

# Application of Generalized Stochastic Petri Nets (GSPN) in Modeling and Evaluating a Resource Sharing Flexible Manufacturing System

Aryanejad Mir Bahador Goli, and Zahra Honarmand Shah Zileh

**Abstract**—In most study fields, a phenomenon may not be studied directly but it will be examined indirectly by phenomenon model. Making an accurate model of system, there is attained new information from modeled phenomenon without any charge, danger, etc... there have been developed more solutions for describing and analyzing the recent complicated systems but few of them have analyzed the performance in the range of system description. Petri nets are of limited solutions which may make such union. Petri nets are being applied in problems related to modeling and designing the systems. Theory of Petri nets allow a system to model mathematically by a Petri net and analyzing the Petri net can then determine main information of modeled system's structure and dynamic. This information can be used for assessing the performance of systems and suggesting corrections in the system. In this paper, beside the introduction of Petri nets, a real case study will be studied in order to show the application of generalized stochastic Petri nets in modeling a resource sharing production system and evaluating the efficiency of its machines and robots. The modeling tool used here is SHARP software which calculates specific indicators helping to make decision.

**Keywords**—Flexible manufacturing system, generalized stochastic Petri nets, Markov chain, performance evaluation.

## I. INTRODUCTION

**F**LEXIBLE manufacturing systems include a set of computerized numerical controlling machines and supporting stations which connected to an automatic transportation system and controlled by a central computer. Key elements of this producing system include:

- Automated machines with reprogramming ability;
- Automatically developing the tools for material transportation;
- Automated controlling system.

Designing the flexible producing systems and then analyzing their performance is one of the problems attended long time ago by researchers.

Using Petri net to modeling the flexible manufacturing systems provide the possibility to consider integrally the effects of different elements of producing system (elements like machinery, robots, parts, etc...) and determine the improved exploiting program in the planning phase.

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Using this method, it is also possible to not only consider the time deterministically for calculating the performance of producing unit, but also consider the changes of time in the calculations while are inevitable.

The most fundamental question is: can we use of Petri net to model a producing system. Researchers believe that as there has been allocated a time to Petri nets proportional to the time for activity of each producing unit, if time for all elements of producing unit follow normal distribution or exponential distribution (or finite time) one can model it using Petri net algorithm and as such conditions can be made in a flexible manufacturing system, so we can use of Petri net algorithm for this reason.

Petri nets were introduced for the first time by Carl Adam Petri in 1962 which described cause and effect relations without intending the time considerations. Hullti continued this study on the project for information system theory of applied data. Most developments on symbolization and recent considerations of Petri nets are of results of this project. This project indicated that how Petri net can model and analyze concurrent and consistent elements and compounds, other studies have also attracted the researchers' attention in the Masachusset Researching Institute. "Structures' determination" group, leaded by Deniss, was the origin of most studies and publications in the field of Petri nets [15].

Introducing the time concepts in Petri nets was suggested by C.Ramachandoni, P.M.Merlin, D.S.Farber and J.Sifakis years later it. First definition of Petri nets along with possible time presented by F.Symons, G. Florin, S. Natkinand M. Malloy. Thisplan discussed the possibility of relation between Petri nets and concepts of performance evaluation based on the approach for modeling the stochastic processes. In a developing approach by Malloy, there was combined probable time with deterministic null delays, so both time changes and system's logical changes were described by model, thus such modeling called General Stochastic Petri Nets [1].

This study aims to present a method by which a comprehensive and accurate plan may be obtained for a production system in the planning phase with optimal efficiency. On the other hand, as a production system is going to upgrades to a flexible manufacturing cell, the suggested flexible manufacturing cell is studying as a dynamic system, using a generalized stochastic Petri nets and then it's performance such as machinery efficiency, robots efficiency, inventory in the manufacturing process size and buffers size

will be calculated and then most optimal option will be selected for implementing.

## II. STRUCTURAL COMPONENTS OF PETRI NETS

Structural components of Petri nets include places, transitions and arcs. Places apply for stating the possible states of the system. Transitions apply for stating the approaches which can change the state of systems. Arcs can determine the relation between states and events. Tokens are indistinguishable tokens which located in places and used for stating the states of systems which usually called Petri net marking. If for example a place indicates a provision, it must include a token of null mark. If there is a token in the place, the provision is true otherwise it is false. If a place also defines a situation, the number of marks may be used to determine the situation.

Petri net model is graphically indicated by such procedure that places indicated by circles, transitions by bar and tokens by black spots in the places [1, 2, 3, 4].

Transitions can be divided into two parts: temporal transitions and immediate transitions. Firing a transition in the Petri network model is related to the event changing the state of a real system. This changing in the state can be occurred due to one of the following reasons: either by examining the accuracy of some of conditions in the system or by ending some of activities. Regarding to the second case, transitions remarkably be used for modeling the activities. Thus, when transition is fired, then complementing the activity will be stated. Therefore, temporal concept can naturally be related to the transitions. Temporal transitions are indicated by boxes in the figure, but all events happening in a model of real system may not be related to the time consuming processes. A model of a multi processor system for example, is usually ignored of time for staring activities comparing to the time for doing other activities. Integrating non-temporal transitions in the temporal Petri net models can define a kind of transitions called "immediate". As soon as becoming powerful, immediate transitions will be fired without delay. Immediate transitions thus are preferred than temporal transitions in the Petri nets.

## III. SYSTEM DYNAMIC

In this discussion, there will be dealt with changes and dynamics of markings in Petri nets. These are controlled by the firing transitions and cause losing or adding tokens.

Enabling Rule and Firing Rule are related to transitions. Enabling rule determine situations by which transitions are allowed to fire. Firing rule determine marking changes obtained by firing the transitions. On the other hand, enabling rule determines situations by which a transition allowed to fire and firing rule determine the transformation made by transition.

Both rules are made by arcs. In enabling rule, input and inhibitory arcs are involved, while in the firing rule, input and output arcs are involved.

*Definition1.* A transition is said to be enabled if and only if:

The number of tokens in the input place is greater or equal with frequency of related arc and the number of tokens in the inhibitory place is smaller than arc frequency:

$$\begin{aligned} \forall P \in \bullet t, M(P) &\geq I(t, p), \\ \forall P \in {}^0 t, M(P) &< H(t, p) \end{aligned} \quad (1)$$

As indicated in Fig. 1, transition  $t$  is enabled if and only if the number of tokens in the place  $P_1$  is greater or equal with  $n$  and the number of tokens in the place  $P_2$  is smaller of equal with  $M$ .

When transition  $t$  is fired, from a place in the set  $\bullet t$ , by the number of frequency of arcs connecting it to the place  $t$ , it will clear the tokens and will add to any place in the output set of  $t$ , by the number of frequencies related to the output of transition  $t$  [1].

*Definition2.* Firing the transition  $t$  is enabled in the marking  $M$ , so it makes marking  $M'$  and may be given by [1]:

$$M' = M + O(t) - I(t) \quad (2)$$

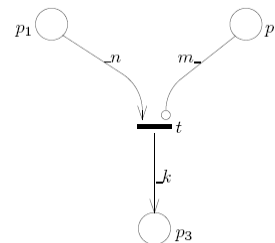


Fig. 1 Enabling and Firing Rules

## IV. SITUATIONS COMMON IN CHANGING THE PETRI NET SYSTEM

### A. Conflict

Petri nets are most suitable for describing the conflicts. Conflict indeed makes a choice determining with Petri net. Fig. 2 indicates that when transition  $t_1$  is firing, a token will be located in the place  $P$  and both transitions  $t_2$  and  $t_3$  will be enabled. When one of them is fired, it will remove the token from place  $P$  and thus another transition will be unable.

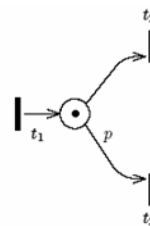


Fig. 2 An example of conflict situation in Petri nets

### B. Concurrency

Concurrency is known by parallel activities.

**Definition3.**  $t_1, t_m$  is said to be concurrent only and only if both are enabled in marking  $M$  and not in conflict with each other.

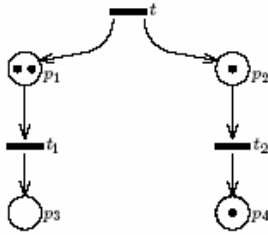


Fig. 3 An example of concurrency situation in Petri nets

### C. Confusion

In marking  $M$  in where  $P_1, P_2$  are marked, transitions of  $t_2, t_1$  are concurrent. If first  $t_1$  is fired, transitions  $t_2, t_3$  will be in conflict together. While, if  $t_2$  is fired initially, there is no conflict. Thus as indicated, if there is begun from marking  $P_1 + P_2$  and attained to marking  $P_4 + P_3$ , it is not necessary to solve a conflict. This is corresponded to the order of firing two transitions  $t_1, t_2$  concurrently. This is called confusion and makes complicated problems [1].

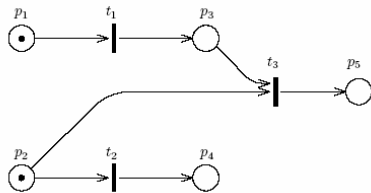


Fig. 4 An example for confusion situation in Petri networks

### D. Mutual Exclusion

As models for Petri networks are used for validity and accuracy of systems, it is important knowing this subject that when two transitions are not enabled in marking together.

**Definition4.** For any Petri net system, transitions  $t_1, t_2$  are in mutual exclusion if:

$$\exists M \in RS(M_0) : t_1 \in E(M) \text{ and } t_m \in E(M) \quad (3)$$

## V. DISCRETE-STATE STOCHASTIC PROCESSES RELATED TO A GENERALIZED STOCHASTIC PETRI NET

Regarding to the property of lack of memory in the exponential distribution function for firing delays, one can indicated that members of reachability set of a generalized stochastic Petri net system are analogous with states space of a Continuous Time Markov Chains (CTMC). Continuous time Markov chain related to a generalized stochastic Petri net system can be obtained from following rules:

1) For continuous time Markov chains, states space of  $S=[S_i]$  based on a generalized stochastic Petri net equals with the reachability set:  $(M_i \rightarrow S_i)$

2) Rate of transition from state  $S_i$  (related to marking  $M_i$ ) to the state of  $S_j$  (related to marking  $M_j$ ) is obtained from summation of firing rate of transitions which are enabled in  $M_i$  and their firing produce the marking  $M_j$ .

Algorithm for automatically producing the matrix  $Q$  (matrix for rate of state transition) for continuous time Markov chain considering  $w_k$  as the rate for firing the transition  $T_k$  and considering  $E_j(M_i) = \{h : T_h \in E(M_i) \wedge E_i[T_h > M_j]\}$  as set of transitions which their firing will take the net from marking  $M_i$  to marking  $M_j$ , is equal with:

$$q_{ij} = \left\{ \sum_{T_k \in E_j(M_i)} w_k \quad i \neq j \right\} \quad (4)$$

$$q_{ii} = \{q_i \quad i = j\}$$

Where

$$q_i = \sum_{T_k \in E(M_i)} w_k$$

Vector  $\eta$  is the steady state probabilities distribution on the markings of generalized stochastic Petri net given by:

$$\begin{cases} \eta Q = 0 \\ \eta 1^T = 0 \end{cases} \quad (5)$$

Where  $0$  is a line vector equal with size  $\eta$  with all its elements equal with zero and  $1^T$  is a column vector equal with  $\eta$  in size with all its elements equal with 1.

## VI. CASE STUDY

### A. Acknowledging with the Producing Process of Studied Cell

Industrial Poly-Met factory is of the polymer devices manufactures in Iran. One of its products is a heater frame of GLX405 (kind of Iranian automobile).



Fig. 5 A heater Frame of GLX405

Required raw material is Polypropylene which after becoming hot, is transferred by an operator to an injection machine ( $M_1$ ) and then injected to its mold. After the completion of operation, the injected pieces are transferred by

an operator to a drilling Machine ( $M_2$ ) and after completion of machine operation; products are moved by an operator to the next production department.

The factory manager is going to improve the production process by purchasing three new automatic robots. So it is very important to evaluate the new production process whether this cause to an increasing in efficiency or not.

The new process is modeled as a flexible manufacturing cell. The shape of related cell has indicated in Fig. 6. As shown, this cell includes two machines  $M_1$  and  $M_2$  with three robots  $R_1$ ,  $R_2$  and  $R_3$ .

Robot  $R_1$  is transferring the raw material from the storing place to machine  $M_1$  and maintains the part in a suitable place for operation in the place of machine  $M_1$ . After completing the related operation by Machine  $M_1$  on the raw material, this robot will be freed from and can repeat the operation again.

Part pulling out of the machine  $M_1$  (semi-manufactured part) may pass one of the following routes based on accessing to the machine  $M_2$  or not accessing to it: either machine  $M_2$  is ready to accept the semi-manufactured machine, thus robot  $R_2$  can bring the semi-manufactured part near the machine  $M_2$  or machine  $M_2$  is occupied with conducting the operation on another semi-manufactured part, then part must be transferred to the buffer  $B_1$ . This transition is done by robot  $R_2$  and then robot  $R_2$  will be freed from. After machine  $M_2$  being released, semi-manufactured part will be transferred to machine  $M_2$  by robot  $R_2$  from buffer  $B_1$  to be operated.

To maintain the semi-manufactured part in the place of machine  $M_2$  and in a suitable situation Robot  $R_3$  is used to operate on it and after operation by machine  $M_2$  on the semi-manufactured part, robot  $R_3$  will pull out the product from machine  $M_2$  and transfer it to the buffer  $B_2$  to be transferred to the next cell.

It is indicated that robot  $R_2$  is a shared source between machine  $M_1$ , machine  $M_2$  and buffer  $B_1$ , while such situation lacks in robots  $R_1$  and  $R_3$ . Based upon this case, there are two different policies for using robots  $R_1$  and  $R_3$  comparing with robot  $R_2$ : robot  $R_1$  is used for replacing the raw materials to machine  $M_1$  and maintaining the raw materials in the suitable situation in the place of machine  $M_1$  and robot  $R_3$  is also used for maintaining the semi-manufactured part in its suitable situation in the place of machine  $M_2$  and then transferring the

product from machine  $M_2$  to buffer  $B_2$ .

This is while robot  $R_2$  is used to transition semi-manufactured part from machine  $M_1$  to the place of machine  $M_2$  as well as transferring it from machine  $M_1$  to buffer  $B_1$  and transferring it from buffer  $B_1$  to machine  $M_2$ . Robot  $R_2$  is freed after transferring the part to the machine  $M_2$ .

#### *B. Suggested Model for Flexible Produced Cell using Generalized Stochastic Petri Net*

Petri net system of production system studied is indicated in Fig. 7. Above mentioned model includes 18 places and 14 transitions. Details of each place and transition have indicated in the following tables:

TABLE I  
PROPERTIES OF PLACES

rank	Place name	Descriptions
1	$P_0$	Presence of raw materials in the warehouse
2	$P_1$	Readiness of robot $R_1$ to replace raw materials from warehouse to machine $M_1$
3	$P_2$	Readiness of machine $M_1$ to inject raw material
4	$P_3$	Entrance of raw materials to machine $M_1$
5	$P_4$	Presence of semi-conducted part (but not determined whether goes to $M_2$ or $B_1$ )
6	$P_5$	Readiness of robot $R_2$
7	$P_6$	Readiness of machine $M_2$ for receiving the semi-manufactured part
8	$P_7$	Presence of semi-manufactured part to be transferred to machine $M_2$
9	$P_8$	Accessibility of semi-manufactured part to be delivered to machine $M_2$ from $M_1$
10	$P_9$	Presence of semi-manufactured part to be transferred to buffer $B_1$
11	$P_{10}$	Not completeness of semi-manufactured asset in the buffer $B_1$
12	$P_{11}$	Accessibility of semi-manufactured part for being delivered to machine $M_2$ from buffer $B_1$

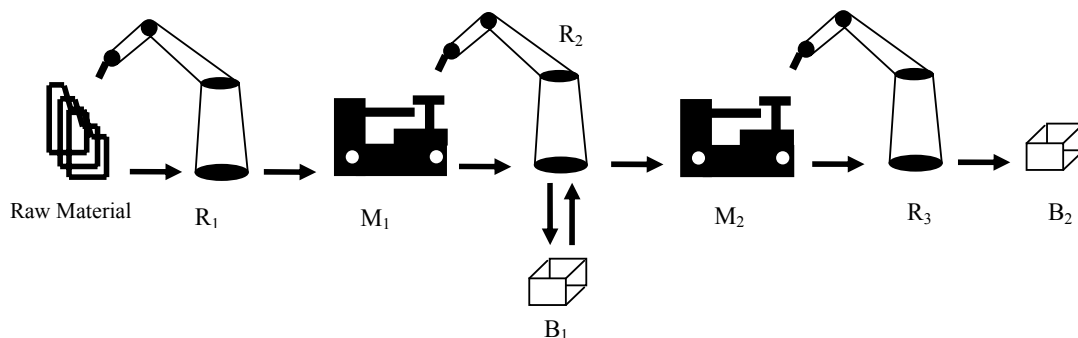


Fig. 6 Suggested Flexible Manufacturing Cell

13	P <sub>12</sub>	Accessibility of semi-manufactured part in the place of machine M <sub>2</sub>
14	P <sub>13</sub>	Accessibility of product to being transferred to buffer B <sub>2</sub>
15	P <sub>14</sub>	Presence of product to being delivered to buffer B <sub>2</sub>
16	P <sub>15</sub>	Readiness of robot R <sub>3</sub>
17	P <sub>16</sub>	Readiness of product to being transferred to the next produced cell
18	P <sub>17</sub>	Presence of semi-manufactured cell to being transferred to buffer B <sub>1</sub>

TABLE II  
PROPERTIES OF TEMPORAL TRANSITION

rank	Place name	descriptions
1	T <sub>0</sub>	Transferred raw material to machine M <sub>1</sub>
2	T <sub>1</sub>	Operation on raw material using machine M <sub>1</sub>
3	T <sub>2</sub>	Transferred semi-manufactured part from machine M <sub>1</sub> to buffer B <sub>1</sub>
4	T <sub>3</sub>	Transferred semi-manufactured part from machine M <sub>1</sub> to machine M <sub>2</sub>
5	T <sub>4</sub>	Delivering semi-manufactured part to machine M <sub>2</sub> from machine M <sub>1</sub>
6	T <sub>5</sub>	Delivering the semi-manufactured part to buffer B <sub>1</sub>
7	T <sub>6</sub>	Delivering semi-manufactured part to machine M <sub>2</sub> from buffer B <sub>1</sub>
8	T <sub>7</sub>	Operation on the semi-manufactured part by machine M <sub>2</sub>
9	T <sub>8</sub>	Transferred product from machine M <sub>2</sub> by robot R <sub>3</sub>
10	T <sub>11</sub>	Transferred product to machine M <sub>2</sub> from buffer B <sub>1</sub>

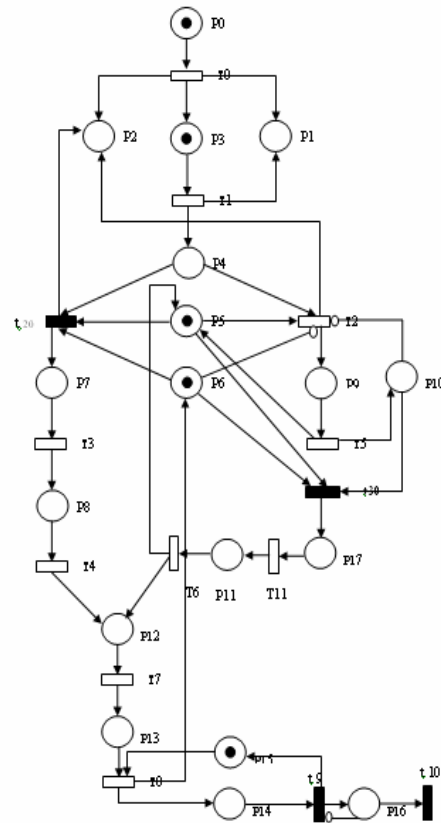


Fig. 7 Petri net system of studied product

TABLE III  
PROPERTIES OF IMMEDIATE TRANSITION

rank	Place name	descriptions
1	T <sub>20</sub>	transferred semi-manufactured part from machine M <sub>1</sub> to machine M <sub>2</sub>
2	T <sub>30</sub>	transferred semi-manufactured part to machine M <sub>2</sub> from buffer B <sub>1</sub>
3	T <sub>9</sub>	Delivering the product to buffer B <sub>2</sub>
4	T <sub>10</sub>	transferred the product to the next produced cell

### C. Evaluating the Efficiency of Studied Flexible Produced System

To evaluate the efficiency of studied flexible produced system ii is used an approach of generalized stochastic processes Petri net. In this approach there is used both temporal transitions and immediate transitions. Thus, model is able to describe complement of operations by temporal transitions and systems' logical behavior by immediate transitions. Properties of temporal and immediate transitions are as follows:

TABLE IV  
RATE OF TEMPORAL TRANSITIONS FIRING

Rank	Transition name	Period proportional to transferring based on min	Rate of transition firing based on min
1	$T_0$	5	0.2
2	$T_1$	50	0.02
3	$T_2$	5	0.02
4	$T_3$	5	0.02
5	$T_4$	5	0.02
6	$T_5$	5	0.02
7	$T_6$	5	0.02
8	$T_7$	45	0.022
9	$T_8$	5	0.02
10	$T_{11}$	15	0.067

TABLE V  
LEVELS FOR PRIORITY OF IMMEDIATE TRANSITIONS

Rank	Place name	Level of priority
1	$T_{20}$	4
2	$T_{30}$	3
3	$T_9$	1
4	$T_{10}$	2

To determine the cause for allocating different levels of priority to immediate transitions it is better to determine conflict transitions and this include:  $ECS = \{t_{20}, t_{30}, t_2\}$ .

To remove this conflict there is used from allocating the different levels of priority and inhibitor arcs such that presence of inhibitor arcs from place  $p_6$  to temporal transition  $T_6$ , it will practically remove the conflict of  $T_2$  with  $T_{20}$  and  $T_{30}$  because these three transitions are common in the places of  $P_4$ ,  $P_5$  and  $P_6$  but presence of inhibitory arc from transition  $T_2$  means that for  $T_2$  being enabled, it is necessary there is no token in the place of  $P_6$ ; in this case, transitions  $T_{20}$  and  $T_{30}$  will be unable and thus conflict between  $T_2$  and transitions of  $T_{20}$  and  $T_{30}$  will be removed.

In the process for providing the semi-manufactured parts also to operate by machine  $M_2$  on it, one can use from semi-manufactured part or buffer  $B_1$  or machine  $M_1$  (transitions  $T_3$  and  $T_{11}$ ). Thus, there is conflict between these two transitions. In addition, because both of them include temporal transitions, so any transition with short period, will be fired earlier and results in another transition be unable forever. On the other hand, these two transitions are enabled together and if each one has short period, will be fired and make another transition unable. To remove this problem, there has been used from immediate transitions with different priority levels. To allocate priority there are two different approaches:

1) Economy

Semi-manufactured part will be transferred from machine  $M_1$  not from buffer  $B_1$ . Thus, it will avoid machine  $M_1$  to be idle.

2) Reducing the inventory while manufacturing

Semi-manufactured part will be transferred from buffer  $B_1$  not from machine  $M_1$ . In this case, although machine  $M_1$  is idle, but in contrast the size of inventory will be reduced while manufacturing. This approach is used when there is less space. Priority level of  $T_{10}$  has always priority on  $T_9$ . Its reason is that product will immediately be transferred to the next producing cell and remaining process will not be ceased due to its shortage.

There has been used first approach for studied flexible produced cell. To evaluate the efficiency of related system, it is necessary to initially examine possible states of the system. As mentioned previously, accessing set (set for possible states) must be limited. Accessing set for above mentioned system has been attained and indicated in Table VI. To determine the rate of system transformation, Fig. 8 indicates accessing graph. To calculate limit states, there is used Sharp software. In this study, the aim is calculating the efficiency of machines  $M_1$  and  $M_2$  and robots  $R_1$ ,  $R_2$  and  $R_3$ .

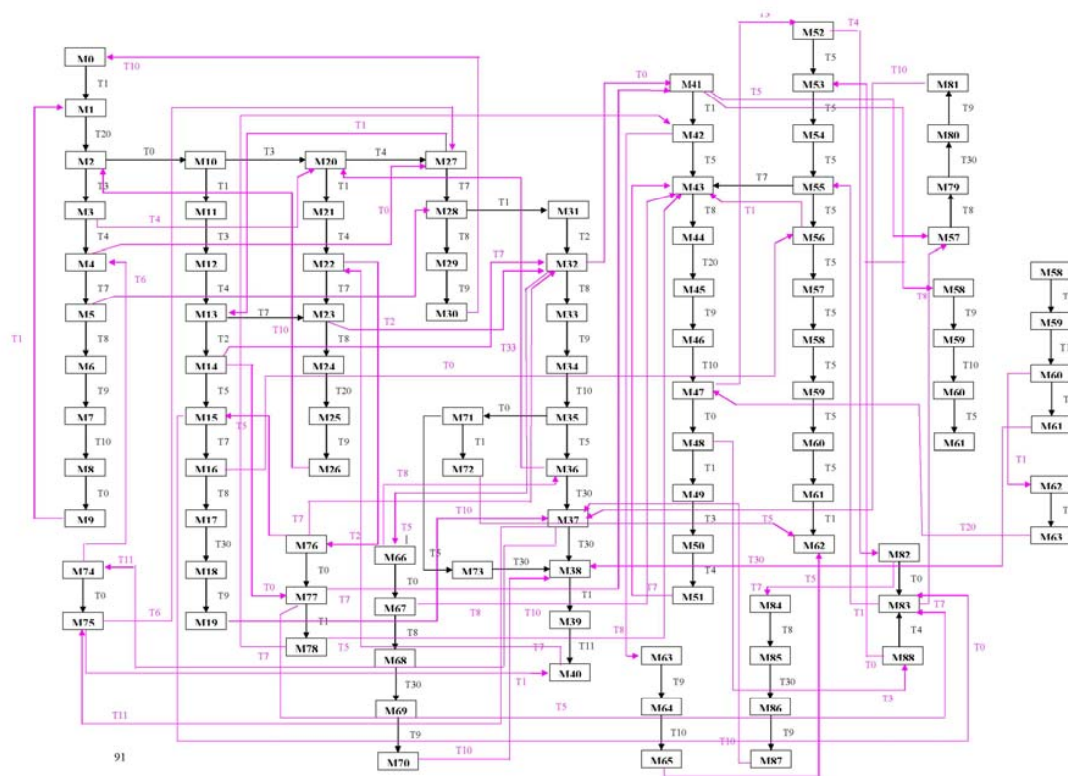


Fig. 8 Reachability graph of suggested flexible manufacturing cell

In related system, there is several factors influence on the system performance:

- Rate of firing attributed to each temporal transition;
- Priority level attributed to each immediate transition;
- The initial marking system;
- Size of buffers  $B_1$  and  $B_2$ .

In this study first three factors have been valued and considered as known and there is studied the effect of changes in fourth factor on the efficiency of machines and robots. The rate of firing the temporal transitions has been indicated in table as well as priority level of immediate transitions. Initial marking has been assumed as following:  $M_0 = P_0 + P_3 + P_5 + P_6 + P_{15}$ . The size of buffers  $B_1$  and  $B_2$ ,  $n$  and  $m$ , have initially considered equal with 1 and thus the amount of mean limit probabilities of the number of tokens will be calculated for each place and its results are as in Table VII.

TABLE VIII  
AMOUNT OF STEADY STATE OF MEAN NUMBER OF TOKEN  
IN PLACE  $P_{10}$  BASED ON  $M=3$

Amounts of n	Probabilities of Mean Number of Token in Place $P_{10}$
1	0.363382522
2	0.752247715
3	1.11667184
5	1.76002002
8	2.5163808
9	2.72668649
10	2.90998133
12	3.21796736

TABLE VI  
REACH ABILITY SET OF SUGGESTED FLEXIBLE MANUFACTURING CELL

Marking No.	P0	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17
0	1			1		1	1									1		
1	1	1			1	1	1									1		
2	1	1	1		1	1	1	1								1		
3	1	1	1						1							1		
4	1	1	1			1			1				1			1		
5	1	1	1			1								1		1		
6	1	1	1			1	1								1			
7	1	1	1			1	1									1	1	
8	1	1	1			1	1									1		
9	1			1		1	1									1		
10	1			1		1	1	1								1		
11	1	1			1			1								1		
12	1	1			1			1	1							1		
13	1	1			1	1							1			1		
14	1	1	1							1			1			1		
15	1	1	1			1					1		1			1		
16	1	1	1			1					1		1	1		1		
17	1	1	1			1	1				1				1			
18	1	1	1			1	1				1				1			1
19	1	1	1			1									1	1	1	
20	1	1	1													1	1	1
21	1			1					1							1		
22	1	1			1				1							1		
23	1	1			1	1							1			1		
24	1	1			1	1								1		1		
25	1	1			1	1	1								1			
26	1	1	1					1							1			
27	1	1	1					1							1			
28	1			1		1							1			1	1	
29	1			1		1								1		1		
30	1			1		1	1								1			
31	1			1		1	1								1		1	
32	1	1			1	1								1		1		
33	1	1	1							1				1		1		
34	1	1	1				1			1					1			
35	1	1	1				1			1					1		1	
36	1	1	1				1			1					1			
37	1	1	1			1	1				1				1			
38	1			1											1			1
39	1	1			1										1		1	
40	1	1			1							1			1			
41	1			1						1				1		1		
42	1	1			1					1				1		1		
43	1	1			1	1				1				1		1		
44	1	1			1	1	1				1				1			
45	1	1	1					1			1				1			
46	1	1	1					1			1				1		1	
47	1	1	1					1			1				1			
48	1			1				1			1				1			
49	1	1			1			1			1				1			
50	1	1			1				1		1				1			
51	1	1			1	1					1		1		1			
52	1	1	1						1		1				1			
53	1			1					1		1				1			
54	1	1			1				1		1				1			
55	1	1			1	1					1		1		1			
56	1			1		1					1			1		1		
57	1			1			1			1					1			
58	1			1			1			1					1		1	
59	1			1			1			1					1			
60	1			1		1	1				1				1			
61	1	1			1	1	1			1					1			
62	1	1			1	1	1				1				1			
63	1	1			1	1	1			1					1			
64	1	1			1		1			1					1		1	
65	1	1			1		1			1					1			
66	1	1	1			1					1			1		1		
67	1			1		1					1			1		1		
68	1			1		1	1				1				1			
69	1			1											1			
70	1			1											1		1	1
71	1			1			1			1					1			
72	1	1			1		1			1					1			
73	1			1		1	1				1				1			
74	1	1	1									1			1			
75	1			1								1			1			
76	1	1	1							1			1		1			
77	1			1						1			1		1			
78	1	1			1					1				1	1			
79	1			1		1	1				1				1			
80	1			1											1			1
81	1			1											1		1	1
82	1	1	1			1					1		1		1			
83	1			1		1					1		1		1			
84	1	1	1			1					1			1	1			
85	1	1	1			1	1				1				1			
86	1	1	1												1			1
87	1	1	1												1		1	1
88	1	1	1						1		1				1		1	1



TABLE VII  
AMOUNTS OF STEADY STATE OF MEAN FOR NUMBER OF TOKENS IN EACH  
PLACE BASED ON  $N=1$  AND  $M=1$

Place name	Limit Possibilities of Mean for Number of Tokens in Each Place
$P_0$	1
$P_1$	0.383086974
$P_2$	0.616913026
$P_3$	0.616913026
$P_4$	0.321395671
$P_5$	0.766258576
$P_6$	0.163023275
$P_7$	0.074391686
$P_8$	0.0371745840
$P_9$	0.0245167182
$P_{10}$	0.363414407
$P_{11}$	0.0245167187
$P_{12}$	0.561830029
$P_{13}$	0.0669219896
$P_{14}$	0.124828685
$P_{15}$	0.875171315
$P_{16}$	0.0493530425
$P_{17}$	0.0731842350

Steady state of mean number of token for example in the place  $P_1$  is equal with 0.383086974; it means robot  $R_1$  is free in 38.31% of working time and are occupied in 61.69% working time. For example limit probability of mean number of token is also 0.766258576 in the place of  $P_5$  and are occupied in 23.37% of working time.

As indicated, the mean number of token in the place  $P_0$  is 1; this is because in this model it is assumed that raw material is always accessible with no loss.

Now the optimal amount of buffer  $B_1$  size is determined regarding to two amounts for buffer  $B_2$  ( $m=3$  and  $m=5$ ). For this reason, there have been calculated amounts of limit probability for mean number of token in  $P_0$  for different amounts of  $n$  and then there will be dealt with analyzing resulted data. For  $m=3$ , results are as Table VIII.

Figures of above mentioned table mean that for example if the size of buffer  $B_1$  be equal with 1, averagely 0.36 of semi-manufactured part will be existed in this buffer.

Now, the amount of  $m$  is assumed equal with 5 and above mentioned calculations will be repeated:

TABLE IX  
AMOUNT OF STEADY STATE OF MEAN NUMBER OF TOKEN IN PLACE  $P_{10}$   
BASED ON  $M=5$

Amounts of $n$	Probabilities of Mean Number of Token in Place $P_{10}$
1	0.363382522
2	0.752247715
3	1.11667184
5	1.76002002
8	2.5163808
9	2.72668649
10	2.90998133
12	3.21796736

It can be indicated that the result has been obtained based on  $m=5$  has no difference with results form  $m=3$ . This is logical answer, because the size of buffer  $B_2$  has no effect on the size of buffer  $B_1$  and its effect on the place  $P_{16}$  and it will affect on the performance of next produced cell.

Diagram for steady state of mean number of token in place  $P_{10}$  based on the size of buffer  $B_1$  is indicated as following:

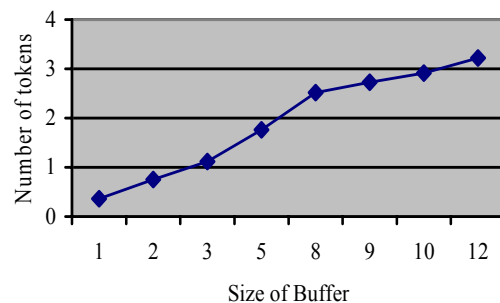


Fig. 9 Mean number of semi-manufacturing parts existing in the buffer  $B_1$  based on the size of buffer

Considering to above mentioned diagram, one can see that the slop of diagram is most in points 3, 5 and 8 so these three points will be examined. If the size of buffer  $B_1$  is equal with 3, the mean number of semi-manufacturing parts in buffer equals with 1.11667184. If the size of buffer  $B_1$  is equal with 5, the mean number of semi-manufacturing parts in buffer equals with 1.76002002. If the size of buffer  $B_1$  is equal with 8 mean number of semi-manufacturing parts in buffer equals with 2.5163808. it is indicated that increasing the buffer from 3 to 5, mean number of semi-manufacturing parts existing in the buffer will be increased to the amount of 0.643 and if this amount increases from 5 to 8, mean number of semi-manufacturing parts existing in the buffer will be increased to the amount of 0.756.

To determine the optimal size of buffer  $B_1$  regarding to the limited size of accessing space and by increasing the size of

buffer, the mean number of semi-manufacturing parts will not be increased considerably, so the optimal size of buffer  $B_1$  has been considered equal with 3.

Now, regarding the optimal amount  $n=3$  for buffer  $B_1$ , one can calculate the efficiency of existing machines. It must be mentioned that presence of token in places of  $P_1$ ,  $P_2$ ,  $P_5$ ,  $P_6$  and  $P_{15}$  indicates accessibility of robot  $R_1$ , machine  $M_1$ , machine  $M_2$ , Robot  $R_2$  and robot  $R_3$  respectively. Lack of token then in such places means they are idle.

TABLE X  
AMOUNT OF STEADY STATE OF MEAN NUMBER OF TOKEN IN EACH PLACE  
BASED ON  $M=3$

Place name	Limit probability for voiding the place
P1	0.649388960
P2	0.93506114
P3	0.268615865
P4	0.88673137
P5	0.589388960

## VII. CONCLUSION

As indicated above, in the studied flexible manufactured cell, aiming to reducing the used space, the amount of inventory in the manufacturing flow and inventory of product will be determined by efficiency of machinery and robots.

The importance of above mentioned study is that in the designing phase, all possible states of studied system and influence of each element on system has been examined and finally the most optimal option for conduction may be selected. Thus, in the planning phase, dynamic of system has also been considered and the probability of anticipating the different states resulting in different functions of system will be obtained and it is also possible to prevent undesirable states.

The main application of this algorithm is for systems where the time of its elements follows normal distribution or exponential distribution; for example, in designing the telecommunication circuits, computer networks and automated producing systems.

Limitations for applying this method include lack of software for simulation of Petri algorithm. Based on extended number of possible states of a system (number of possible states in mentioned model in this study is 88), calculation is very time consuming; therefore, it is needed a software for simulation and calculation of possible different states of system; but unfortunately there are limited software designed based on this algorithm.

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