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Approximate Solution of Some Mixed Boundary Value Problems of the Generalized Theory of Couple-Stress Thermo-Elasticity

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***Abstract*—**We have considered the harmonic oscillations and general dynamic (pseudo oscillations) systems of theory generalized Green-Lindsay of couple-stress thermo-elasticity for isotropic, homogeneous elastic media. Approximate solution of some mixed boundary value problems for finite domain, bounded by the some closed surface are constructed**.**

***Keywords*—**The couple-stress thermo-elasticity, boundary value problems.

I. Introduction

PROBLEMS of the connected theory of couple-stress thermo-elasticity are dynamic problems [1]. The general theory of these dynamic problems, involving the proof of the basic existence and uniqueness theorems, is developed on the assumption that the boundaries of the domains under consideration are fixed in the finite part of the space. For the general case when the boundary or some pieces thereof extend to infinity, the boundary and initial-boundary problems of elasticity were studied but little and the progress achieved hitherto in this direction is limited to some particular results. These results are of a more general nature in the case when the boundary of an infinite domain is composed of certain systems of planes or of systems of straight segments (in the plane case); we mean here problems for the half-space and the half-plane, problems for some other parts of the space and the plane.

Assuming the existence of a solution in some, sufficiently wide, class of functions, many problems of such kind may be solved explicitly and verified by a direct substitution. If the results of the verification are favorable, we may expect the solution to be one of the possible solutions in a given class. Such "uncertainty", however, will disappear if the corresponding uniqueness theorems are proved.

Problems of couple-stress thermo-elasticity for which it appears possible to obtain such results are rather great in number. Here will be considered problems for finite domain, bounded by the some closed surface. They are solved explicitly by using the potential method.

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II. Notations and Definitive Concepts

Introduce the notations: Let E_3be three-dimensional Euclidean space, $x = (x_j)$; $y = (y_j)$; $j = 1,2,3$ - points of this space, $D_k \subset E_3$- finite domain, bounded by the closed surface $S_k \subset L_{(2)}(\alpha), \alpha > 0, k = o,\dots,m$ i.e. by the surface with a continuous curvature [1]; $S_k \cap S_j = \emptyset, \alpha > 0, k, j = o,\dots,m$ surface S_0contains all these surfaces; $\overline{D}_k = D_k \cup S_k$; $S = \cup_{k=0}^{m} S_k, D^+ = D_0 \backslash - \cup_{k=1}^{m} \overline{D}_k$; i.e. D^+- finite connected domain with the surface S; $D^- = E_3 \backslash - \cup_{k=1}^{m} \overline{D}_k$- infinite connected domain with the surface $S' = \cup_{k=1}^{m} S_k$.

The model of partial differential equations of the harmonic oscillations and general dynamic (pseudo oscillations) systems of theory generalized Green-Lindsay couple-stress thermo-elasticity for isotropic homogeneous elastic media has the form [2]:

$$\begin{aligned}&(\mu + \alpha)\Delta u + (\lambda + \mu - \alpha) graddivu + 2\alpha rot\omega - \gamma_\tau gradu_7 - \varsigma\tau^2 u = h^{(1)}\\&(\nu + \beta)\Delta\omega + (\varepsilon + \nu - \beta) graddiv\omega + 2\alpha rotu + (I\sigma^2 - 4\alpha)\omega = h^{(2)}\\&\Delta u_7(x,t) + \frac{i\sigma}{\wp_\tau} u_7 + i\sigma divu = h_7\end{aligned} \qquad (1)$$

where,
$u = (u_1, u_2, u_3)$ is the displacement vector,$\omega = (\omega_1, \omega_2, \omega_3)$ is the rotation vector, u_7 is the Temperature variation,$\varsigma, \lambda, \mu, \alpha, I, \varepsilon, \nu, \beta, \gamma_\tau, \wp_\tau, \eta$are elastic and thermal constants of the domain, $\Delta$is the three-dimensional Laplacian operator; σin general, complex parameter; the case$\sigma = p > 0$ corresponds to the harmonic oscillations, while the case $\sigma = i\tau,\ \tau = \sigma + iq, q > 0$ corresponds to the general dynamic problems [1], [2];

$$H = (h^{(1)}, h^{(2)}. h_7) = (h_1, h_2, h_3, .., h_7) \subset C^{0,\alpha}(\bar{D}^+), \qquad \alpha > 0$$

is a given vector of Helder's class.

III. Statement Problem

Problem $M^+(\sigma)$. It is required to find in $D^+$the regular vector $U = (u, \omega, u_7)$- solution of the (1) system with the boundary conditions:

$$\{u(y)\}^+ = F^{(k)}(y), \{\omega(y)\}^+ = G^{(k)}(y), \{u_7(y)\}^+ = f^{(k)}(y),\quad y \in S_k, k = \overline{0, m_1} \qquad (2)$$

$$\{T_1(\partial_y, n)U(y)\}^+ = F^{(k)}(y), \{T^4(\partial_y, n)\omega(y)\}^+ = = G^{(k)}(y), \left\{\frac{\partial u_7(y)}{\partial n}\right\}^+ = f^{(k)}(y), y \in S_k, \qquad (3)$$
$$k = \overline{m_1 + 1, m_2}$$

$$\{u(y)\}^{+}=F^{(k)}(y),\{\omega(y)\}^{+}=G^{(k)}(y),\left\{\frac{\partial u_7(y)}{\partial n}\right\}^{+}=f^{(k)}(y),$$
$$y\in S_k, k=\overline{m_2+1,m_3} \quad (4)$$

$$\{T_1(\partial_y,n)U(y)\}^{+}=F^{(k)}(y),\{T^4(\partial_y,n)\omega(y)\}^{+}=G^{(k)}(y),$$
$$\{u_7(y)\}^{+}=f^{(k)}(y), y\in S_k, k=\overline{m_3+1,m_4} \quad (5)$$

Here

$$F^{(k)}(y)=\left(F^{(k)}_{(1)}(y),F^{(k)}_{(2)}(y),F^{(k)}_{(3)}(y)\right),$$
$$G^{(k)}(y)=\left(G^{(k)}_{(1)}(y),G^{(k)}_{(2)}(y),G^{(k)}_{(3)}(y)\right), f_7^{(k)}(y)\in S_k, k=\overline{0,m}$$

are corresponding, given vector-functions and scalar-functions of classes:

$$F^{(k)}(y),G^{(k)}(y),f^{(k)}(y)\in, C^{1,\alpha}(S_k),\alpha>0, y\in S_k, k=\overline{0,m_1}$$

$$F^{(k)}(y),G^{(k)}(y),f^{(k)}(y)\in, C^{0,\alpha}(S_k),\alpha>0, y\in S_k, k=\overline{m_1+1,m_2}$$

$$F^{(k)}(y),G^{(k)}(y)\in C^{1,\alpha}(S_k), f^{(k)}(y)\in C^{0,\alpha}(S_k)\ \ \alpha>0, y\in S_k,$$
$$k=\overline{m_2+1,m_3}$$

$$F^{(k)}(y),G^{(k)}(y)\in C^{0,\alpha}(S_k), f^{(k)}(y)\in C^{1,\alpha}(S_k),\alpha>0, y\in S_k,$$
$$k=\overline{m_3+1,m}$$

$T_1U=T^{(1)}u+T^{(2)}\omega-\gamma_\tau n u_7$ is the force-stress vector, $T^{(4)}(\omega)$ is the couple-stress vector [1]-[3], $n=(n_1,n_2,n_3)$ is the unit normal vector to the surface S, at the point y, outward toD^{+}; $m_i, i=1,2,3$ are any natural numbers, satisfying the conditions$0\le m_1\le m_2\le m_3\le m$.

Further, differential operators of size 7x7, corresponding to the boundary conditions (3), (4), (5), we shall denote as$R(\partial_y,n), Q(\partial_y,n), P(\partial_y,n)$:

$$R(\partial_y,n)U(y)=\left(T_1(\partial_y,n)U(y),T^4(\partial_y,n)\omega(y),\frac{\partial u_7(y)}{\partial n}\right),$$
$$Q(\partial_y,n)=\left(u(y),\omega(y),\frac{\partial u_7(y)}{\partial n}\right),$$
$$P(\partial_y,n)=\left(T_1(\partial_y,n)U(y),T^4(\partial_y,n)\omega(y),u_7(y)\right)$$

Uniqueness and existence theorems for the problem $M^{+}(\sigma)$ in the works [2], [5] are proved. In the given paper we construct approximate solutions by the generalized Fourier's series method [1], [3].

IV. Approximate Solution

Let us construct auxiliary domains and surfaces. Let $\widetilde{D}_k$be the domain - subset ofD_k: $\widetilde{D}$, $\widetilde{D}_0$be the domain which contains D_0: $\widetilde{D}_0\supset\overline{D}_0$, $\tilde{S}_k$ be sufficiently smooth surface - boundary of $\widetilde{D}_k, k=\overline{0,m}$; $\tilde{S}=\cup_{k=0}^{m}\tilde{S}_k$, $\tilde{S}'=\cup_{k=1}^{m}\tilde{S}_k$; $\{x_k\}_{k=1}^{\infty}$ be everywhere accounted set of the points.

Introduce the matrix $M(y-x,\sigma,\gamma_\tau)=\|M_1,M_2,M_3\|_{7x7}$ determined by the formula:

$$M(y-x,\sigma,\gamma_\tau)=\begin{cases}\Phi(y-x,\sigma,\gamma_\tau), y\in\cup_{k=0}^{m_1}S_k\\ R(\partial_y,n)\Phi(y-x,\sigma,\gamma_\tau), y\in\cup_{k=m_1+1}^{m_2}S_k\\ Q(\partial_y,n)\Phi(y-x,\sigma,\gamma_\tau), y\in\cup_{k=m_2+1}^{m_3}S_k\\ P(\partial_y,n)\Phi(y-x,\sigma,\gamma_\tau), y\in\cup_{k=m_3+1}^{\mathrm{m}}S_k\end{cases}$$

Here $x\in E_3$, $\Phi(y-x,\sigma,\gamma_\tau)$ is the matrix of fundamental solutions of system of the equations (1), which is constructed by the elementary functions [4]. There are proved next theorems:

Theorem 1. Accounted set of the vectors $\{M^j(y-x^k,i\tau,\gamma_\tau)\}_{k=1}^{\infty}, j=1,2,3, y\in S=\cup_{k=0}^{m_1}S_k, \mathrm{Re}\tau>0$ is linearly independent and full in the space $L_2(s)$.

Theorem 2. Accounted set of the vectors $\{M^j(y-x^k,p,\gamma_\tau)\}_{k=1}^{\infty}, j=1,2,3, y\in S=\cup_{k=0}^{m_1}S_k, p>0$ is linearly independent and full in the space $L_2(S)$, if p doesn't equal to proper numbers of the homogeneous problem $M_0^{+}(p)$.

Let $U(x)=U(x,\sigma)$ be the direct solution of the problem $M^{+}(\sigma)$. Using the theorems 1 and 2, by the method of potentials and integral equations [1], [6] can be proved the theorem:

Theorem 3. If $\sigma=i\tau,\mathrm{Re}\tau>0$ or $\sigma=p, p>0$, then for any $\varepsilon>0$ can be found the natural number N_0 such, that when $N>N_0$, in any domain $\overline{D}'\subset D^{+}$, uniformly holds the inequality

$$|U(x)-U^N(x)|<\varepsilon, x\in\overline{D}'$$

where,

$$U^N(x)=\sum_{k=1}^{N}\sum_{j=1}^{k}X_k\,a_k^j\Phi^{e_j}\left(x-x^{\left[\frac{j+2}{3}\right]},\sigma,\gamma_\tau\right)-\frac{1}{2}\int_{D^{+}}\Phi(y-x,\sigma,\gamma_\tau)H(y)\,dy,$$
$$e_k=k-3\left[\frac{k-1}{3}\right], X_k=\int_S X(y)\,\varphi^k(y)ds$$

$X(y)$Is the determined vector, which is expressed by the boundary data of the problem;

$$\varphi^k(y)=\sum_{j=1}^{k}a_k^j\,\psi^j(y), k=\overline{1,\infty}, y\in S$$

is orthonormal system of vectors on S; a_k^j are coefficients of the orthonormalization; $\psi^k(y)=M^{e_k}\left(y-x^{\left[\frac{k+2}{3}\right]},\sigma,\gamma_\tau\right), k=\overline{1,\infty}$.

V. Result and Discussion

Above indicated theorems can be generalized for outward problems$M^{-}(i\tau), M^{-}(p)$ and here the approximate solution of the problem $M^{-}(p)$can be constructed for any$p>0$. Also, can be investigated additive questions, connected to the behavior of the solution at infinite.

Using received results, further investigations have been shown, that the same methods: potential methods, theory of singular integral equations integral transforms of Laplace-Melline and other, also able us to construct approximate solutions for other mixed boundary value problems bounded by the multiply closed surface. Here we shall not discuss this in more detail

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