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Angles of Arrival Estimation with Unitary Partial Propagator

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***Abstract*—**In this paper, we investigated the effect of real valued transformation of the spectral matrix of the received data for Angles Of Arrival estimation problem. Indeed, the unitary transformation of Partial Propagator (UPP) for narrowband sources is proposed and applied on Uniform Linear Array (ULA).

Monte Carlo simulations proved the performance of the UPP spectrum comparatively with Forward Backward Partial Propagator (FBPP) and Unitary Propagator (UP). The results demonstrates that when some of the sources are fully correlated and closer than the Rayleigh angular limit resolution of the broadside array, the UPP method outperforms the FBPP in both of spatial resolution and complexity.

***Keywords*—**DOA, Uniform Linear Array, Narrowband, Propagator, Real valued transformation, Subspace, Unitary Operator.

I. Introduction

DIRECTION OF ARRIVAL estimation (DOA) of impinging electromagnetic or acoustic waves on antenna arrays is an important problem in many fields [1] such as sonar, phased array radar, radio astronomy, Electronic Surveillance Measure (ESM), submarine acoustics, Global System for Mobile (GSM) positioning, geophysics and so on .

Many DOA estimation methods have been proposed in the last two decades such as Multiple Signal Classification (MUSIC) [2], Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [3] which are subspace based techniques by using the eigen-value decomposition (EVD) or the Singular Value Decomposition (SVD) of the cross correlation matrix of the received signals from the radiating sources in order to construct the sets of signal and noise subspaces. However, when the number of antenna elements is large, the subspace based techniques may require extensive computational load. To overcome this problem, many fast DOA techniques have been established like the Propagator Method (PM) [4], [5], [9], [10] without eigen-decomposition where the steering matrix of the array is divided into two sub- matrices enabling the computation of the propagation operator. In the other hand, when the noise of the antenna sensors is not spatially and temporally uniform, which can be caused by many factors such as temperature, humidity, pressure and vibration, the PM method may fail to detect the Angles Of Arrival (AOA) of the incoming signals, for this reason, a modified version of the PM method [6], we call it Partial Propagator, is derived by using the off diagonal sub-matrices of the steering matrix, which make it efficient in the presence of colored noise.

In this paper, we present real valued Unitary Partial Propagator (UPP) using unitary transformation matrices in case of incoherent and coherent narrowband sources to estimate theirs azimuth angles. The simulation results are illustrated to test the efficiency of the proposed transformation that outperforms the Partial Propagator even if the Forward Backward averaging technique is applied to it, moreover the computational complexity of the (UPP) is reduced by factor of four at least [7].

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II. Parametric Data Model

We consider a Uniform Linear Array (ULA) consisting of N isotropic and identical antenna elements as illustrated in Fig. 1 and P sources emitting monochromatic electromagnetic waves with the same carrier frequency f_c. After the acquisition of K snapshots, the received signal vector $X(t) = [X_1(t), X_2(t), \dots, X_N(t)]^T$, with the index $t = 1,2,\dots,K$ can be written as follows :

$$X(t) = A(\theta)s(t) + n(t) \tag{1}$$

where $A(\theta) = [a_1(\theta), a_2(\theta), \dots, a_P(\theta)]^T$ is the array steering matrix, taking the right most element of the ULA as the phase reference, as illustrated in Fig. 1, the steering vector is $a(\theta_i) = \left[1, e^{-j\mu_i}, \dots, e^{-j(N-1)\mu_i}\right]^T$, $\mu_i = 2\pi c^{-1} f_c d \sin\theta_i$, d is the inter-element distance and c is the speed of the electromagnetic wave $c = 2.99e+8\, m/s$. The sources waveforms and the zero mean ergodic Gaussian noise are denoted respectively by $s(t) = [s_1(t), s_2(t), \dots, s_P(t)]^T$ and $n(t) = [n_1(t), \dots, n_N(t)]^T$. The cross correlation matrix of the received signals can be expressed as:

$$R_{xx} = E\{X(t)X^H(t)\} = A(\theta)R_{ss}A^H(\theta) + \sigma^2 I_N \tag{2}$$

where R_{ss} is the covariance matrix of the source signals, I_N is the identity matrix, σ^2 is the noise variance and $(.)^H$ denotes the conjugate transpose. If we suppose that the number of sensors satisfy the condition N > 2P, the set of P steering vectors is linearly independent and the sources signals are not correlated, the array steering matrix can be partitioned into three sub-matrices as follows:

$$A(\theta) = [A_1(\theta), A_2(\theta), A_3(\theta)]^T \tag{3}$$

$A_1(\theta), A_2(\theta)$ are of size PxP while $A_3(\theta)$ is (N-2P)xP matrix. We compute the following partial cross correlation matrices:

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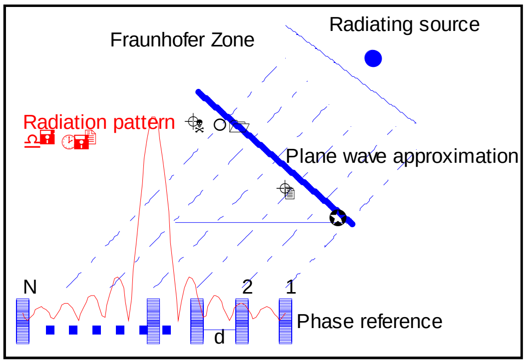

Fig. 1 Phase interferomerty with broadside linear array

$$R_{12} = E\{X(1{:}P,t)X^H(P+1{:}2P,t)\} = A_1R_{ss}A_2^H \quad (4)$$

$$R_{31} = E\{X(2P+1{:}N,t)X^H(1{:}P,t)\} = A_3R_{ss}A_1^H \quad (5)$$

$$R_{32} = E\{X(2P+1{:}N,t)X^H(P+1{:}2P,t)\} = A_3R_{ss}A_2^H \quad (6)$$

such that $X(i{:}j,t)$ (i.e t=1,..,K) is the i^{th} to the j^{th} rows of $X(t) \in \mathbb{C}^{N\times K}$. The sub-matrix $A_3(\theta)$ can be calculated by two different equations:

$$R_{32}R_{12}^{-1}A_1 = A_3R_{ss}A_2^HA_2{}^{-H}R_{ss}^{-1}A_1{}^{-1}A_1 = A_3 \quad (7)$$

$$R_{31}R_{21}^{-1}A_2 = A_3R_{ss}A_1^HA_1{}^{-H}R_{ss}^{-1}A_2{}^{-1}A_2 = A_3 \quad (8)$$

Combining (7) and (8) gives:

$$R_{32}R_{12}^{-1}A_1 + R_{31}R_{21}^{-1}A_2 = 2A_3 \quad (9)$$

Using matrix augmentation yields to:

$$[R_{32}R_{12}^{-1}|R_{31}R_{21}^{-1}| - 2I_{N-2P}]A(\theta) = 0_{(N-2P)\times P} \quad (10)$$

where I_{N-2P} is an identity matrix, hence we can construct the matrix Q^H that is orthogonal to the steering matrix $A(\theta)$: $Q^HA(\theta) = 0_{(N-2P)\times P}$ .The angular spectrum is given by:

$$f(\theta) = \|Q^Ha(\theta)\|^{-2} \quad (11)$$

Our contribution, in this paper, lies in the use of real valued covariance matrix R_R instead of complex valued one R_{xx}, that will reduce the computational complexity of the DOAs estimation and enhance the resolution capability of correlated sources. We begin with the definition of complex unitary matrices, let $n \in \mathbb{N}^*$, for $N = 2n$ we define:

$$Q_N = \frac{1}{\sqrt{2}}\begin{pmatrix} I_n & jI_n \\ J_n & -jJ_n \end{pmatrix} \quad (12)$$

For $N = 2n + 1$ we define:

$$Q_N = \frac{1}{\sqrt{2}}\begin{pmatrix} I_n & 0_{n\times 1} & jI_n \\ 0_{1\times n} & \sqrt{2} & 0_{1\times n} \\ J_n & 0_{n\times 1} & -jJ_n \end{pmatrix} \quad (13)$$

where I_n is an n identity matrix, $j = \sqrt{-1}$ and J_n is the flipped identity matrix in the left right side direction also called exchange matrix. The unitary matrix satisfies the following properties: $J_NQ_N^* = Q_N$ and $Q_N^HQ_N = J_N$. Next, the following theorem [7] is used for the real valued transformation of the spectral matrix R_{xx}.

Theorem 1: For any Hermitian persymmetric matrix $H \in \mathbb{C}^{N\times N}$ such that $J_NH^*J_N = H$, the matrix $W = Q_N^HHQ_N$ is real and symmetric (i.e $W \in \mathbb{R}^{N\times N}$ & $W = W^T$) .

Proof is presented in [8], using the relations $J_NJ_N = I_N$ and $J_NR_{xx}^*J_N = R_{xx}$ we have:

$$\begin{aligned}(Q_N^HR_{xx}Q_N)^* &= Q_N^TR_{xx}^*Q_N^* = Q_N^TJ_NJ_NR_{xx}^*J_NJ_NQ_N^* \\ &= Q_N^TJ_NR_{xx}J_NQ_N^* = Q_N^HR_{xx}Q_N\end{aligned}$$

However, the matrix R_{xx} in (2) is Hermitian but not persymmetric, the persymmetrization property can be realized with forward backward averaging technique [7] :

$$R_{FB} = \frac{1}{2}(R_{xx} + J_NR_{xx}^*J_N) \quad (14)$$

The real valued matrix is given by:

$$\begin{aligned}R_R &= Q_N^H\left\{\frac{1}{2}(R_{xx} + J_NR_{xx}^*J_N)\right\}Q_N \quad (15)\\ &= \frac{1}{2}(Q_N^HR_{xx}Q_N + Q_N^HJ_NR_{xx}^*J_NQ_N) \\ &= \frac{1}{2}(Q_N^HR_{xx}Q_N + Q_N^{H*}R_{xx}^*Q_N^*) \\ &= Real\{Q_N^HR_{xx}Q_N\}\end{aligned}$$

The partial matrices described in (4), (5) and (6) are extracted from the real covariance matrix defined in (15) to construct a real valued set Q_R as in (10) and the new spectrum $f_R(\theta)$ is computed using a new steering vector $a'(\theta) = Q^Ha(\theta)$ in (11).

III. Simulation Results

This section is divided into two parts, first we evaluate the performance of the proposed DOA function with fixed Signal to Noise Ratio (SNR), in the second time we investigate the performance in the presence of fully correlated sources while considering the inter-element distance as function of the wavelength $d = f(\lambda)$.

A. Modified DOA Function with Uncorrelated Sources

In this part, we consider a Uniform Linear Array (ULA) consisting of N=11 isotropic and identical sensors with the same excitation current and zero phase, the inter-element distance is half the wavelength. We assume that there are P=4 narrowband and almost equally powered radiating sources (Power ~ 1watt) that are statistically independent of each other and impinge on the array from directions $\theta_1 = -65°, \theta_2 = 21°, \theta_3 = 25°$ and $\theta_4 = 51°$. The sources signals are complex ergodic Gaussian processes with number of snapshots K=200 and the carrier frequency is fc = 500Mhz which makes the length of the array ULA = 3m. Fig. 2 represents the average

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of L=200 of Monte Carlo runs of the Unitary Partial Propagator (UPP) when SNR=2dB.

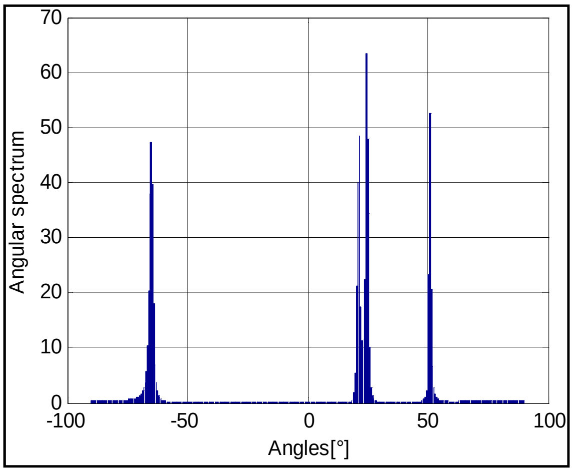

Fig. 2 Unitary Partial Propagator (UPP), SNR=2dB

Using the same conditions, we compare the UPP function with the Unitary Propagator function (UP) and the complex valued Partial Propagator (PP), Fig. 3 represents the average of L=200 runs of the three spectra.

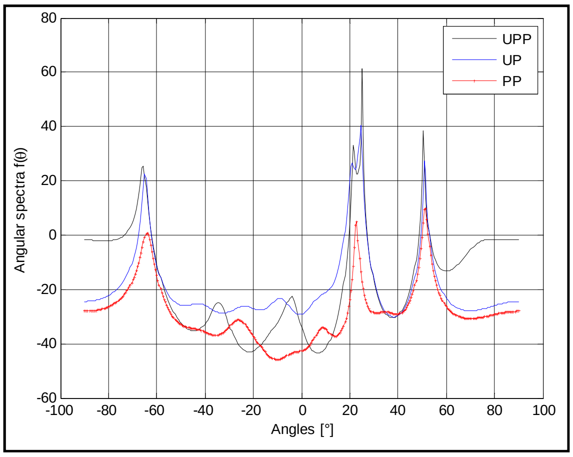

Fig. 3 DOA functions with SNR=2dB

The Unitary transformation of both the Propagator (UP) and Partial Propagator (UPP) has increased theirs resolution power while the Partial Propagator failed to separate the sources located at [21°, 25°]. In terms of computational complexity, the Unitary Propagator (UP) has $q(NPK + O(P^3))$ with q=0.25, the Partial Propagator (PP) takes $P(N-P)K + O(P^3)$ while the Unitary Partial Propagator (UPP) has the lower complexity which is q($P(N-P)K + O(P^3)$).

B. Modified DOA Function with Correlated Sources

The problem of correlated sources is treated with many solutions, the two well known methods are the Forward Backward (FB) averaging technique and the spatial smoothing of the cross correlation matrix R_{xx}. To evaluate the Unitary Partial Propagator, we compare it with Forward Backward Partial Propagator (FBPP) where the decorrelation operation is applied using equation (14) before extracting the partial sub-matrices. In this part, we consider the second and the third sources, whose Angles Of Arrival (AOA) are respectively $\theta_2 = 21°, \theta_3 = 25°$, to be fully correlated $s_2(t) = s_3(t)$ and $s_1(t) \neq s_2(t) \neq s_4(t)$.

The objective of this paragraph is to investigate if there exists an ensemble of values of the inter-element distance in terms of wavelength such that the UPP and FBPP spectra can detect the fully correlated sources. Indeed we consider the inter-element distance as a function of the wavelength $d = f(\lambda)$ with the bounds d_{min}/λ=0.1 and d_{max}/λ=1.

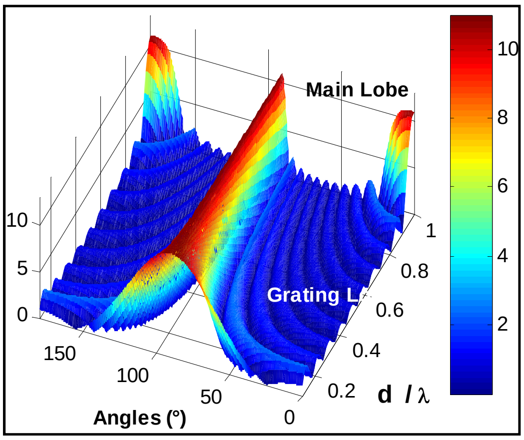

Fig. 4 Array Factor of broadside ULA, d = f(λ)

The variation of the inter-element distance impacts the radiation pattern of the Uniform Linear Array (ULA) precisely the main lobe, Fig. 4 represents the radiation pattern of the broadside ULA ($\theta_{max} = \pi/2$) as function of d. The Rayleigh limit of angular resolution when $d = \lambda/2$ is $\theta_{HPBW} \cong 10°$ where $\theta_{HPBW} = \lambda/\, d(N-1)$. The FBPP function is evaluated by taking the average spectrum of L=150 Monte Carlo runs for every value of d, as illustrated in the Fig. 5.

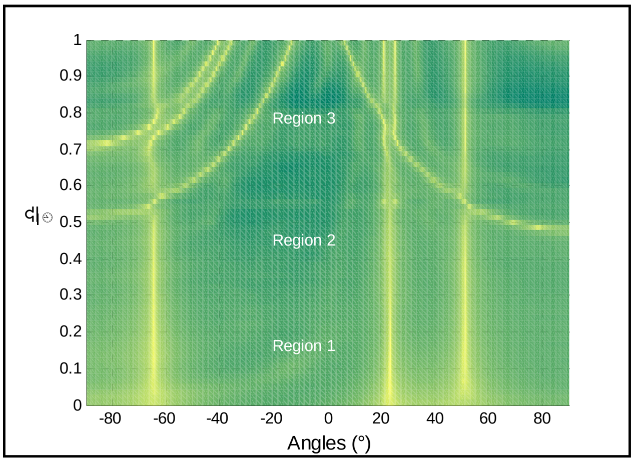

Fig. 5 Forward Backward Partial Propagator in dB, d = f(λ)

The first and the fourth sources are well detected and the spectrum is operational in the range $d \in [0.2\lambda, 0.5\lambda]$, the correlated sources are not discerned but located as single source, staring from d =0.8λ the correlated source are well located but side lobes in the spectrum prevent its efficiency . In Fig. 6, we represent the result of the UPP DOA function which is performing, in the range $d \in [0.4\lambda, 0.5\lambda]$ unlike the FBPP, and successful in discerning the fully correlated sources.

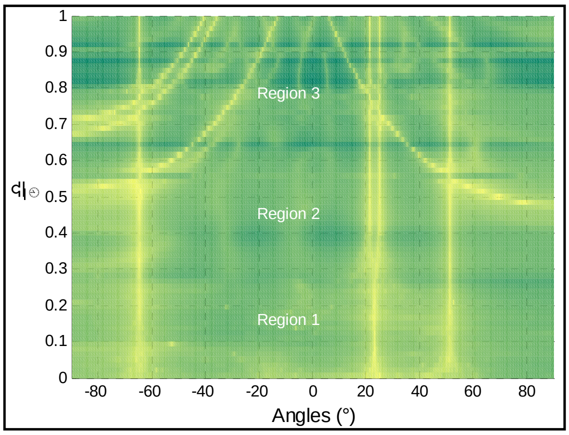

Fig. 6 Unitary Partial Propagator in dB, d = f(λ)

IV. Conclusion

In this paper, we have proposed a unitary transformation of Partial Propagator (UPP) operator for narrowband Direction Of Arrival (DOA) estimation using Uniform Linear Array (ULA). In the first time we have compared the UPP spectrum with two functions: the Partial Propagator (PP) and the Unitary Propagator (UP). In the second experiment, we have assumed that half of the sources are fully correlated and we evaluated the (UPP) with the Forward Backward Partial Propagator (FBPP) while the Signal to Noise Ratio is fixed, theoretical results proved that the UPP is successful in discerning the correlated sources.

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