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Attitude Stabilization of Satellites Using Random Dither Quantization

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***Abstract*—**Recently, the effectiveness of random dither quantization method for linear feedback control systems has been shown in several papers. However, the random dither quantization method has not yet been applied to nonlinear feedback control systems. The objective of this paper is to verify the effectiveness of random dither quantization method for nonlinear feedback control systems. For this purpose, we consider the attitude stabilization problem of satellites using discrete-level actuators. Namely, this paper provides a control method based on the random dither quantization method for stabilizing the attitude of satellites using discrete-level actuators.

***Keywords*—**Quantized control, nonlinear systems, random dither quantization.

I. Introduction

THE quantization of control inputs occurs in many systems equipped with discrete-level actuators. The control signals are also quantized in communication networks. Thus, the quantized control of systems is one of the most important research topics in recent years.

Recently, the random dither quantization method that transforms a given continuous-valued signal to a discrete-valued signal by adding artificial random noise to the continuous-valued signal before quantization has been proposed in [1]. Model predictive control [2]-[4], also known as receding horizon control [5]-[10], is a kind of optimal feedback control and the so-called stochastic model predictive control [11]-[14] has been applied to the quantized control of systems with random dither quantizer in [15]. It has been shown that the random dither quantization method exhibits much better performance than the simple uniform quantization method for linear feedback control systems. Hence, this paper focuses on the feedback control systems with random dither quantization method.

The control performance of quantized control of systems using random dither quantizer has been well analyzed for linear feedback control systems. However, the random dither quantization method has not yet been applied to nonlinear feedback control systems. The objective of this paper is to verify the effectiveness of random dither quantization method for nonlinear feedback control systems.

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For this purpose, we analyze the control performance of quantized control of nonlinear systems using the random dither quantizer. So far, several kinds of nonlinear feedback control problems have been studied [16]-[19]. In this study, we focus on the class of attitude control problems of satellites in which the control of nonlinear dynamics is taken into account. Motivated by the fact that the actuators such as thrusters that are often used for attitude control of satellites yield discrete-level inputs, we apply the random dither quantization method to the nonlinear feedback control of satellite attitude. The main contribution of this study is to verify the effectiveness of random dither quantization method for attitude stabilization of satellites.

This paper is organized as follows. In Section II, we introduce some notations and the system model of satellites. In Section III, we formulate the control problem of satellite attitude with quantized control inputs. The main results are provided in Section IV. Finally, some concluding remarks are given in Section V.

II. Notations and System Model

First, we introduce some notations that are adopted throughout this paper. Let the set of real numbers be denoted by $\mathbb{R}$. Let the set of non-negative real numbers be denoted by $\mathbb{R}_+$. Let $t \in \mathbb{R}_+$ denote the temporal variable. Next, we introduce the system model of satellites. Let us consider a rigid satellite in an inertial reference frame and let $\omega_1(t)$, $\omega_2(t)$, and $\omega_3(t)$ denote the angular velocity components along a body fixed reference frame having the origin at the center of gravity and consisting of three principal axes. The Euler's equations for the rigid body with three independent controls aligned with three principal axes are

$$J_1\dot{\omega}_1(t) = (J_2 - J_3)\omega_2(t)\omega_3(t) + u_1(t) \tag{1a}$$
$$J_2\dot{\omega}_2(t) = (J_3 - J_1)\omega_3(t)\omega_1(t) + u_2(t) \tag{1b}$$
$$J_3\dot{\omega}_3(t) = (J_1 - J_2)\omega_1(t)\omega_2(t) + u_3(t) \tag{1c}$$

where $J_1 > 0$, $J_2 > 0$, and $J_3 > 0$ denote the principal moments of inertia and $u_1(t)$, $u_2(t)$, and $u_3(t)$ denote the control torques. Let us introduce the inertia ratios I_1, I_2, I_3 defined as follows:

$$I_1 = \frac{J_2 - J_3}{J_1}$$
$$I_2 = \frac{J_3 - J_1}{J_2}$$
$$I_3 = \frac{J_1 - J_2}{J_3}$$

Using inertia ratios I_1, I_2, and I_3, the system model (1) can be rewritten as follows:

$$\dot{\omega}_1(t) = I_1\omega_2(t)\omega_3(t) + \frac{u_1(t)}{J_1} \tag{2a}$$

$$\dot{\omega}_2(t) = I_2\omega_3(t)\omega_1(t) + \frac{u_2(t)}{J_2} \tag{2b}$$

$$\dot{\omega}_3(t) = I_3\omega_1(t)\omega_2(t) + \frac{u_3(t)}{J_3} \tag{2c}$$

Let a unit vector along the Euler axis be denoted by

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix},$$

where e_1, e_2, and e_3 are the direction cosines of the Euler axis relative to the body fixed control axes. The four elements called the quaternions are defined as follows:

$$q(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \end{bmatrix} = \begin{bmatrix} e_1 \sin\left(\frac{\theta(t)}{2}\right) \\ e_2 \sin\left(\frac{\theta(t)}{2}\right) \\ e_3 \sin\left(\frac{\theta(t)}{2}\right) \\ \cos\left(\frac{\theta(t)}{2}\right) \end{bmatrix},$$

where $\theta(t)$ denotes the rotation angle about the Euler axis. Then we have the following relation.

$$q_1^2(t) + q_2^2(t) + q_3^2(t) + q_4^2(t) = 1. \tag{3}$$

It is known that the quaternion kinematic differential equations are given by

$$\begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \dot{q}_3(t) \\ \dot{q}_4(t) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_3(t) & -\omega_2(t) & \omega_1(t) \\ -\omega_3(t) & 0 & \omega_1(t) & \omega_2(t) \\ \omega_2(t) & -\omega_1(t) & 0 & \omega_3(t) \\ -\omega_1(t) & -\omega_2(t) & -\omega_3(t) & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \end{bmatrix}. \tag{4}$$

Let the state vector $x(t) \in \mathbb{R}^7$ be defined by

$$x(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \end{bmatrix}.$$

Using the state vector $x(t)$, the rotational equations of motion of a rigid satellite about principal axes are described by

$$\dot{x}_1(t) = \frac{1}{2}(x_5x_4 - x_6x_3 + x_7x_2), \tag{5a}$$

$$\dot{x}_2(t) = \frac{1}{2}(x_5x_3 + x_6x_4 - x_7x_1), \tag{5b}$$

$$\dot{x}_3(t) = \frac{1}{2}(-x_5x_2 + x_6x_1 + x_7x_4), \tag{5c}$$

$$\dot{x}_4(t) = \frac{1}{2}(-x_5x_1 - x_6x_2 - x_7x_3), \tag{5d}$$

$$\dot{x}_5(t) = I_1x_6(t)x_7(t) + \frac{u_1(t)}{J_1}, \tag{5e}$$

$$\dot{x}_6(t) = I_2x_7(t)x_5(t) + \frac{u_2(t)}{J_2}, \tag{5f}$$

$$\dot{x}_7(t) = I_3x_5(t)x_6(t) + \frac{u_3(t)}{J_3}. \tag{5g}$$

III. Design of Feedback Control System

In this section, we design the feedback control system for stabilizing the rotational motion of a satellite. Let the target state denoted by $\bar{x}$ be set as

$$\bar{x}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Here, we introduce the control law as follows:

$$u_1(t) = -k_1x_1 - c_1x_5, \tag{6a}$$

$$u_2(t) = -k_2x_2 - c_2x_6, \tag{6b}$$

$$u_3(t) = -k_3x_3 - c_3x_7, \tag{6c}$$

where k_1, k_2, k_3 and c_1, c_2, c_3 are positive constants. Next, we examine the stability of the target state using the control inputs (6). Let the following positive-definite function as a Lyapunov function.

$$E = \frac{J_1}{2k_1}x_5^2 + \frac{J_2}{2k_2}x_6^2 + \frac{J_3}{2k_3}x_7^2 + q_1^2 + q_2^2 + q_3^2 + (q_4 - 1)^2 \tag{7}$$

Suppose that k_1, k_2, k_3 are selected so as to satisfy the following condition:

$$\frac{J_2 - J_3}{k_1} + \frac{J_3 - J_1}{k_2} + \frac{J_1 - J_2}{k_3} = 0. \tag{8}$$

Then, the following condition holds true.

$$\dot{E} \le 0 \tag{9}$$

Consequently, based on the Lyapunov stability theory, we can see that if condition (8) is satisfied, then the equilibrium point $\bar{x}$ is globally asymptotically stable for any positive constants c_1, c_2, c_3. Next, we introduce the simple uniform quantizer defined by

$$v(t) = q\left(u(t)\right), \tag{10}$$

where q denotes the static nearest-neighbor quantizer toward $-\infty$ with the quantization interval d as shown in Fig. 1 of [15]. Furthermore, we introduce the random dither quantizer defined by

$$v(t) = q\left(u(t) + \eta(t)\right), \tag{11}$$

where $\eta(t)$ is an independent and identically distributed random variable with the uniform probability distribution on $[-d/2, d/2)$.

A schematic view of quantized control system using the simple uniform quantizer (10) is shown in Fig. 1. In contrast, a schematic view of quantized control system using the random dither quantizer (11) is shown in Fig. 2.

Hereafter, we consider the attitude control problem of satellites with quantized control inputs governed by the

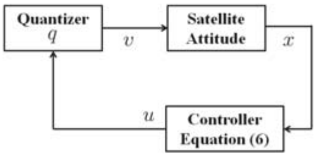

Fig. 1 A schematic view of system using the simple uniform quantizer (10)

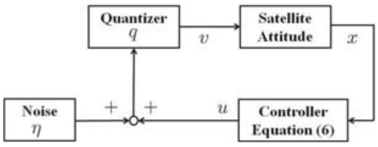

Fig. 2 A schematic view of system using the random dither quantizer (11)

following equations:

$$\dot{x}_1(t) = \frac{1}{2}(x_5x_4 - x_6x_3 + x_7x_2), \tag{12a}$$

$$\dot{x}_2(t) = \frac{1}{2}(x_5x_3 + x_6x_4 - x_7x_1), \tag{12b}$$

$$\dot{x}_3(t) = \frac{1}{2}(-x_5x_2 + x_6x_1 + x_7x_4), \tag{12c}$$

$$\dot{x}_4(t) = \frac{1}{2}(-x_5x_1 - x_6x_2 - x_7x_3), \tag{12d}$$

$$\dot{x}_5(t) = I_1x_6(t)x_7(t) + \frac{v_1(t)}{J_1}, \tag{12e}$$

$$\dot{x}_6(t) = I_2x_7(t)x_5(t) + \frac{v_2(t)}{J_2}, \tag{12f}$$

$$\dot{x}_7(t) = I_3x_5(t)x_6(t) + \frac{v_3(t)}{J_3}. \tag{12g}$$

IV. Main Results

In this section, we show the control performances of satellites with quantized controls governed by (12) for both cases of the SUQ (simple uniform quantizer) and the RDQ (random dither quantizer). The parameters employed in the numerical simulations are as follows: $J_1 = 1$, $J_2 = 2$, $J_3 = 3$, $k_1 = k_2 = k_3 = 1$, $c_1 = c_2 = c_3 = 1$, $d = 1$. Time responses of the state x of quantized control system (12) for both cases of the SUQ and the RDQ are shown in Figs. 3–9. Those figures verify the effectiveness of the proposed RDQ method. We can see from Figs. 3–9 that the proposed RDQ method exhibits much better performance than the SUQ method for attitude stabilization of a satellite. Figs. 10–12 show the difference between the quantized control inputs using the SUQ method and the RDQ method. We can see from Figs. 10–12 that the quantized control inputs using the proposed RDQ method are well adjusted to decrease the quantization errors rather than the ones using the SUQ method.

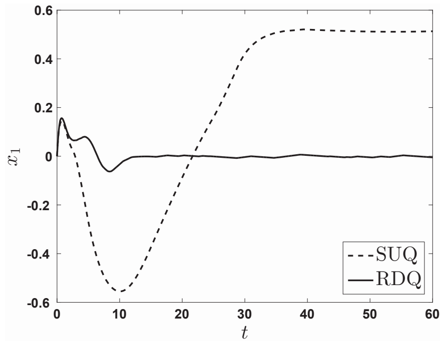

Fig. 3 Time responses of x_1 for both cases of SUQ and RDQ

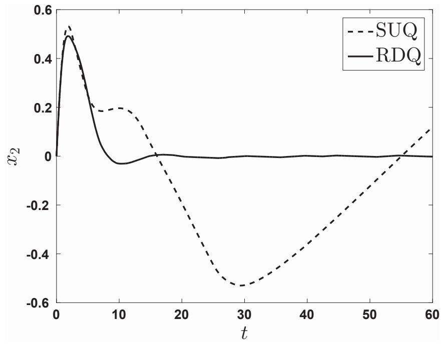

Fig. 4 Time responses of x_2 for both cases of SUQ and RDQ

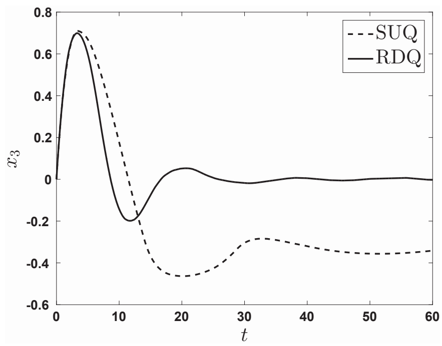

Fig. 5 Time responses of x_3 for both cases of SUQ and RDQ

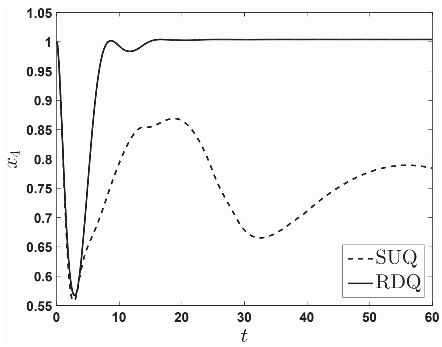

Fig. 6 Time responses of x_4 for both cases of SUQ and RDQ

V. Conclusion

In this study, we have examined a design method of quantized control systems for nonlinear dynamics. The

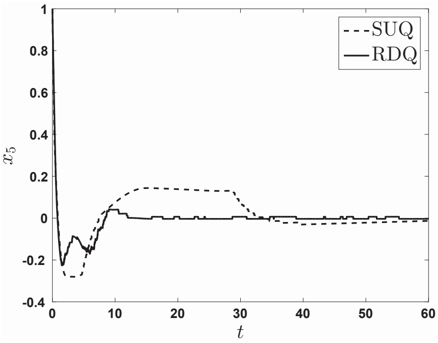

Fig. 7 Time responses of x_5 for both cases of SUQ and RDQ

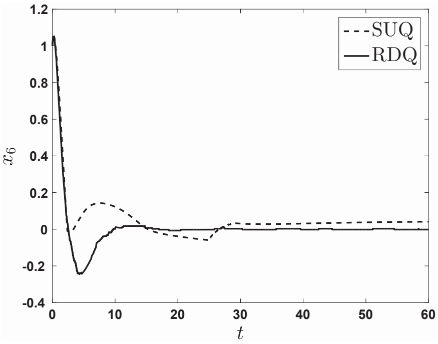

Fig. 8 Time responses of x_6 for both cases of SUQ and RDQ

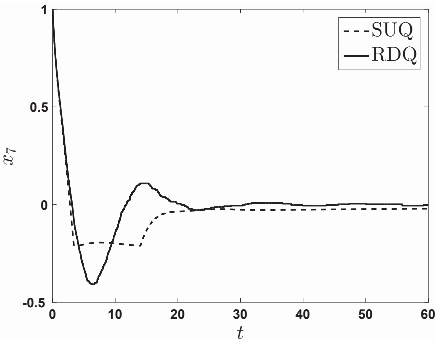

Fig. 9 Time responses of x_7 for both cases of SUQ and RDQ

approach shown here is based on the random dither quantization method that transforms a given continuous-valued signal to a discrete-valued signal by adding artificial random noise to the continuous-valued signal before quantization. It has been shown that the random dither quantization method exhibits much better performance than the simple uniform quantization method for attitude control of satellites. The results of numerical simulations were provided to verify the effectiveness of the proposed method. It is known that not only quantization errors but also time delays may cause instabilities of control systems and lead to more complex analysis [20]-[25]. The stabilization problem of random dither quantized systems with time delays is a possible future work.

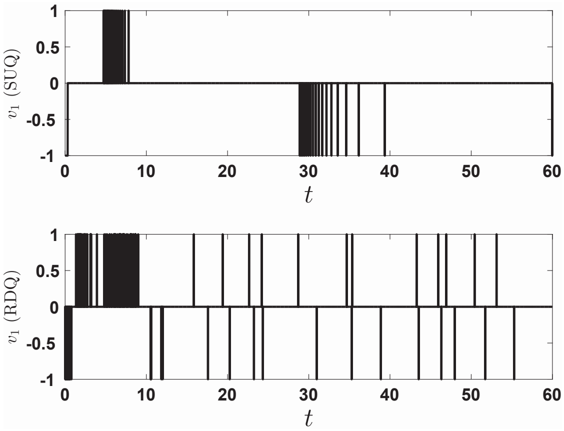

Fig. 10 Time responses of v_1 for both cases of SUQ and RDQ

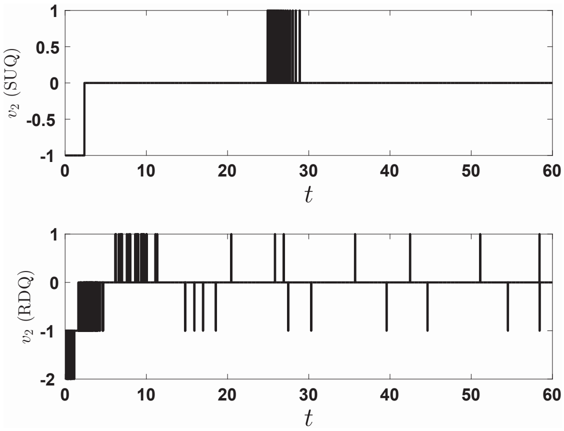

Fig. 11 Time responses of v_2 for both cases of SUQ and RDQ

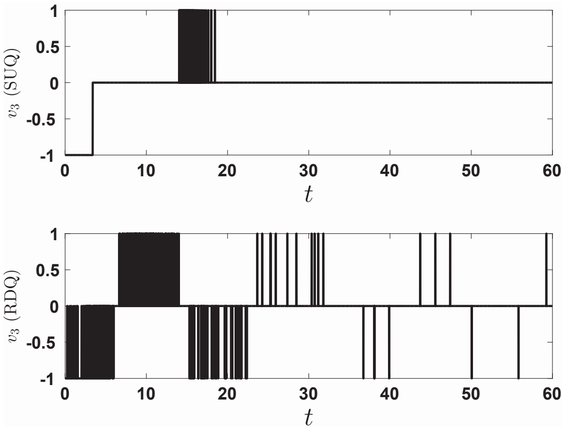

Fig. 12 Time responses of v_3 for both cases of SUQ and RDQ

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