

# Application of Adaptive Neuro-Fuzzy Inference System in the Prediction of Economic Crisis Periods in USA

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**Abstract**—In this paper discrete choice models, Logit and Probit are examined in order to predict the economic recession or expansion periods in USA. Additionally we propose an adaptive neuro-fuzzy inference system with triangular membership function. We examine the in-sample period 1947-2005 and we test the models in the out-of sample period 2006-2009. The forecasting results indicate that the Adaptive Neuro-fuzzy Inference System (ANFIS) model outperforms significant the Logit and Probit models in the out-of sample period. This indicates that neuro-fuzzy model provides a better and more reliable signal on whether or not a financial crisis will take place.

**Keywords**—ANFIS, discrete choice models, financial crisis, US economy

## I. INTRODUCTION

ONE major challenge of macroeconomists and financial managers is the prediction of financial crisis and economic recessions and expansions periods. The subprime mortgage crisis which took place in USA and became apparent in 2007 has led to great weakness in the financial system and the financial industry regulation. Various approaches have been developed and applied in financial, banking and currency crises. One of these approaches is the application of Logit and Probit models [1]-[5]. The methodology of Probit and Logit models allows for statistical testing, identifying the sign, the magnitude and the marginal distributions of the explanatory variables to the onset of crisis. On the other hand this approach confronts the problem of misspecification errors and serial correlation.

Another approach which has been used in the crisis prediction is the noise-to-ratio model [6]-[7]. The advantage of this approach is that we can directly rank the possible candidate variables as potential crisis periods, but it does not allow for statistical testing and it is not possible to examine the magnitude of each explanatory variable to crisis phenomena

Since 1990 new approaches have been introduced in economics and finance, like neural networks, fuzzy logic and genetic algorithms. We propose the scientific findings and

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methods of artificial intelligence because most studies have found superior results, especially in stock prediction, economic data prediction and in prediction of bankruptcies than the common Logit models and Multiple Discriminant Analysis among others [8]-[13].

## II. METHODOLOGY

### A. Binary Logit and Probit Regressions

The logistic distribution is defined as [14]:

$$\Pr ob(Y = 1 | x) = \frac{e^{x'\beta}}{1 + e^{x'\beta}} = \Phi(x'\beta) \quad (1)$$

The marginal partial effects of explanatory variables are given by:

$$\frac{\partial E[y | x]}{\partial x} = \Phi(x'\beta)[1 - \Phi(x'\beta)]\beta \quad (2)$$

Logit model can be written a general form regression as:

$$y = \alpha + \sum_{i=1}^n \beta_i x_i + \varepsilon \quad (3)$$

, where variable  $y$  is a binary dummy variable taking value 1 if the economy is on crisis or economic recession period and value zero otherwise (no crisis period),  $x_i$  indicates the explanatory variables,  $\alpha$  is the constant,  $\beta_i$  are the regression estimators.

Next we present the Probit regression [14]. More specifically is defined as:

$$\Phi^{-1}(p_i) = Z = \alpha + \sum_{i=1}^n \beta_i x_i + \varepsilon \quad (4)$$

, where  $\Phi^{-1}(p_i)$  is the inverse cumulative distribution function (CDF) of the standard normal,  $\alpha$ ,  $\beta_i$  and  $x_i$  are defined as in (3). Also it can be written as:

$$\Pr(y = 1 | x) = \Phi(x_i \beta_i) \quad (5)$$

The inverse cumulative distribution function (CDF) is

$$p = \Phi(Z) = \int_{-\infty}^Z \frac{1}{\sqrt{2\pi}} \exp(-u^2 / 2) du \quad (6)$$

The Log-Likelihood function for Probit is:

$$\ln L = \sum w_i \ln \Phi(x_i \beta_i) + \sum w_i \ln(1 - \Phi(x_i \beta_i)) \quad (7)$$

The classification of dummy variable is based on the definition by National Bureau of Economic Research (NBER), where a recession begins when the economy reaches a peak of activity. Based on the definitions of NBER the most important and conceptual measures of the economic activity is employment rate and the domestic production. Furthermore, according to NBER the last economic recession began in December of 2007 so we include this sample in the fourth quarter of 2007 as we use in our analysis quarterly data. The prediction or the classification percentage is done based on the estimated coefficients from the in-sample period each time using as the cut-off point the value of 0.5. For the forecasting and the classification performance of the binary discrete choice models is:

If  $y^* > 0.5$ , then the economy is on the financial or economic crisis period

If  $y^* \leq 0.5$ , then the economy is not on crisis period.

Variable  $y^*$  denotes the predicted values.

#### B Adaptive neuro-fuzzy fuzzy inference system (ANFIS)

The reason why we propose neuro-fuzzy logic is that the traditional classification of one and zero can be misleading. More specifically in economics economic depressions are discriminated from economic recessions. For example the current crisis of 2007 and the crisis of 1929 are considered as economic depressions, so we take value 1 in the dummy. But the crises of 1973-195 and 1979-1981 which are owned to petroleum crisis are considered as economic recessions and not depressions, but still the dummy takes value 1. But it should be noticed that the impact of economic recessions and depressions have different impact in gross domestic product and unemployment rate among other macroeconomic and microeconomic variables. So a better classification could be for example 0.8-1 for economic depressions and 0.6-0.8 for economic recessions. On the other hand the post-economic recovery period is very different from an economic expansion period, where dummy variable takes the value 0.

We incorporate three linguistic terms {low,medum,high}. More linguistic terms can be introduced, as very low and very high, but the forecasting performance is almost the same, indicating that we can simplify the procedure by taking less linguistic terms and less rules. The rules are 9 because we

have two inputs with three linguistic terms and it is  $3*3=9$ . These rules are:

*IF GDP is low AND unemployment rate is low THEN  $f_1=p_1x_1 + q_1x_2 + r_1$*

*IF GDP is low AND unemployment rate is medium THEN  $f_2=p_2x_1 + q_2x_2 + r_2$*

*IF GDP is low AND unemployment rate is high THEN  $f_3=p_3x_1 + q_3x_2 + r_3$*

*IF GDP is medium AND unemployment rate is low THEN  $f_4=p_4x_1 + q_4x_2 + r_4$*

*IF GDP is medium AND unemployment rate is medium THEN  $f_5=p_5x_1 + q_5x_2 + r_5$*

*IF GDP is medium AND unemployment rate is high THEN  $f_6=p_6x_1 + q_6x_2 + r_6$*

*IF GDP is high AND unemployment rate is low THEN  $f_7=p_7x_1 + q_7x_2 + r_7$*

*IF GDP is high AND unemployment rate is medium THEN  $f_8=p_8x_1 + q_8x_2 + r_8$*

*IF GDP is high AND unemployment rate is high THEN  $f_9=p_9x_1 + q_9x_2 + r_9$*

, where GDP denotes the gross domestic product growth rate. Basically, there are two types of fuzzy set operation that are usually used in the antecedent rule, which are *AND* and *OR*. Mathematically, the *AND* operator can be realized using *Min* or *Product* operation while *OR* can be realized using *Max* or *Algebraic* sum operator. We choose the *AND* operator and we take the *Product* operator instead to *Min* operator to avoid monotonic results. Each rule has 2 parameters and plus the constant there will be  $(3^3)$  27 parameters.

Jang [15] and Jang and Sun [16] introduced the adaptive network-based fuzzy inference system (ANFIS). This system makes use of a hybrid learning rule to optimize the fuzzy system parameters of a first order Sugeno system. Each rule has two parameters and plus the constant there will be  $3*9=27$  consequent parameters. The steps for ANFIS system computation are:

In the first layer we generate the membership grades

$$O_i^1 = \mu_{A_i}(x_1), \mu_{B_i}(x_2) \quad (8)$$

, where  $x_1$  and  $x_2$  are the inputs. In layer 2 we generate the firing strengths or weights

$$O_i^2 = w_i = \prod_{j=1}^m (\mu_{A_j}(x_1), \mu_{B_j}(x_2)) = \text{andmethod}(\mu_{A_j}(x_1), \mu_{B_j}(x_2)) = \text{product}(\mu_{A_j}(x_1) \cdot \mu_{B_j}(x_2)) \quad (9)$$

In layer 3 we normalize the firing strengths. Because we have nine rules will be:

$$O_i^3 = \bar{w}_i = \frac{w_i}{w_1 + w_2 + \dots + w_8 + w_9} \quad (10)$$

In layer 4 we calculate rule outputs based on the consequent parameters.

$$O_i^4 = y_i = \bar{w}_i f = \bar{w}_i (p_i x_1 + q_i x_2 + r_i) \quad (11)$$

In layer 5 we take:

$$O_i^5 = \sum_i \bar{w}_i f_i = \frac{\sum_i \bar{w}_i f_i}{\sum_i \bar{w}_i} \quad (12)$$

In the last layer the consequent parameters can be solved for using a least square algorithm as:

$$Y = X \cdot \theta \quad (13)$$

, where X is the matrix

$$X = [w_1 x + w_1 + w_2 x + w_2 + \dots + w_9 x + w_9] \quad (14)$$

, where x is the matrix of inputs and  $\theta$  is a vector of unknown parameters as:

$$\theta = [p_1, q_1, r_1, p_2, q_2, r_2, \dots, p_9, q_9, r_9]^T \quad (15)$$

, where T indicates the transpose.

For the first layer and relation (8) we use the triangular membership function. The triangular function is defined as:

$$\mu_{ij}(x_j; a_{ij}, b_{ij}) = \begin{cases} 1 - \frac{|x_j - a_{ij}|}{b_{ij}/2}, & \text{if } |x_j - a_{ij}| \leq \frac{b_{ij}}{2} \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

, where  $a_{ij}$  is the peak or center parameter and  $b_{ij}$  is the spread or support parameter. We use error back propagation algorithm with steepest descent method in order to find the

optimum parameters a and b. The peak parameter update for the triangle membership function is:

$$a_{ij}(n+1) = a_{ij}(n) - \eta_\alpha \cdot \frac{\partial E}{\partial a_{ij}} \quad (17)$$

, where  $\eta_\alpha$  is the learning rate for the parameter  $a_{ij}$  and E is the error functions which is:

$$E = \frac{1}{2} (y - y^t)^2 \quad (18)$$

, where  $y^t$  is the target-actual and y is ANFIS output variable. The chain rule used in order to calculate the derivatives and update the membership function parameters are [17]-[19]:

$$\frac{\partial E}{\partial a_{ij}} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial y_i} \cdot \frac{\partial y_i}{\partial w_i} \cdot \frac{\partial w_i}{\partial \mu_{ij}} \cdot \frac{\partial \mu_{ij}}{\partial a_{ij}} \quad (19)$$

The partial derivatives are derived below:

$$\frac{\partial E}{\partial y_{ij}} = y - y^t = e \quad (20)$$

$$\frac{\partial \bar{w}_i}{\partial w_i} = \frac{r_i - y}{\sum_{i=1}^n w_i} \quad (21)$$

$$\frac{\partial w_i}{\partial \mu_{ij}(x_j)} = \frac{w_i}{\mu_{ij}(x_j)} \quad (22)$$

$$\frac{\partial \mu_{ij}}{\partial a_{ij}} = \begin{cases} \frac{2|x_j - a_{ij}|}{b_{ij}}, & \text{if } |x_j - a_{ij}| \leq \frac{b_{ij}}{2} \\ 0, & \text{if } |x_j - a_{ij}| > \frac{b_{ij}}{2} \end{cases} \quad (23)$$

$$\frac{\partial \mu_{ij}}{\partial b_{ij}} = \frac{1 - \mu_{ij}(x_j)}{b_{ij}} \quad (24)$$

After some derivations and substituting into the update equation (17) we have relations (25)-(26)

$$\frac{\partial E}{\partial a_{ij}} = y - y^t \frac{(r_i - y)}{\sum_{i=1}^n w_i} \cdot \frac{w_i}{\mu_{ij}(x_j)} \cdot \frac{2|x_j - a_{ij}|}{b_{ij}} \quad (25)$$

$$\frac{\partial E}{\partial b_{ij}} = y - y^t \frac{(r_i - y)}{\sum_{i=1}^n w_i} \cdot \frac{w_i}{\mu_{ij}(x_j)} \cdot \frac{1 - \mu_{ij}(x_j)}{b_{ij}} \quad (26)$$

For the RHS membership functions we have:

$$\frac{\partial E}{\partial r_i} = \frac{\partial E}{\partial y} \frac{\partial y}{\partial y_i} \cdot \frac{\partial y_i}{\partial r_i} \quad (27)$$

$$\frac{\partial y_i}{\partial r_i} = \overline{w_i} \quad (28)$$

, resulting in the following gradient:

$$\frac{\partial E}{\partial w_i} = y - y^t \overline{w_i} \quad (29)$$

The update equations for  $a_{ij}$  are,  $b_{ij}$  are respectively

$$\alpha_{ij}(n+1) = \alpha_{ij}(n) - \eta_a \cdot y - y^t \frac{(r_i - y)}{\sum_{i=1}^n w_i} \cdot \frac{w_i}{\mu_{ij}(x_j)} \cdot \frac{2|x_j - a_{ij}|}{b_{ij}} \quad (30)$$

$$b_{ij}(n+1) = b_{ij}(n) - \eta_b \cdot y - y^t \frac{(r_i - y)}{\sum_{i=1}^n w_i} \cdot \frac{w_i}{\mu_{ij}(x_j)} \cdot \frac{1 - \mu_{ij}(x_j)}{b_{ij}} \quad (31)$$

The learning rates for parameters  $a$ ,  $b$  and  $RHS$  are set up at 0.1, 0.01 and 0.05 respectively. The initial parameters  $\alpha_{ij}$  before the training process, for gross domestic, and based on the observed values are set up at -0.01, 0.01 and 0.03 for linguistic terms low, medium and high respectively. In the case of unemployment rate parameters  $\alpha_{ij}$ , are set up at 5, 7 and 9 for linguistic terms low, medium and high respectively, while parameters  $b_{ij}$  are set up at 0.025 and 2.5 for gross domestic and unemployment rate respectively. The number of maximum epochs is 50.

### III. DATA

We estimate the period 1947-2005 and we examine the in-sample forecasting performance. Then we apply all the models to compare their predicting performance for the period 2006-2009. The data source is the *Federal Reserve Bank of St. Louis* and the *National Bureau of Economic Research*. The choice of variables is based on various research papers and studies [1,2,5], as also based on National Bureau of Economic Research (NBER), which defines real GDP, real income unemployment rate, industrial production and retail sales as the most important factors defining the economic activity in US economy. Moreover we try all the candidate variables and we choose the most significant. Specifically we use the same variables in Logit and Probit regressions.

TABLE I  
VARIABLES USED IN ESTIMATED REGRESSIONS

|                       |   |
|-----------------------|---|
| Industrial production | Unemployment rate                         |
| Inflation rate        | Total investments at all commercial banks |

|   |                               |
|---|-------------------------------|
| Total borrowings of depository institutions from federal reserve system | Oil prices                    |
| Interest rates of 3-monthly treasury bills                              | Bank prime loan rate          |
| Public debt   | Balance of accounts           |
| Total loans at all commercial banks                                     | Gross domestic product growth |

### IV. EMPIRICAL RESULTS

From the variables of Table I we used these ones found statistically significant and improve the forecasting performance. These are the industrial production, bank prime loan rate, balance of accounts and the gross domestic product growth rate. We do not present the estimated regression results because we are only interesting for the forecasting performance.

In Table II we present the correctly percentage or the forecasts of Logit regression for the in-sample period 1947-2005, while in the Table II the forecasts of the same model for the out-of sample period 2006-2009 are provided. We observe that the forecasting performance in the in-sample period is relatively significant, while in the out-of sample is too poor. More specifically, with Logit model we successfully predict at 40.00 per cent the crisis periods and 66.67 the no crisis periods. In Tables IV and V we present the forecasts of Probit regression in the in-sample and out-of sample period respectively. We observe that in the in-sample period 1947-2005 Probit presents slightly superior results with Logit model. In the out-of sample period 2006-2009 Probit predicts 96.85 per cent correct the no crisis periods and 70.00 per cent the crisis periods, which is much higher to the respective predicted percentage 40.00 per cent of Logit model.

From Table VI we observe that ANFIS has an overall success of 85.59 per cent in the in-sample period, where we predict 84.81 per cent the no crisis periods in relation to 95.81 and 96.85 of Logit and Probit models respectively. So the first conclusion is that discrete choice models, Logit and Probit, have a very high performance in predicting the no crisis periods in the in-sample period. ANFIS predicts 88.88 per cent the crisis periods in the in-sample period in relation to 60.00 and 64.44 of Logit and Probit models respectively. This indicates that ANFIS outperforms significant the other two models. In Table VII we see that ANFIS has an overall success of 87.50 per cent in the out-of sample period, while the respective percentage of Logit and Probit models is 50.00 and 68.75 respectively. Additionally Logit model has a very poor predicting performance for the crisis periods, with only 40.00 per cent success, while Probit presents a higher correct percentage at 70.00 per cent, but still low in relation with 90.00 correct per cent of ANFIS. On the other hand both Logit and Probit models present exactly the same forecasting performance concerning the prediction of no crisis periods in the out-of-sample interval, which is 66.67, significant lower

from the respective percentage of ANFIS which is 83.33. Moreover Root Mean squared Error (RMSE) and Mean Absolute Error (MAE) values, in ANFIS estimation, are significant lower in the out-of-sample periods than the respective values of Logit and Probit estimations.

TABLE II  
PREDICTION RESULTS OF BINARY LOGIT REGRESSION  
FOR IN-SAMPLE PERIOD

|                    |        |      |        |
|--------------------|--------|------|--------|
| Crisis             | 27     | 18   | 60.00  |
| No Crisis          | 8      | 183  | 95.81  |
| Overall percentage |        |      | 88.98  |
| MAE                | 0.1141 | RMSE | 0.3377 |

TABLE III  
PREDICTION RESULTS OF BINARY LOGIT REGRESSION FOR  
OUT-OF-SAMPLE PERIOD

|                    |        |      |        |
|--------------------|--------|------|--------|
| Crisis             | 4      | 6    | 40.00  |
| No Crisis          | 2      | 4    | 66.67  |
| Overall percentage |        |      | 50.00  |
| MAE                | 0.3750 | RMSE | 0.6123 |

TABLE IV  
PREDICTION RESULTS OF BINARY PROBIT REGRESSION  
FOR IN-SAMPLE PERIOD

|                    |        |      |        |
|--------------------|--------|------|--------|
| Crisis             | 29     | 16   | 64.44  |
| No Crisis          | 6      | 185  | 96.85  |
| Overall percentage |        |      | 90.68  |
| MAE                | 0.1032 | RMSE | 0.3052 |

TABLE V  
PREDICTION RESULTS OF BINARY PROBIT REGRESSION  
FOR OUT-OF-SAMPLE PERIOD

|                    |        |      |        |
|--------------------|--------|------|--------|
| Crisis             | 7      | 3    | 70.00  |
| No Crisis          | 2      | 4    | 66.67  |
| Overall percentage |        |      | 68.75  |
| MAE                | 0.1250 | RMSE | 0.3535 |

TABLE VI  
PREDICTION RESULTS OF ANFIS  
FOR IN-SAMPLE PERIOD

|                    |        |      |        |
|--------------------|--------|------|--------|
| Crisis             | 40     | 5    | 88.88  |
| No Crisis          | 29     | 162  | 84.81  |
| Overall percentage |        |      | 85.59  |
| MAE                | 0.1213 | RMSE | 0.3396 |

TABLE VII  
PREDICTION RESULTS OF ANFIS  
FOR OUT-OF-SAMPLE PERIOD

|                    |        |      |        |
|--------------------|--------|------|--------|
| Crisis             | 9      | 1    | 90.00  |
| No Crisis          | 1      | 5    | 83.33  |
| Overall percentage |        |      | 87.50  |
| MAE                | 0.0733 | RMSE | 0.1618 |

## V. CONCLUSIONS

In this paper we examined and applied three different approaches in financial crisis prediction modeling. We have shown that the correctly classification percentage and the forecasting performance of Logit regression is very poor in the out-of sample period, while Probit regression exhibits higher forecasting performance to Logit in both in-sample and out-of sample periods. On the other hand the forecasting performance of ANFIS with triangular membership function is much more significant in the out-of sample period than traditional discrete choice Logit and Probit models. This indicates the superiority of fuzzy logic and artificial intelligence models suggesting that is a powerful tool for the economic policy and decision makers. Our proposal is that both methodologies can be useful. The discrete choice models can be helpful to examine the magnitude and the sign of each independent variable, while ANFIS can be very useful for forecasting purposes. Furthermore, genetic algorithms can be applied instead to error backpropagation we used in this study and might have superior results. Additionally, we examined only one membership function, while also other fuzzy membership functions can be applied, as the Gaussian or trapezoidal among others. Finally, more inputs can be obtained, but this is not absolutely necessary that it will improve the forecasts.

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