International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:11, 2010

Application of Pearson Parametric Distribution Model in Fatigue Life Reliability Evaluation

E. A. Azrulhisham, Y. M. Asri, A. W. Dzuraidah, A. H. Hairul Fahmi

***Abstract*—**The aim of this paper is to introduce a parametric distribution model in fatigue life reliability analysis dealing with variation in material properties. Service loads in terms of response-time history signal of Belgian pave were replicated on a multi-axial spindle coupled road simulator and stress-life method was used to estimate the fatigue life of automotive stub axle. A PSN curve was obtained by monotonic tension test and two-parameter Weibull distribution function was used to acquire the mean life of the component. A Pearson system was developed to evaluate the fatigue life reliability by considering stress range intercept and slope of the PSN curve as random variables. Considering normal distribution of fatigue strength, it is found that the fatigue life of the stub axle to have the highest reliability between 10000 – 15000 cycles. Taking into account the variation of material properties associated with the size effect, machining and manufacturing conditions, the method described in this study can be effectively applied in determination of probability of failure of mass-produced parts.

***Keywords*—**Stub axle, Fatigue life reliability, Stress-life, PSN curve, Weibull distribution, Pearson system

I. Introduction

STUB axle is a part of the constant-velocity (CV) system assembly. The CV system transfer engine power from the transaxle to the wheels and the function of the stub axle is to support this power transmission so that the automotive can move forward or backward by rotating the wheel. Repeated impact on the stub axle will applies cyclic load to the component when driving through a bumpy road. This cyclic load will affect the fatigue life of the stub axle. In this case, the fatigue life reliability assessment of these components with respect to fatigue failure is of great importance for the safety, efficiency and availability of the system.

In terms of fatigue life analysis of a mechanical structure, response of structure or components towards load patterns is usually expressed as a strain or stress time history. In case where the scatter in fatigue life was neglected it is sufficient to know the relationship between load and life using typical SN relationship. In this approach, fatigue life is predicted by associating the information from the cycle counting (typically represented by the rainflow matrices) of the variable amplitude service loads and the material properties of the component represented by SN curve [1].

However, in terms of mass production, fatigue properties of material used in the fabrication of components cannot be exactly consistent in quality even if ordering of the material is made with the same material specification. Material properties of components used in the fabrication cannot be exactly consistent due to uncertainties associated with the size effect, machining and manufacturing conditions. These uncertainties factors should be considered as random variables that results in variation of the fatigue life curves.

Statistical trends about the fatigue life can be acquired from fatigue experiment. The stair-case method is the most well known procedure to obtain an estimate of the mean value and the standard deviation [2]. This approach is inappropriate due to the increasing pressures of shortened development cycles and the desire to save costs since it require long lasting test in order to obtain a reasonable confidence level.

In dealing with variation of the fatigue life due to uncertainties in mechanical properties, several researchers and organizations over the last 50 years have accumulated statistical distribution of material property data. However, property data is still not available for many materials or is not made generally available by the manufacturer of the product [3]. In general, the variation in material properties which characterized the fatigue life curve of the material is assumed to be normally distributed for it is a reasonable model for many processes or physical properties [4], [5]. Although this may be considered to be reasonable, it should be recognized that the actual distribution function is not really known [6].

In this study, variation in the slope and intercept of the fatigue life curve of a stub axle which characterized the deviation in fatigue life is selected as random variables. Pearson parametric statistical model is used to provide approximate of random variables based on the distribution properties of the fatigue life. Fatigue life of the component under random loading conditions is estimated using rainflow cycle counting, PSN curve, and cumulative damage accumulation method. Distribution family of fatigue life estimates by variation in fatigue life curves can be identified using Pearson's criterion. Probability density function of the fatigue life estimates is calculated using statistical moments of

E. A. Azrulhisham is with the Malaysia France Institute, Universiti Kuala Lumpur , 43650 Selangor Malaysia (phone: +603-8926-2022; fax: +603-8925-8845; e-mail: engku@mfi.unikl.edu.my).

M. Y. Asri is with the Faculty of Mechanical Engineering, Universiti Teknikal Malaysia, 76109, Malacca, Malaysia (e-mail: mohd.asri@utem.edu.my).

A. W. Dzuraidah is with the Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, 43650 Selangor Malaysia (e-mail: dzuraida@eng.ukm.my).

A. H. Hairul Fahmi is with the Engineering Division, Perusahaan Otomobil Nasional Sdn Bhd (PROTON) 40918 Selangor Malaysia (e-mail: fahmiah@proton.com).

the identified distribution and the fatigue life reliability is then calculated from the obtained probability density function.

II. METHODOLOGY

A. Finite Element Analysis and Materials

In this study, critical stress location of the stub axle was identified by developing Finite Element (FE) model based on MSC/PATRAN™ and MSC/NASTRAN™. The inputs to the process are an FE model of the component, a set of cyclic material properties and a set of sinusoidal excitation bending loads ranging from 1000N to 7000N. Imported solid model of the stub axle from CATIA™ was meshed using second order tetrahedral element (TET10) topology and linear static analysis of model was performed using MSC/NASTRAN™. The results were then evaluated using MSC/PATRAN™. The critical stress location of the stub axle is shown in Figure 1.

Fig. 1 Critical stress location

The stub axle material is medium carbon steel JIS S48C. The chemical composition and mechanical properties for the JIS S48C steel are shown in Tables 1 and 2, respectively.

TABLE I
CHEMICAL PROPERTIES OF JIS S48C

Element	C	Cr	Si	Mn	Fe	Cu
Max %	0.47	0.18	0.25	0.85	98.07	0.08

TABLE II
MECHANICAL PROPERTIES OF JIS S48C

Ultimate tensile strength	752.18 MPa
Yield strength	578.32 MPa
Modulus of elasticity	200000 MPa
Density	7.850E-06 kg/mm^3

B. Monotonic Tension Test

A sample of ten units of stub axle was subjected to a set of four different level of cyclic bending fatigue load. This is achieved by clamping the four mounting points at the base of the spindle with a 2-ton clamping mechanism. The cylindrical end of the stub axle was attached to a load arm, which will be connected to a motor with an eccentric mass to induce a moment. A mixture of zinc oxide powder with glycerin was painted on the critical stress location in order to ease the detection of crack initiation.

C. Vehicle Instrumentation

Mechanical and structural behaviors of components subjected to the desired load patterns were observed using micro measurement strain gauges. Strain gauges were strategically positioned at critical stress location of the stub axle to directly reflect the input loads experienced by the component. In addition to strain gauges, accelerometers and displacement transducers were used for data acquisition during the road load data acquisition.

D. Road Load Data Acquisition

Loading sequences in terms of load-time histories of proving ground are acquired using the Road Load Data Acquisition (RLDA). The RLDA activity was established using a vehicle equipped with electronic data acquisition system (EDAQ). Figure 2 illustrated the EDAQ in the form of instrumented suspension system which consists of accelerometers and force transducers which are capable of sensing inclination, vibration and shock experiences by the vehicle's components as it progress along the path of proving ground. In this study, the instrumented vehicle was driven over 1.44 kilometers of the British Millbrook accredited proving ground Belgian pave driving range in order to measure the response-time history. Due to the severe suspension input received, the vehicle was driven with a constant speed of 50 km/h. The Belgian road is commonly used for testing vehicle durability since it has 100 times the severity in comparison with general roads [7]. Several passes of proving ground road surface were collected to ensure a statistically valid and representative sample of data. The specimen responses are simultaneously recorded as a time history on the EDAQ.

Fig. 2 Instrumented suspension system

E. Durability Test Rig

The acquired time history from the proving ground data acquisition is utilized in the system and component level fatigue durability test using spindle coupled full vehicle road simulator. In this study the MTS 320 multi-axial spindle coupled road simulator shown in Figure 3 was used for the laboratory testing. This system allows the excitation of each of the six degree of freedom which is translation in x, y, z and rotation around these axes with the simulation range of up to 50 Hz. Remote Parameter Control RPC® iterative deconvolution technique was used in order to accurately replicate the load time history obtained from the proving ground. The replicated stub axle load-time history for a segment of 1.44km Belgian Pave is shown in Figure 4.

Fig. 3 MTS 320 multi-axial spindle coupled road simulator

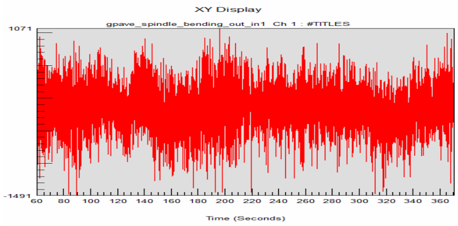

Fig. 4 Replicated stub axle load-time history

F. Fatigue Life Estimation

In this study, fatigue analysis software the nCode Glyphworks™ is applied to predict the fatigue life of the components by combining the information from loads obtained from the road simulator and material properties of the component by fatigue damage accumulation theory. In this case, fatigue life estimates of the stub axle were determined by stress-life (SN) method employing Palmgren-Miner rule along with rainflow cycle counting procedure. This approach estimates number of amplitudes of blocks can be applied before failure occurs. Segmentation of the load- time data was done by implementing a rainflow cycle extraction algorithm in order to segment the load-time histories into maximum and minimum amplitude as well as number of occurrences for certain amplitude ranges. Figure 5 shows the load- time data segmentation in the form of rainflow cycle matrix for the stub axle. Fatigue life of the stub axle was then estimated by combining information from the rainflow cycle extraction of the service loads and the fatigue life curve of the component material.

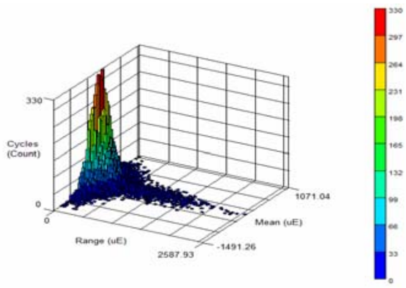

Fig. 5 Rainflow cycle matrix for the stub axle loads

III. RESULTS AND DISCUSSION

A. Probabilistic SN Curve (PSN)

A sample of ten units of stub axle was subjected to monotonic tension test and the result is shown in Table 3. Distribution of fatigue life (crack initiation cycles) was identified using three criterions which is the average goodness-of-fit, plot normalization, and log likelihood function with respective decision weights of 50%, 20% and 30%. It is found that two-parameter Weibull distribution function provides the best fit to crack initiation cycle at each stress levels.

TABLE III
RESULTS OF MONOTONIC TENSION TEST

Sample #	Load Amplitude (N)	Stress Amplitude (MPa)	Crack Initiation Cycle
1	5886	141.15	24697
2	4905	117.63	46286
3	4905	117.63	48667
4	4905	117.63	61399
5	3924	94.10	104205
6	3924	94.10	124484
7	3924	94.10	138049
8	2943	70.57	870777
9	2943	70.57	653570
10	2943	70.57	426669

The probability distribution function (PDF) of two-parameter Weibull distribution is represented by Equation 1 where α and β is scale and shape parameters, respectively.

$$f(x) = \frac{\beta}{x}\left(\frac{x}{\alpha}\right)e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \qquad (1)$$

The result of monotonic tension test was divided in terms of number of crack initiation cycles corresponding to each stress level. The scale and shape parameter of Weibull distribution for each stress levels is then computed using Bernard's median rank and regression analysis. The result is shown in Table 4.

TABLE IV
WEIBULL DISTRIBUTION PARAMETERS

Stress Amplitude (MPa)	Crack Initiation Cycles	Scale Parameter, α	Shape Parameter, β
70.57	426669	739930	2.68
	653570		
	870777		
94.10	104205	129910	6.79
	124484		
	138049		
117.63	46286	55181	7.07
	48667		
	61399		

The probability of failure and the probability of survival for two-parameter Weibull distribution are given by Equation 2 and Equation 3, respectively. In this study, as shown in Figure 6, probabilistic stress-life (PSN) plots were drawn for the values of P_{10}, P_{50} and P_{90} (or R_{90}, R_{50} and R_{10}). The median life value (50% life) is given by the PSN plot of P_{50} (or R_{50}).

$$F_{f(x)} = 1 - e^{-(\frac{x}{\alpha})^{\beta}} \quad (2)$$

$$F_{f(s)} = 1 - F_{f(x)} \quad (3)$$

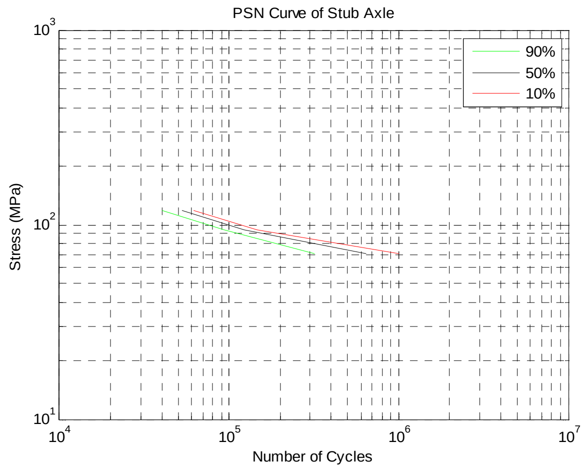

Fig. 6 Probabilistic stress-life (PSN) plots

B. Fatigue Life Reliability

The mean value of material property of the stub axle (represented by PSN plot of 50% survival) has been obtained by a set of monotonic tension test. As a result, the experimental data have the standard deviation and it is difficult to ensure that the actual material used in the fabrication of the stub axle is closely matched to the known mean value. In this case, it is necessary to evaluate the degree of reliability of the estimated fatigue life of the component.

In the case of large number of components, variation in the fatigue life curve which characterized the uncertainties appearing in mechanical properties is known to influence the fatigue performance [8]. The distribution properties of the fatigue life curve can be taken from the expert judgments reported in various literatures [9].

The most well known and classical distribution function is the normal distribution function which characterized by the mean value and the standard deviation. The coefficient of variation which is a normalized measure of dispersion of a probability distribution is known often from experience and depends on the uniformity of the quality of the component [10]. Table 5 shows the coefficient of variation for various materials compiled from a number of sources [3].

TABLE V
COEFFICIENT OF VARIATION FOR VARIOUS MATERIALS

Material Type	Coefficient of Variation
Carbon Steel	0.01 – 0.03
Nodular Cast Iron	0.04
Titanium	0.09
Aluminum	0.03

The fatigue life reliability was evaluated by developing a Pearson statistical model of selected random variables. The Pearson system which is a parametric family of distributions can be used to model a broad scale of distributions with excellent accuracy [11]. Four statistical moments which is the mean, standard deviation, skewness and kurtosis were selected as the first to fourth statistical moments of the Pearson system. Three levels and weight, with respect to each variable, were used in the fatigue life prediction

In the case of stress-life method, the primary factor which influences the fatigue life is the SN curve. In this case, elastic modulus and density is not seriously affects the fatigue life as compared to the SN curve [8]. In the developed Pearson model, three levels and weight with respect to each variable were used to predict the fatigue life. The selected variables are the slope, *n* and the stress range intercept, *a* of the mean life probabilistic SN curve as shown in Figure 7.

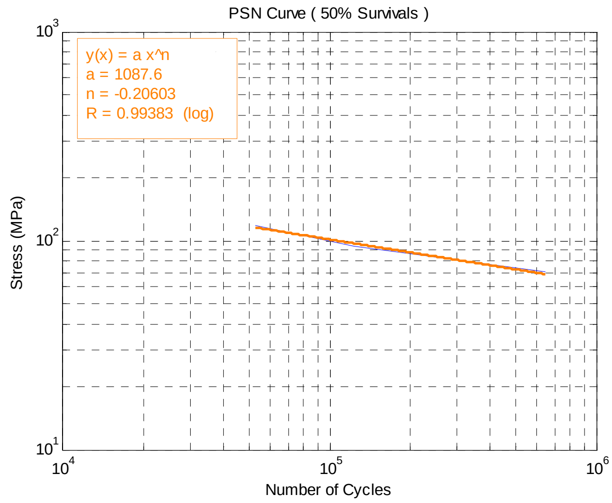

Fig. 7 Mean life probabilistic SN curve

In the case of carbon steel, the variation in material property is typically assumed normally distributed for it is a reasonable model for many natural processes or physical properties [5]. Consequently, the two selected variables are assumed to be the normal distribution with a coefficient of variation of 0.01.

The levels ($l_{1\text{-}3}$) and weights ($w_{1\text{-}3}$) of each variable can be calculated based on the defined moments using Equations (4) and (5). The calculated levels and weights of the random variables are shown in Table 6.

$$\{l_1, l_2, l_3\}^T = \begin{bmatrix} \mu + \frac{\sqrt{\beta_1}\sigma}{2} - \frac{\sigma}{2}\sqrt{4\beta_2 - 3\beta_1} \\ \mu \\ \mu + \frac{\sqrt{\beta_1}\sigma}{2} + \frac{\sigma}{2}\sqrt{4\beta_2 - 3\beta_1} \end{bmatrix} \quad (4)$$

$$\{w_1, w_2, w_3\}^T = \begin{bmatrix} \dfrac{(4\beta_2 - 3\beta_1) + \sqrt{\beta_2}\sqrt{4\beta_2 - 3\beta_1}}{2(4\beta_2 - 3\beta_1)(\beta_2 - \beta_1)} \\ \dfrac{\beta_2 - \beta_1 - 1}{\beta_2 - \beta_1} \\ \dfrac{(4\beta_2 - 3\beta_1) - \sqrt{\beta_2}\sqrt{4\beta_2 - 3\beta_1}}{2(4\beta_2 - 3\beta_1)(\beta_2 - \beta_1)} \end{bmatrix} \quad (5)$$

TABLE VI
LEVEL AND WEIGHT OF RANDOM VARIABLES

Variable	$Level_{1\text{-}3}$	$Weight_{1\text{-}3}$
Stress range intercept, *a*	1068.76	0.1667
	1087.60	0.6667
	1106.44	0.1667
Slope, *n*	0.2025	0.1667
	0.2060	0.6667
	0.2096	0.1667

Since two variables were selected (stress range intercept, *a* and slope, *n*), a total of nine fatigue lives and their weight can be calculated as shown in Table 7. Each fatigue life of the stub axle was calculated by the linear damage rule stress-life method using the stress range intercept, *a* and slope *n* of Table 6 and cycle of the loads obtained from the road simulator. The fatigue life weights are calculated by multiplying each weight (Table 6) with respect to *a* and *n.*

TABLE VII
FATIGUE LIFE RESULTS AND WEIGHTS

a	*n*	Fatigue Life (Cycles)	Weight
1068.76	0.2025	1.373E+04	0.02778
1068.76	0.2060	1.101E+04	0.11111
1068.76	0.2096	8.869E+03	0.02778
1087.60	0.2025	1.499E+04	0.11111
1087.60	0.2060	1.184E+04	0.44444
1087.60	0.2096	9.501E+03	0.11111
1106.44	0.2025	1.618E+04	0.02778
1106.44	0.2060	1.276E+04	0.11111
1106.44	0.2096	1.016E+04	0.02778

The first to fourth statistical moments of the Pearson system were calculated using Equation (6). Table 8 shows the first through fourth moments of the probability density function calculated using nine fatigue life estimates of the stub axle. Equation (7) represents the Pearson's criterion for fixing the distribution family based on the selected statistical moments. The type of the Pearson system and probability density function differs depending on the value of K as shown in Table 9.

$$\{\mu_g, \sigma_g, \sqrt{\beta_{1g}}, \beta_{2g}\}^T = \begin{bmatrix} \sum_{i=1}^{m} q_i \bullet g(L_i) \\ \left\{\sum_{i=1}^{m} q_i (g(L_i) - \mu_g)^2\right\}^{\frac{1}{2}} \\ \left\{\sum_{i=1}^{m} q_i (g(L_i) - \mu_g)^3\right\} / \sigma_g^3 \\ \left\{\sum_{i=1}^{m} q_i (g(L_i) - \mu_g)^4\right\} / \sigma_g^4 \end{bmatrix} \quad (6)$$

TABLE VIII
MOMENTS OF THE FATIGUE LIFE DATA

Mean (μ_g)	11983.97
Standard deviation (σ_g)	1675.19
Skewness ($\sqrt{\beta_{1g}}$)	0.5227
Kurtosis (β_{2g})	3.2035

$$K = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \quad (7)$$

TABLE IX
TYPE OF PEARSON SYSTEM AND PROBABILITY FUNCTION

Type I	Type II	Type III	Type IV	Type V	Type VI	Type VII
K < 1	K = 0, $\beta_1 = 0$, $\beta_2 < 3$	K = ∞, $2\beta_2 = 3\beta_1 - 6 = 0$	0 < K < 1	K = 1	K > 1	K = 0, β1 = 0, β2 > 3
Normal / Beta	Special case of Type I	Chi-square / Gamma	Cauchy	Inverse-gamma	Beta-prime / F	Student's t

Based on the moments calculated in Table 8, it is found that the value of K = - 0.3851 which represents Type I of the Pearson system. The probability density function of the Beta distribution was calculated using MATLAB® statistical toolbox and the probability density function of the stub axle fatigue life is shown in Figure 8.

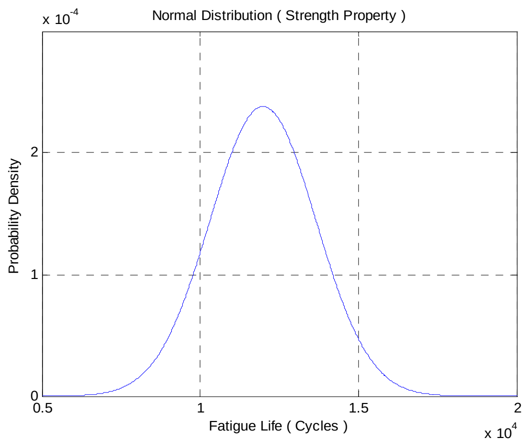

Fig. 8 Probability density function of stub axle

Fatigue life range calculated from the Beta distribution is distributed from 5000 to 20000 cycles. The fatigue life reliability of the stub axle is shown in Table 10. The fatigue life of the stub axle is found to have the lowest reliability between 15000 and 20000 cycles. The highest fatigue life reliability is recorded for 10000 – 15000 cycles.

TABLE X
FATIGUE LIFE RELIABILITY OF STUB AXLE

Cycles	5000 – 10000	10000 - 15000	15000 - 20000
Reliability	0.1182	0.8459	0.0358

IV. CONCLUSION

In this study, the fatigue life of the stub axle is predicted for a passenger car and the predicted fatigue life reliability is evaluated by considering the variations in material properties. The slope and intercept of the mean life SN curve, which mostly affects the fatigue life results, are selected as random variables in the Pearson fatigue life reliability evaluation. It is found that the fatigue life of the stub axle to have the lowest reliability between 15000 and 20000 cycles. The highest reliability is recorded for cycles between 10000 and 15000 cycles which include the 11984 cycles calculated by the mean value of material property.

The use of a statistical method to evaluate the expected life has the advantage that replacement time and failure probability of the parts can be predicted in advance. For example, assuming that a stub axle life has 20000 cycles which is the cycle range of lowest reliability, the vehicle will be safe to travel 288000 km at a speed of 50 km/hour. Since the Belgian Pave has 100 times the severity of the general road, the life of the stub axle is relatively long, compared with the life cycle of the general vehicle.

The method described in this study can be effectively applied in the determination of probability of failure of mass-produced parts where lack of uniformity in quality of the material procured is the main challenge. In this study, the use of a statistical method to evaluate the expected life of an automotive component has the advantage in estimating the replacement time and failure probability of the component.

ACKNOWLEDGMENT

The presented methodologies are parts of research project 'Reliability Prediction in Automotive Component Design' which is supported by the Research and Vehicle Engineering Division, Perusahaan Otomobil Nasional Sdn Bhd (PROTON). This project is partly funded by the Malaysian Ministry of Science, Technology and Innovation (MOSTI) e-science research grant 01-01-02-SF03066.

REFERENCES

[1] R.I. Stephens, A. Fatemi, R.R. Stephens and H.O. Fuchs, *Metal Fatigue in Engineering,* 2nd Edition, Wiley Interscience, New York, 2001.
[2] A guide for fatigue testing and statistical analysis of fatigue data. American Society for Testing and Materials, Philadelphia, ASTM STP No. 91-A; 1963
[3] J.D. Booker, M. Raines and K.G. Swift, *Designing Capable and Reliable Products,* Butterworth-Heinemann, Oxford, 2001.
[4] P.W. Hovey, A.P. Berens and D.A. Skinn, "Risk analysis for aging aircraft", *Flight Dynamic Directorate,* vol. 1, Wright Laboratory, Ohio, October 1991.
[5] R.E. Melchers, *Structural Reliability Analysis and Prediction,* 2nd Edition, John Wiley & Sons Ltd, Chichester, 1999
[6] J. Schijve, "Statistical distribution functions and fatigue of structures", *International Journal of Fatigue,* vol. 7, no. 9, pp. 1031-1039, 2005.
[7] K.J. Jun, T.W. Park, S.H. Lee, S.P. Jung and J.W. Yoon, "Prediction of fatigue life and estimation of its reliability on the parts of an air suspension system", *International Journal of Automotive Technology,* vol. 9, no. 6, pp. 741-747, 2008.
[8] J.A. Bannantine, J.J. Comer and J.L. Handrock, *Fundamentals of Metal Fatigue Analysis,* Prentice Hall, New Jersey, 1989.
[9] B. Sudret, Z. Guede, P. Hornet, J. Stephan and M. Lemaire, "Probabilistic assessment of fatigue life including statistical uncertainties in the SN curve", in *Transactions of the 17th International Conference on Structural Mechanics in Reactor Technology,* Prague, Czech Republic, August 2003.
[10] G. Genet, *A Statistical Approach to Multi-Input Equivalent Fatigue Loads for the Durability of Automotive Structures,* Chalmers University of Technology and Goteborg University, Goteborg, Sweden, 2006.
[11] G.J. Hahn and S.S. Shapiro, *Statistical Models in Engineering,* John Wiley and Sons, New York, 1967.