Auto Tuning of PID Controller for MIMO Processes

M. J. Lengare, R. H. Chile, L. M. Waghmare and Bhavesh Parmar

***Abstract*—**One of the most basic functions of control engineers is tuning of controllers. There are always several process loops in the plant necessitate of tuning. The auto tuned Proportional Integral Derivative (PID) Controllers are designed for applications where large load changes are expected or the need for extreme accuracy and fast response time exists. The algorithm presented in this paper is used for the tuning PID controller to obtain its parameters with a minimum computing complexity. It requires continuous analysis of variation in few parameters, and let the program to do the plant test and calculate the controller parameters to adjust and optimize the variables for the best performance. The algorithm developed needs less time as compared to a normal step response test for continuous tuning of the PID through gain scheduling.

***Keywords*—**Auto tuning; gain scheduling; MIMO Processes; Optimization; PID controller; Process Control.

I. INTRODUCTION

IN industrial control systems, Proportional-Integral-Derivative (PID) [1, 2, and 3] is extensively used to optimize the processes by constantly monitoring the process variable for any oscillation around the desired output. In present, the control adjusts the PID parameters to eliminate these. This type of tuning is ideal, where load characteristics change while the process is running.

In the relay-based auto tuning, a relay is used to excite the process dynamic information. The limiting factor for this tuning method is the time required to achieve ultimate gain. Adaptive control method is also not the solution to all control problems therefore a special kind of open-loop adaptation or change of PID parameter through gain scheduling is employed. [4]

The identification of the suitable scheduling variables is based on knowledge of the process, example is production rate. When scheduling variables have been determined, the regulator parameters are calculated for different operating conditions, using suitable methods. The regulator is thus tuned or calibrated for each operating condition.

The proposed method is an extension of the Basilio and Matos method of Design of PID Controllers [5], which is suitable for plants with both monotonic and underdamped step

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responses. The advantage of the method is that the control process remains in a closed loop while the parameters are continually modified to keep the process tuned within prescribed ranges. In this work, the initial control design is done off-line and the PID coefficients are determined analytically. These parameters are then instantaneously modified online accordingly

The standard PID [6-7] control signals for Fig. 1 are given as:

$$\left[u(t) = K_P e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d \frac{d}{dt}e(t) \right] \tag{1}$$
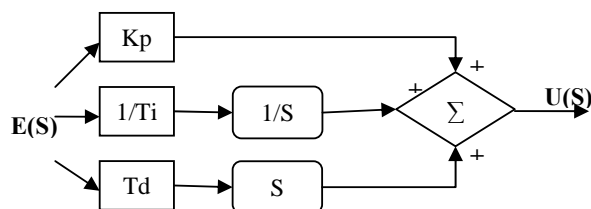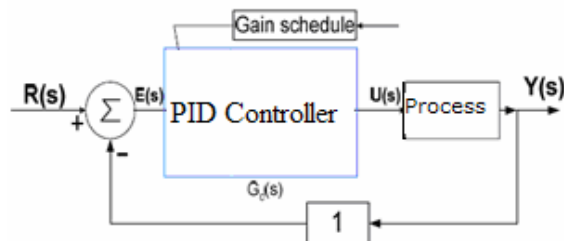

Fig. 1 Standard PID Controller

Fig. 2 Block diagram of system in which influences of parameter variations are compensated by gain scheduling

The equation of PID presented in [5] is appended below:

$$u(t) = K_p[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d \frac{d}{dt}e(t)] \tag{2}$$

Where u (t) and e (t) denote the control and the error signals, respectively, K_p, T_i and T_d are the parameters to be tuned. The corresponding transfer function is given as

$$G_c(s) = K_P\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{3}$$

The main advantage of this algorithm is that the changes made in K_p will accordingly change all the three parameters of the controller. This can be used for online continuous tuning

of the process by employing gain scheduling technique as shown in Fig. 2.

This paper is organized as follows. The tuning of PID controllers for plants with both monotonic and underdamped step responses is covered in Section II. Simulation results are given in Sections III. Finally, conclusions are drawn in Section IV.

II. DESIGN OF PID CONTROLLERS

Initially, the design of PID controllers for processes with monotonic step response is modeled by second-order systems. Let $G_p(s)$ denote the transfer function of the process to be controlled, and assume that the system has a step response with the same shape as that shown in Fig. 3.

Then $G_p(s)$ may be modeled as [8]

$$G_p(s) = \frac{K}{(\tau s+1)^2} \qquad (4)$$

The relation of time constant 'τ', in (4) for a step with amplitude A is given in [8], as below:

$$y(t) = KA\left(1 - \frac{1}{\tau}t e^{-\frac{1}{r}t} - e^{-\frac{1}{r}t}\right), t \geq 0 \qquad (5)$$

Therefore, since $y_\infty = KA$, then

$$A_O = \int_0^\infty [KA - y(t)]dt = KA\int_0^\infty \left(1 - \frac{1}{\tau}t e^{-\frac{1}{r}t} - e^{-\frac{1}{r}t}\right)dt \qquad (6)$$
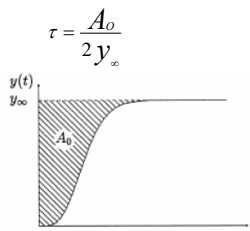
Then

$$\tau = \frac{A_O}{2 y_\infty} \qquad (7)$$

Fig. 3 System with monotonic step response used for identification of the process

A. Design PID Controllers for Processes with Monotonic Step Response

Consider the feedback system of Fig. 2 and suppose that $Gp(s)$ is given by (4). When the controller to be designed is a PID, then (3) can be rewritten as

$$G_c(s) = \overline{K}_p \frac{(s+z_1)(s+z_2)}{s} \qquad (8)$$

Where $-z_1$ and $-z_2$ are the controller zeros with the assumption that $|z_1| > |z_2|$.

$$\overline{K}_p = K_p T_d, z_1 + z_2 = \frac{1}{T_d}, and\ z_1 z_2 = \frac{1}{T_i T_d} \qquad (9)$$

The correct choice of K_p, T_i, and T_d is made with the help of the root-locus diagram. The open-loop transfer function is given as:

$$G(s) = G_p(s)G_c(s) = \frac{K\overline{K}_p (s+z_1)(s+z_2)}{s(\tau s+1)^2} \qquad (10)$$

By placing $-z_1$ at $-1/\tau$ and $-z_2$ at $-1.5/\tau$. One pole cancels and the open-loop transfer function can be given as:

$$G(s) = \overline{K}_p \frac{K\overline{K}_p (s+1.5/\tau)}{\tau^2 s(s+1/\tau)} \qquad (11)$$

Where K_p can be calculated by converting (11) into closed loop polynomial and equating the characteristic equation with 0. Finally the PID parameters in term of k and τ are

$$k_p = \frac{0.6699}{k}, T_i = \frac{5\tau}{3}, T_d = \frac{2\tau}{5} \qquad (12)$$

Thus the PID parameters are computed as under:

Algorithm I
1) Apply a step signal with amplitude A to the plant, and record the response $y(t)$.
2) Compute numerically the area A_0 and the steady-state value y_∞ of $y(t)$ as shown in Fig. 3.
3) Compute $K = y_\infty/A$.
4) Set the controller parameters as $K_p = 0.6699/K$; $T_i = 5A_0/6y_\infty$ and $T_d = A_0 / (5y_\infty)$.

Record the values of K_p against a discrete set of full range of plant operational parameters for the operation of plant within prescribed range and for other values compute K_p through interpolation.

B. Design of PID Controllers for Plants with Underdamped Step Response

Systems with underdamped step response may be approximated by a second-order system with the transfer function [8].
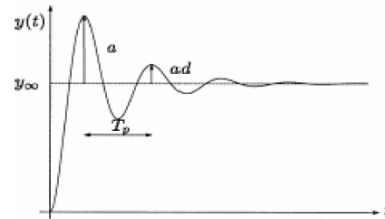

Fig. 4 System with underdamped step response used for identification of the process

In order to obtain a PID controller for this system, it should be noted that (3) can be rewritten as

$$G_c(s) = \frac{K_p T_d}{s}\left(s^2 + \frac{1}{T_d}s + \frac{1}{T_i T_d}\right) \qquad (13)$$

A natural choice for the controller zeros would be such that the numerator polynomial $G_c(s)$ of (13) and the denominator polynomial $G_p(s)$ of (11) cancel. In order for this condition to happen, T_d and T_i must satisfy:

$$T_d = \frac{1}{2\xi \omega_n}, T_i = \frac{2\xi}{\omega_n}, K_p = \frac{4T_i}{K t_s}, \qquad (14)$$

Where t_s are settling time and are calculated from the step response.

The PID parameters can be computed as under:

Algorithm II
1) Apply to the plant a step of amplitude A and record the output $y(t)$, as shown in Fig. 4.

2) Determine y_∞ and the settling time t_{so} of the plant response, the first two peak values M_{p1} and M_{p2}, and the corresponding time instants t_{p1} and t_{p2}

3) Compute $d = (M_{p2} - y_\infty)/(M_{p1} - y_\infty)$ and $T_p = T_{p2} - T_{p1}$

4) Compute $\xi = 1/\sqrt{1 + (2\pi/\ln d)^2}$ and $\omega_n = 2\pi/\left(T_p\sqrt{1-\xi^2}\right)$

5) Compute PID parameters using (14).

With the controller embedded in the real system, determine and record the values of K_p, against a discrete set of full range of plant operational parameters for satisfactory operation. For other values of the plant parameters compute K_p through interpolation.

III. SIMULATION RESULTS FOR THE GAIN SCHEDULING

In the block diagram of the Fig. 2, the unknown plant for monotonic step response is represented by

$$G_p(s) = \frac{1}{(s+1)^s} \tag{15}$$

The plant response to a unit step input is taken [5], the parameters of which are $K_p = 0.846$, $T_i = 6.6667$ and $T_d = 1.6$. The values of the K_p required for variation in process parameters are determined and shown in Table I.

TABLE I
VALUE OF THE K_p REQUIRED FOR DIFFERENT MONOTONIC PLANT PARAMETERS

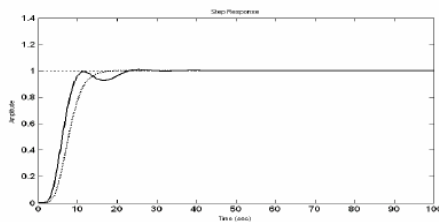| S. No.	K_p	Process ($G_p(s)$)
1	0.846	$\dfrac{1}{(s+1)^8}$
2	1.5	$\dfrac{1}{(s+1.5)(s+1)^7}$
3	3.4	$\dfrac{1}{(s+2)(s+1.5)(s+1)^6}$
4	10	$\dfrac{1}{(s+1.5)(s+2)(s+3)(s+1)^5}$

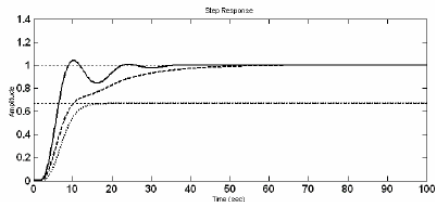Fig. 5a for Table I, S. No. 1

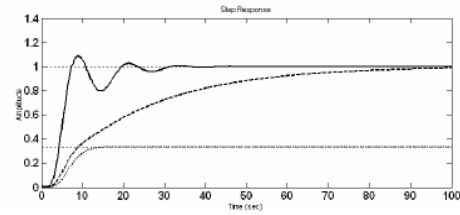Fig. 5b for Table I, S. No. 2

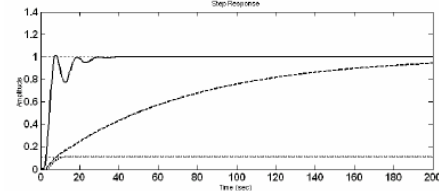Fig. 5c for Table I, S. No. 3

Fig. 5d for Table I, S. No. 4

Fig. 5 Open loop system response (dotted line) and Close loop system response: (dash line) for the plant with $K_p = 0.846$ and solid line for the plant with $K_p = 0.846$, 1.5, 3.4 & 10 respectively for unit step input.

TABLE II
VALUE OF THE K_p REQUIRED FOR THE DIFFERENT UNDERDAMPED STEP RESPONSE PROCESS PARAMETERS

K_p	Process $G_p(s)$
0.04	$\dfrac{28.50s^2 + 6.93s + 18.20}{s^6 + 17.47s^5 + 46.78s^4 + 67.52s^3 + 64.86s^2 + 43.30s + 14.16}$
0.09	$\dfrac{28.50s^2 + 6.93s + 18.20}{s^6 + 17.47s^5 + 48.08s^4 + 89.96s^3 + 119.4s^2 + 97.27s + 34.36}$
0.20	$\dfrac{28.50s^2 + 6.93s + 18.20}{s^6 + 17.47s^5 + 48.99s^4 + 102.1s^3 + 103.1s^2 + 62.52s + 41.66}$

In the block diagram of the process in Fig. 2, for underdamped step response, the unknown plant is represented by:

$$G(s) = \frac{28.50s^2 + 6.93s + 18.20}{s^6 + 17.47s^5 + 46.78s^4 + 67.52s^3 + 64.86s^2 + 43.30s + 14.16} \tag{16}$$

The plant response for under damped step response is taken as given in [5], the parameters of which are $K_p = 0.04$, $T_i = 0.2366$ and $T_d = 4.3251$. The value of K_p is determined by changing system parameters and is shown in Table II.
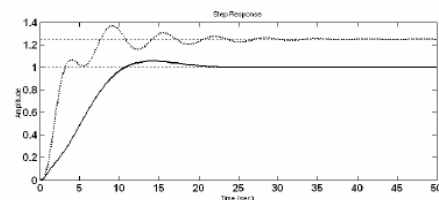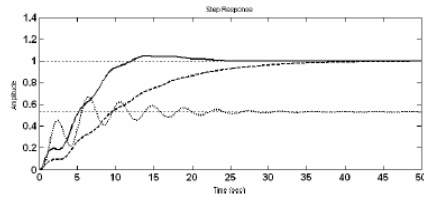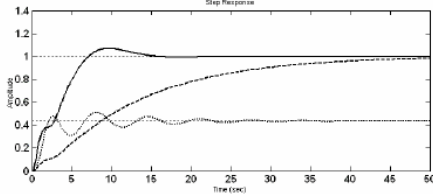

Fig. 6a, Table II, $K_p = 0.04$

Fig. 6b, Table II, $K_p = 0.09$

Fig. 6c, Table II, $K_p = 0.20$

Fig. 6 a, b & c. shows open loop system response (dotted line) and Close loop system response: (dash line) for the plant with $K_p = 0.04$ and solid line for the plant with $K_p = 0.04, 0.09$ & 0.2 for the processes from Table II respectively for step input.

The figures (Fig. 6 a-c) show the response of the system with variation in K_p to compensate for changes in process parameters.

It can be seen from the results of the simulations shown in Figs. 5 & 6 that after initially setting up the system, the changes in system parameters cannot be compensated for the initially tuned PID. However, if K_p is varied through Gain Scheduling according to changes in the plant parameters then the PID will effectively compensates for the changes. The use of Gain Scheduling for this purpose provides an effective means to regulate the online process and an optimum solution.

Initially the tuning of the system is time consuming as the values of the coefficients of PID controller are determined. Then K_p is determined by changing the operational parameters of the plant. These values of K_p are used during on line tuning of the plant through gain scheduling.

IV. CONCLUSION

In this paper, methodology for online tuning PID controllers has been proposed. Like the design technique presented in [5] which use both monotonic and step responses for initial tuning, this paper provides an extension in the technique by employing gain scheduling for instantaneous continuous tuning of the process. Simulations are given to illustrate the efficiency of the methodology.

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