x_j}\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) = \frac{\partial}{\partial x_j}\mu_t\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{2}$$

$$\rho c\left[\frac{\partial \overline{T}}{\partial t} + (\overline{u}_j)\frac{\partial \overline{T}}{\partial x_j}\right] = \frac{\partial}{\partial x_j}\left(k + \frac{c\mu_t}{\mathrm{Pr}_t}\right)\frac{\partial \overline{T}}{\partial x_j} \tag{3}$$

For simplicity, second viscosity effects and body forces are neglected. $\mu_t = \mathrm{Re}_t\,\mu$ is the eddy viscosity, Alammar [11], where $\mathrm{Re}_t = |\overline{u}_i|\rho l_i / \mu$, and l_i is a length scale given by

$$\rho\left[\frac{\partial(l_i)}{\partial t} + (\overline{u}_j)\frac{\partial(l_i)}{\partial x_j}\right] = \frac{\partial}{\partial x_j}\mu\left(\frac{\partial l_i}{\partial x_j}\right) + C_1^2\,\rho\frac{\partial}{\partial x_j}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + C_2\frac{\mu}{C_1^2} \tag{4}$$

Hence, we have three equations for the turbulence length scale with their sources being the average strain rate, along with the molecular viscosity. C1 is a constant length parameter *perhaps* attributed to the fluid. C2 is another constant length parameter attributed to wall roughness.

III. Numerical Procedure

The governing equations were solved with a finite-volume solver using Gauss-Seidel iterative method, in conjunction with second-order schemes. 20,000 structured cells were used with y^+ down to 0.4. No-slip boundary condition was applied at the wall and the inlet turbulence length scale was set to $1.13e^{-3}$m. C_1 and C_2 were $8.06e^{-5}$m and $2.93e^{-9}$m, respectively. $\mathrm{Pr}_t = 0.87$, Redjem-Saad [7].

IV. Results and Discussion

Along with DNS results of Wu and Moin [12], predicted mean velocity distributions are depicted in Fig. 1. The agreement is excellent in all regions, including the laminar sub-layer and buffer and outer layers. It's the strain rate in the length equation which is responsible for energizing the buffer layer. The predicted friction coefficients are the same. Along with DNS results of Satake et al. [8], predicted mean temperature distributions are shown in Fig. 2. Again, the agreement is excellent in all regions. The predicted Nusselt numbers are the same. While 3 equations were used for the length scale, it's tempting to extend (3) to (9) with their sources being the corresponding strain rate in the average momentum. Such equations are expected to yield more accurate results for more complex turbulence involving compressibility and in homogeneity.

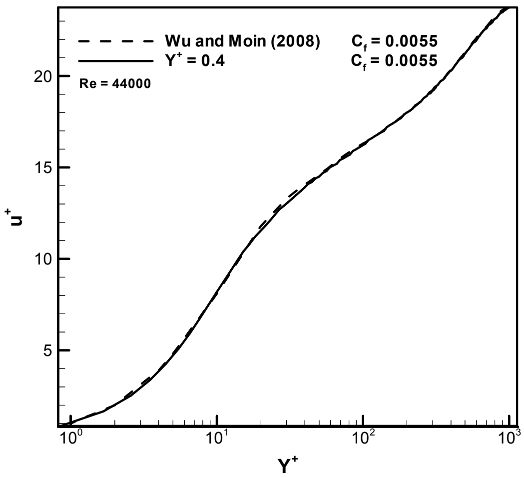

Fig. 1 Mean velocity distribution for Re = 44,000

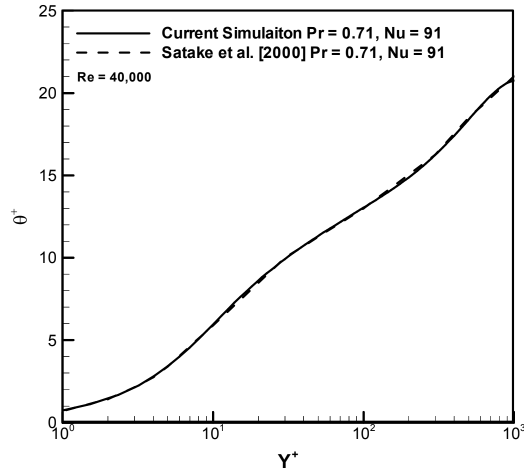

Fig. 2 Mean temperature distribution for Re = 40,000

V. Conclusion

In this paper, the accuracy of a three-equation turbulence model was assessed. Using the model, average turbulent flow and heat transfer through a pipe was simulated for Reynolds number of 40,000 and 44,000. Model results for mean axial velocity and temperature were compared with DNS results. The agreement was excellent. The new model is should be tested for more complex turbulence involving separation, compressibility, and inhomogeneity

Nomenclature

b a non-dimensional function of wall roughness

c specific heat, J/(kg K)
D pipe diameter, m
d normal distance from the wall, m
f friction factor = $\tau_w / 0.5\rho U^2$
h heat transfer coefficient = $q/(T_w - T_b)$, $[W/(m^2 \cdot K)]$
k thermal conductivity, $[W/mK]$
p pressure, Pa
q heat flux through the wall, $[W/m^2]$
Re Reynolds number $= U\rho D/\mu$
Re_t non-dimensional parameter $= |\overline{u}_i| \rho d / \mu$
RS Reynolds stress $= -\rho\overline{u'v'}/\tau_w$, Pa
T_b bulk temperature = $\int_0^R uTrdr / \int_0^R urdr$, K
T_τ friction temperature $= q/(\rho c U^*)$, K
T_w wall temperature, K
U area-average velocity, m/s
U^* friction velocity $= \sqrt{\tau_w/\rho}$, m/s
$\overline{u}_i$ mean velocity component, m/s
u mean axial velocity, m/s
u^+ normalized mean axial velocity = u/U^*
x axial distance, m
y^+ non-dimensional wall distance = $rU^*\rho/\mu$
r radial distance, m

Greek symbols

μ fluid dynamic viscosity, $N \cdot s/m^2$
μ_t eddy viscosity, $N \cdot s/m^2$
ρ fluid density, kg/m^3
$\theta^+ = (T_w - T)/T_\tau$
τ_w wall shear stress, Pa.

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