International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:13, No:7, 2019

Assessment of Analytical Equations for the Derivation of Young's Modulus of Bonded Rubber Materials

Z. N. Haji, S. O. Oyadiji, H. Samami, O. Farrell

Abstract**—**The prediction of the vibration response of rubber products by analytical or numerical method depends mainly on the predefined intrinsic material properties such as Young's modulus, damping factor and Poisson's ratio. Such intrinsic properties are determined experimentally by subjecting a bonded rubber sample to compression tests. The compression tests on such a sample yield an apparent Young's modulus which is greater in magnitude than the intrinsic Young's modulus of the rubber. As a result, many analytical equations have been developed to determine Young's modulus from an apparent Young's modulus of bonded rubber materials. In this work, the applicability of some of these analytical equations is assessed via experimental testing. The assessment is based on testing of vulcanized nitrile butadiene rubber (NBR70) samples using tensile test and compression test methods. The analytical equations are used to determine the intrinsic Young's modulus from the apparent modulus that is derived from the compression test data of the bonded rubber samples. Then, these Young's moduli are compared with the actual Young's modulus that is derived from the tensile test data. The results show significant discrepancy between the Young's modulus derived using the analytical equations and the actual Young's modulus.

Keywords**—**Bonded rubber, quasi-static test, shape factor, apparent Young's modulus.

I. Introduction

RUBBER materials are considerably used as engineering materials in mechanical, civil, and aerospace engineering applications to control vibrations and noise. This is not only due to their capability to extend (or compress) hyperelastically to high strain, but also due to their ability to dissipate energy owing to their viscoelastic characteristics.

In principle, the properties of rubber products, such as rubber isolators and seals, can be controlled (or improved) in two ways: chemically by creating a rubber compound, which involves mixing and heating various ingredients together and mechanically by geometrical design of the product according to the required specifications for an application. However, the flexibility of controlling properties chemically is limited because it depends on the percentage of added ingredients which produces a desirable change in a property but often has a negative effect on other properties. Therefore, engineers have headed for changing the physical properties of rubber products through geometrical and boundary conditions of rubber products in applications as a second way of modifying rubber properties. For instance, bonding a layer of rubber between rigid surfaces changes the compression stiffness by orders of magnitude.

The knowledge of the intrinsic material properties, namely Young's modulus, damping factor and Poisson's ratio, is fundamental in predicting the vibration response of rubber products using analytical or numerical method. These properties need to be determined experimentally. One of the common experimental test procedures uses a bonded rubber sample in which a layer of the rubber material is bonded to rigid metal end plates. Compression tests on such a sample yield an apparent Young's modulus which is greater in magnitude than the intrinsic Young's modulus of the rubber material because of the restrained motion of the upper and lower bonded plate surfaces [1]-[8]. As a result, many studies have been carried out to produce an applicable mathematical model for the bonded rubber layers [3]-[12], [14].

Theoretical relationships, which are based on the theory of a train-independent Young's modulus and the theory of bonded rubber at a small strain, have been applied to the characterization of the load-deflection compression of rubbers at high strains. The mathematical expressions cover various rubber geometries such as rubber blocks, rings, spheres and rollers [5], [6]. In [9] the linear theory of elasticity was used to predict the behavior of constrained elastic cylinders under axial loads. A set of infinite orthogonal Bessel and trigonometric functions was used. The presented solutions and analysis satisfied all boundary conditions of constrained elastic cylinders.

For a bonded rubber with a circular cross-section between rigid plates, an analytical equation was derived in [5] in the form:

$$E_a = E(1 + \beta S^2) \quad (1)$$

where E_a is apparent modulus without bulk modulus effect, E is Young's modulus, β is a numerical constant and S is the ratio of one loaded surface to the force-free surface, so called shape factor. The shape factor of a cylindrical rubber of diameter d and height l is equal to $d/4l$; the shape factor of a rectangular rubber block with sides a and b and thickness t is equal to $ab/2t(a+b)$.

Z. N. Haji is with School of Mechanical, Aerospace & Civil Engineering, University of Manchester, Manchester M13 9PL, uk and with Farrat Isolevel Limited, Balmoral Road, Altrincham, Cheshire WA15 8HJ, UK (phone: +44(0)1613062352; e-mail: zyad.haji@manchester.ac.uk).
S. O. Oyadiji is with School of Mechanical, Aerospace & Civil Engineering, University of Manchester, Manchester M13 9PL, UK (phone: +44(0)1612754348; e-mail: s.o.oyadiji@manchester.ac.uk).
H. Samami is with Farrat Isolevel Limited, Balmoral Road, Altrincham, Cheshire WA15 8HJ, UK (phone: +44(0)1619241600; e-mail: KS@farrat.com).
O. Farrell is with Farrat Isolevel Limited, Balmoral Road, Altrincham, Cheshire WA15 8HJ, UK (phone: +44(0)7887760085; e-mail: OF@farrat.com).

The contribution of the bulk modulus to the total deformation of the compressed bonded rubber becomes noticeable when the shape factors S becomes very large. Therefore, [5] derived an equation to account for this effect:

$$\frac{1}{E'_a} = \frac{1}{E(1+\beta S^2)} + \frac{1}{K} \tag{2}$$

where K is the Bulk modulus and E'_a is apparent modulus with bulk modulus effect

Reference [5] derived (1) and (2) based on the assumptions that: (a) the material is virtually incompressible in terms of volume, (b) the cross-section of the block is normal to the direction of the applied load and remains plane and horizontal through the deformation, (c) the free surfaces bulge in parabolic forms. However, Horton et al. omitted the assumption that the free vertical surfaces manifest parabolic profile and developed a new expression as [10], [11]:

$$\frac{1}{E'_a} = \frac{1}{E}\left(1 - S\sqrt{\frac{2}{3}}\tanh\left(\frac{1}{S}\sqrt{\frac{3}{2}}\right)\right) + \frac{1}{K} \tag{3}$$

Then, Horton et al. [10] found a satisfactory agreement between the results of (3) and its approximated expression:

$$\frac{1}{E'_a} = \frac{1}{E(1.2+2S^2)} + \frac{1}{K} \tag{4}$$

For $B = \infty$ and incompressible rubber block with circular or square shape, (3) can be expressed as:

$$E_a = \frac{E}{1 - S\sqrt{\frac{2}{3}}thanh\frac{1}{S}\sqrt{\frac{3}{2}}} \tag{5}$$

and (4) thus:

$$E_a = E(1.2 + 2S^2) \tag{6}$$

In the context of Poisson's ratio influence, Williams and Gamonpilas [12] derived analytical equations with Poisson's ratio and shape factor parameters. The derivation of the equation was based on the Timoshenko and Goodier equilibrium equations [13]. This equation has the form:

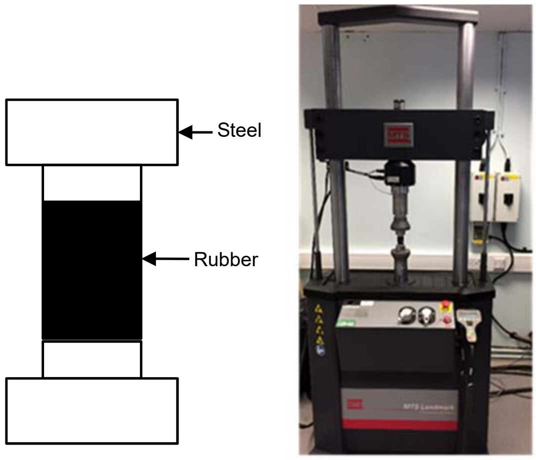
$$\frac{E_a}{E} = \frac{1+3v(\frac{1-v}{1+v})S^2}{1+3v(1-2v)S^2} \tag{7}$$

Reference [7] investigated the aforementioned theoretical equation and evaluated the Young's modulus of rubber-like materials bonded to rigid surfaces. The investigation was carried out theoretically using finite element method (FEM) to predict the frequency response functions (FRFs) of the rubber-like bonded sample. The results showed a significant difference between the Young's modulus determined from the FRFs, predicted by the FEM, and the actual Young's modulus that was used in the FEM analysis.

In this work, the analytical equations (1), (5) and (6) are assessed for the vulcanized nitrile butadiene rubber NBR70. A cylindrical sample of the material is bonded to steel plates and subjected to quasi static compression test. Also, the material is subjected to a tensile test. The analytical equations are used to determine the intrinsic Young's modulus from the apparent Young's modulus derived from the compression test results of the bonded rubber samples which include the effects of shape factor. Then, these values are compared with the intrinsic Young's modulus derived from the tensile test results which have no shape factor effect. The results show significant discrepancy between the Young's modulus derived using the analytical equations and the actual Young's modulus, which is determined from the experimental tensile test results.

II. Experimental Procedures

This investigation is based on testing a vulcanised nitrile butadiene rubber (NBR70) of hardness 70 IRHD under static compression and tensile test deformation methods. All test pieces were cut from the vulcanised rubber NBR70 which comprises of the following ingredients by weight (phr): polymer 41.77, carbon black 41.77, plasticiser 9.86, anti-degradants/activators 3.34, and curatives 3.26.



(a) (b)

Fig. 1 Compression sample test set-up: (a) a bonded compression sample, (b) Sample set up in Material Testing Machine (MTS)

For the compression test, four samples of size 30 mm diameter and 40 mm length, in cylindrical shape, were cut from the vulcanized rubber. The samples were tightly glued to cylindrical steel end plates as shown in Fig. 1 (a). To avoid scragging effect contribution, the samples were mechanically preconditioned up to strain of 30% for 8 consecutive cycles with velocity of 10 mm/min using the MTS at Farrat Isolevel Company (see Fig. 1 (b)).

For the tensile test, four strips each of size 140 mm total length, 100 mm gauge length, 6.2 mm width and 2.2 mm thickness were cut from the vulcanized rubber NBR70 and tested using the tensile test machine type Zwick/Roell -Z010 at the university of Manchester (see Fig. 2). The samples were tested up to 30% strain with velocity 10 mm/min. All the samples for both tensile and compression were cut from one batch in order to eliminate inconsistency that results from using different batches.

(a) (b)

Fig. 2 Tensile sample test set-up: (a) A tensile sample, (b) Sample set up in tensile testing machine Zwick/Roell - Z010

III. Methodology

The average force-deformation curves of four tested samples in each of compression test and tensile test were determined. The Young's modulus was evaluated from apparent compression modules of bonded rubber samples using (1), (5) and (6) of Gent & Lindley [5], Horton et al. [10] and Horton approximated [10], [11], respectively. The calculations of measurements were based on one shape factor value and no bulk modulus contribution. The derived Young's moduli from the analytical equations were compared with the tensile Young's moduli in order to assess the applicability of each of the analytical equation to produce realistic Young's modulus as an intrinsic property. The determination of intrinsic property of rubber materials, namely Young's modulus, using the tensile test is more realistic than using the compression test because the latter is affected by geometrical shape and boundary conditions which produces an apparent Young's modulus that is higher in magnitude than the true Young's modulus.

IV. Results and Discussions

A. Stress-Strain Characteristics

To understand the strength and stiffness of a sample of a material, the stress-strain curve is required which is created from the force and the corresponding measured deformation of the sample of the material during a test. The stress and strain are determined by dividing force by unstrained cross-section area (original area) and change in length by original length, respectively. These are called engineering stress-strain, whereas using strained cross-section area (instantaneous area) results in true stress-strain relationship. Based on the classical elasticity theory, which is specified for small strain limits, the difference between engineering and true stresses is not important since the area does not change notably. In rubber, however, the difference between engineering stress and true stress is very significant [14], [15].

The engineering and true stress-strain curves of both the compression and tensile tests are shown in Figs. 3 and 4 respectively. The data are derived based on the rubber elasticity theory. The expression in the form [14]

$$\sigma_o = \frac{E_a}{3}(\lambda^{-2} - \lambda) \tag{8}$$

is used for engineering compression stress, where compression extension ratio λ = 1- e and e is the engineering strain.

For the true compression stress, (8) is multiplied by λ and gives:

$$\sigma_t = \frac{E_a}{3}(\lambda^{-1} - \lambda^2) \tag{9}$$

where, σ_o and σ_t are the engineering and true stress, respectively.

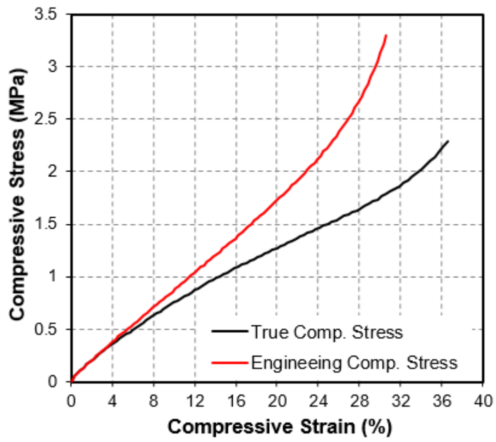


Fig. 3 Engineering and true compression stress- strain relationship

Similarly, (8) and (9) are used for determining tensile engineering and trues stress considering the sign convention rule (also, use Young's modulus, *E*, rather than apparent modulus E_a , the latter only for the bonded compression test). Herein, the material is assumed to be incompressible with the theoretical Poison's ratio $\nu = 0.5$.

Fig 3 shows that the engineering and true compression stresses have no difference until 4% compression strain which agrees with the aforementioned states of the classical elasticity theory for small strains. The difference becomes significant beyond 4% strain and increases with strain. Also, the figure shows that the nonlinear effect becomes prominent after 24% strain. This behaviour can be explained by the fact that when the sample is compressed, the height decreases which gives rise to non-homogenous uniaxial compression distribution in the bonded sample and the influence of the shear and bulk moduli becomes significant. Furthermore, the values of true stresses are less than the engineering stress, which proves that the area of the rubber material changes dramatically when it is subjected to a force.

In the case of tensile test, the results in Fig. 4 show that the tensile stress-strain curves have the same linear trend up to strain of 4%. Then, the curves start diverging from each other so that the true stress varies linearly with the strain, and the engineering stress varies nonlinearly with strain. The figure, in addition, indicates that the stress is uniformly distributed throughout the sample in uniaxial tension over whole range of strain. That is, no effect of the clamped ends on the stress distribution along the sample. The results also prove that area of rubber samples changes substantially under loading because the true stress values are higher than the engineering stress values.

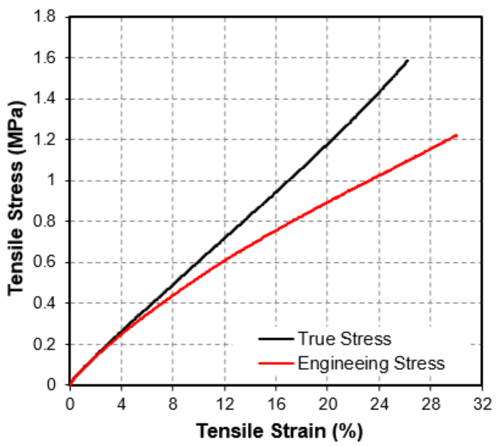


Fig. 4 Engineering and true tensile stress – strain relationship

B. Derived Young's Modulus Properties

The Young's modulus derived using (1), (5) and (6) in Section I and the apparent compression modulus (or effective compression modulus), which was determined from the average engineering stress-strain curves of the compression test, are shown in Fig. 5. The figure shows that all the analytical equations produced a Young's modulus which is less than the apparent modulus. This agrees with the literature that indicates that bonding an elastomer layer to rigid plates increases the stiffness to order of magnitude [1]-[13]. The Young's moduli derived from the analytical equations follow the behaviour of the apparent modulus; it decreases when strain increases up to engineering strain of 28%. Beyond 28% strain, the Young's moduli increased sharply with engineering strain. This is because of the reason that has been mentioned previously: The height of the sample decreases with compression deformation and thereby results in non-homogenous compression distribution throughout the compressed sample. Surprisingly, the discrepancy between the values of compression Young's moduli derived using the analytical equations is very remarkable and creates an ambiguous situation where the correct value of compression Young's modulus is uncertain. This unacceptable difference between the Young's modulus derived using the analytical equations gives rise to an important question to consider: Which of these analytical equations produces the correct Young's modulus?

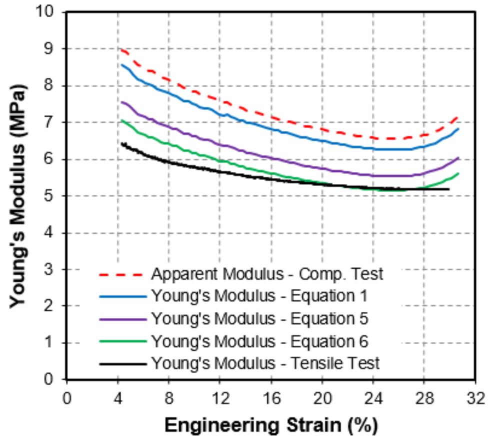


Fig. 5 Comparison of derived Young's moduli with the apparent compression modulus. The results are based on the engineering compression stress – strain data

In order to obtain an answer, tensile test was carried out to produce actual Young's modulus. It is known that tensile test is not severely influenced by geometrical shape and boundary conditions as is the case with compression tests. Fig. 5 shows the comparison of the Young's modulus derived from engineering tensile test data with those derived from engineering compression test data using the analytical equations (hereinafter, the tensile Young's modulus is regarded as the actual Young's modulus). The results show that the actual Young's modulus is less than the apparent modulus and continues to decrease with strain increases. The actual Young's modulus did not manifest a change in its trend beyond 24% strain, whereas the Young's moduli that analytical equations produced started increasing in magnitude after 24% strain. This is because of the boundary conditions of the sample and shape factor effects, which are more influence in compression test than in tensile test. The figure also shows that none of the Young's moduli derived from the compression test data using the analytical equations agreed with the actual Young's modulus except for those derived from (6). Even in this case, however, the agreement is not fairly satisfied but it is better than (1) and (5) for the derivation of Young's modulus from the apparent modulus of the bonded rubber materials.

Similarly, the true stress-strain data of both compression and tensile tests are used to compare the Young's moduli derived from the analytical equations with the actual Young's modulus from tensile test. The results of compression in Fig. 6 show that there is no much perceptible difference between using engineering stress data and true stress data. In addition, Figs. 5 and 6 show that Young's modulus decreases with strain even at small strain where Young's modulus is usually quoted for rubbers. This is rather strange. In fact, Fig. 4 shows that the true tensile stress-strain behaviour is linear up to 26%

strain. This implies that the tensile Young's modulus should remain constant over this range of strain. However, Fig. 5 shows that the tensile Young's modulus decreases with strain which should not be the case. Therefore, this suggests that perhaps the theory of rubber elasticity that was applied in the derivations of (8) and (9) may not be applicable when deriving Young's modulus from experimental data. This may be the case, since the theory of rubber elasticity is based on the micromechanics of rubber. For analysis of real practical samples, a micromechanics approach may be necessary.

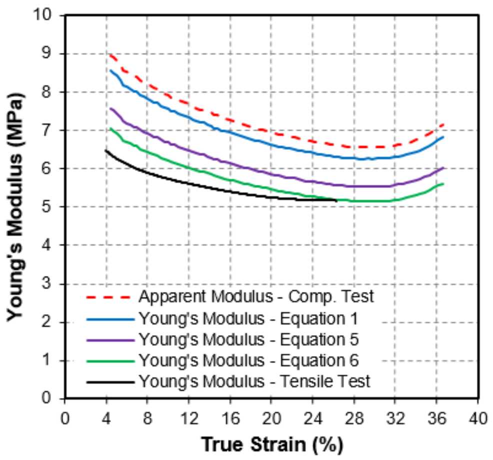


Fig. 6 Comparison of the actual tensile Young's modulus with the derived Young's moduli, and the apparent compression modulus. The results are based on the true stress – strain data

V. Conclusions

The applicability of three analytical equations, which are used to derive Young's modulus from the apparent modulus that is obtained from compression tests on bonded rubber, was investigated experimentally on compression samples of vulcanised rubber NBR70. For comparison, the true (or actual) Young's modulus was derived from experimental tests carried out on a tensile sample. All the analytical equations derived Young's modulus values that were less than the apparent modulus but with very noticeable differences in values between each other. Equation (5) derived the Young's modulus lower than (1) but higher than (6). The derived Young's moduli from all the theoretical equations followed the behaviour of the apparent modulus, which decreases when strain increases.

The actual Young's modulus values that were derived from the tensile test are much lower than the Young's moduli that were derived from all the analytical equations. That is, none of the analytical equations derived Young's modulus values that were close to the actual Young's modulus. Generally speaking, (6) is better than (1) and (5) for the derivation of the Young's modulus of the bonded rubber materials from the apparent compression modulus.

The results suggest that the Young's moduli decrease with strain. However, this does not seem valid because the true tensile stress-strain behaviour is linear up to 26% strain. This requires further investigation in future work.

Acknowledgment

This work is supported by Innovate UK and Farrat Isolevel Ltd.

References

[1] J.C. Snowdon, ''Vibration isolation: Use and characterization'', The Journal of the Acoustical Society of America, vol.66, pp.1245-1274, 1979.

[2] H. C. Tsai and C. C. Lee, ''Compressive stiffness of elastic layers bonded between rigid plates'', International Journal of Solids and Structures, vol. 35, no. 23, pp. 3053-3069, 1998.

[3] C.G. Koh and H.L. Lim, ''Analytical solution for compression stiffness of bonded rectangular layers'', International Journal of Solids and Structures, vol. 38, no. 1, p. 445-455, 2001

[4] H.C. Tsai, ''Compression analysis of rectangular elastic layers bonded between rigid plates'', International Journal of Solids and Structures, vol. 42, no. 11-12, pp. 3395-3410, 2005.

[5] A.N. Gent and, P.B. Lindley, ''The compression of bonded rubber blocks'', Proceedings of the Institution of Mechanical Engineers, vol. 173, no. 1, pp. 111-122, 1959

[6] P.B. Lindley, ''Load-compression relationship of rubber units'', Journal of strain analysis, vol. 1, no. 1, pp. 190-195, 1966.

[7] D. Koblar, J. Skofic, and M. Boltezar, ''Evaluation of the Young's Modulus of Rubber-Like Materials Bonded to Rigid Surfaces with Respect to Poisson's Ratio'', Journal of Mechanical Engineering, vol. 60, no. 7-8,pp. 506-511, 2014

[8] S. Qiao and N. Lu, ''Analytical solutions for bonded elastically compressible layers'', International Journal of Solids and Structures, vol. 58, pp. 353-36, 2015.

[9] S.R. Moghe and H.F. Neef, ''Elastic deformations of constrained cylinders'', Journal of strain analysis, vol. 1, no. 1, pp. 190-195, 1966.

[10] J.M. Horton, G.E. Tupholme and M.J.C. Gover, ''Axial loading of bonded rubber blocks'', Journal of Applied Mechanics, vol. 69, no. 6, pp. 836-843, 2002

[11] J.M. Horton, M.J.C. Gover and G.E. Tupholme, ''Stiffness of Rubber Bush Mountings Subjected to Tilting Deflection'', Rubber Chem. Technol., vol. 73, pp. 619–633, 2000.

[12] J.G. Williams and C, Gamonpilas, ''Using the simple compression test to determine young's modulus, Poisson's ratio and coulomb friction coefficient'', International Journal of Solids and Structures, vol. 45, no. 16, pp. 4448-4459, 2008.

[13] D. Koblar and M. Boltezar, ''Evaluation of the frequency-dependent young's modulus and damping factor of rubber from experiment and their implementation in a finite-element analysis'', Journal of Experimental Techniques, vol. 40, no. 1, pp. 235-244, 2013.

[14] L. R. G. Treloar, ''The physics of rubber elasticity'' 3rd ed., Oxford University Press Inc., New York, pp. 59-62, 1975.

[15] A. L. Gent, ''Engineering with rubber: how to design rubber components'', 3rd ed., Carl Hanser Verlag, Munich 2012.