Analyzing of Temperature-Dependent Thermal Conductivity Effect in the Numerical Modeling of Fin-Tube Radiators: Introduction of a New Method

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Abstract—In all industries which are related to heat, suitable thermal ranges are defined for each device to operate well. Consideration of these limits requires a thermal control unit beside the main system. The Satellite Thermal Control Unit exploits from different methods and facilities individually or mixed. For enhancing heat transfer between primary surface and the environment, utilization of radiating extended surfaces are common. Especially for large temperature differences; variable thermal conductivity has a strong effect on performance of such a surface .In most literatures, thermo-physical properties, such as thermal conductivity, are assumed as constant. However, in some recent researches the variation of these parameters is considered. This may be helpful for the evaluation of fin's temperature distribution in relatively large temperature differences. A new method is introduced to evaluate temperature-dependent thermal conductivity values. The finite volume method is employed to simulate numerically the temperature distribution in a space radiating fin. The present modeling is carried out for Aluminum as fin material and compared with previous method. The present results are also compared with those of two other analytical methods and good agreement is shown.

Keywords—Variable thermal conductivity, New method, Finite volume method, Combined heat transfer, Extended Surface

I. INTRODUCTION

IN recent years, interest in fin with radiation heat transfer has been simulated in space exploration. Radiation is almost the main mechanism by which waste heat from electronic part or other heat generating equipments in satellite can be dissipated. Every satellite has different units which control its performance. One of these subsystems is the thermal control unit devised to dissipate the generating heat The basic mechanism of heat transfer in a space radiator or a fin array is conduction combined with radiation in a nonparticipating medium.

Finned surfaces have enhanced heat transfer mechanism between the primary surface and its surrounding medium. Usually, space radiating fins are used for performing the heat loss by radiation.

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The one-dimensional radiating fins and their heat transfer characteristics have been studied extensively, including references [1] - [6], on the basis of constant thermal conductivity of the fin material. One of the first papers to treat the radiation mode as the sole means of heat dissipation from the faces of a fin was that of Callinan and Berggren [1]. Wilkins Jr. [2] has given expressions for the heat analysis of triangular fins radiating to space at absolute zero. Bartas and Sellers [3] have studied a heat rejecting system consisting of parallel tubes joined by web plates that served as extended surfaces. Stockman et al. [4] have compared one-dimensional and two-dimensional analyses in the central fin-tube radiators and have shown good agreement between one-dimensional and two-dimensional analyses. Cockfield [5] has discussed the role of the radiator as a structural element in the spacecraft applications. Naumann [6] has investigated analytically the thermal design of heat pipe/fin type space radiators for the case of uniformly tapered fins as well as for flat fins with constant thermal conductivity assumption. Whereas, the temperature difference between the fin base and its tip is very high in the actual situation. Hence, the variation of material's thermal conductivity should be taken into consideration.

Arslanturk [7] has evaluated the temperature distribution along a radiating fin by the analytical ADM (Adomian Decomposition Method) method. He has assumed a temperature dependent thermal conductivity of fin material and compared his results with those of Naumann [6] and the agreement is shown to be satisfactory. The Adomian decomposition method provides an analytical solution in terms of an infinite power series. Also, Hosseini and Ghanbarpour [8] have applied the HPM (Homotopy Perturbation Method) method for a variable thermal conductivity, displaying their results to be in good agreement with those of reference [7]. The HPM is a new method to solve a non-linear differential equation analytically and it is on the basis of the perturbation technique. A numerical calculation of temperature distribution analysis is carried out by Bazdidi and Kamrava [9] on the basis of finite volume technique for linear function of temperature-dependent thermal conductivity and good agreement has been shown with three references [6-8].

The present heat transfer analysis of a space radiating fin is carried out numerically by developing a computer code which is based on the finite volume method [10].
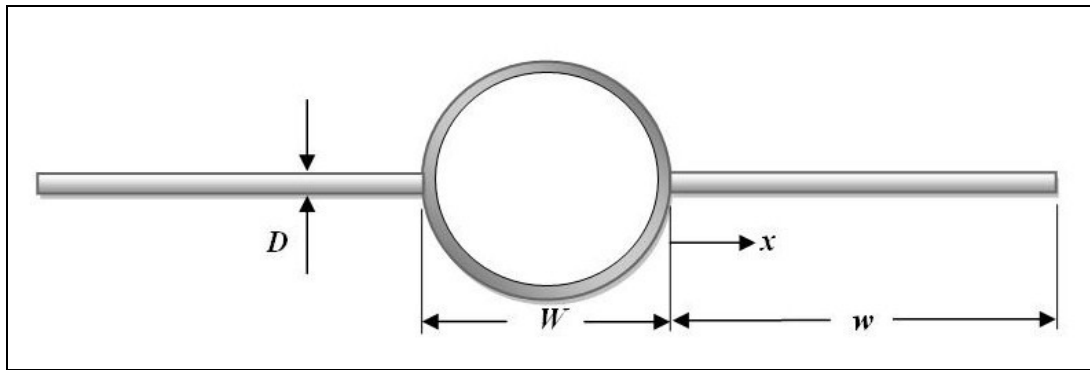
Fig.1 Fin configuration

The governing equation for the one-dimensional configuration is the energy equation encompassing the conduction and thermal radiation modes, under the steady-state condition. The thermal conductivity of fin material is assumed as a linear function of temperature. The contact resistance is regarded as negligible. Also, all radiating surfaces are considered as diffuse and gray. The results on the temperature distribution and fin efficiency are obtained for different geometric and thermal variables with new variable thermal conductivity method.

II. STATEMENT OF THE PROBLEM

The configuration of present radiating fin is shown in Fig. 1. It is assumed that the fin has a length, w and thickness, D connected to a tube at its base which acts as a heat pipe. The fluid flow in pipe is under saturated condition and hence the fluid has a constant temperature. All of the geometrical parameters are measured in meter. The fin base at $x = 0$ is held at constant temperature, T_b and both side surfaces of fin can radiate to outer vacuum space, which is considered at absolute zero temperature. There are no gradients across the thickness of fin and no significant radiation from the edges, because the thickness is assumed to be thin enough.

Also, it is presumed that the fin has diffuse and gray surfaces with a constant emissivity (ε). Radiation exchange between surfaces of pipe and fin is negligible. Assuming $D \ll w$, the problem is solved as one-dimensional heat flow (in the x direction). The governing energy equation is as follows:

$$D \frac{d}{dx} \left[k(T) \frac{dT}{dx} \right] = 2 \varepsilon \sigma T(x)^4 \qquad (1)$$

where, $T(K)$ is temperature, σ is Stefan-Boltzmann constant and $k(W/m.K)$ is the thermal conductivity of fin material.

At moderate to high temperatures such as >100K, the true thermal conductivity, k, of well annealed, high purity (99.99+%) aluminum is relatively insensitive to the impurity level, measured by the residual resistivity, ρ_0 (the resistivity at 0

K).[11] For well-annealed, high-purity aluminum with a ρ_0 of $5.94 \times 10^{-12} \Omega.m$, the recommended value for thermal conductivity is listed in Table 1 [12].

TABLE I
RECOMMENDED THERMAL CONDUCTIVITY OF ALUMINUM [12]

Temperature (T), K	Thermal conductivity (κ), W cm⁻¹ K⁻¹	Temperature (T), K	Thermal conductivity (κ), W cm⁻¹ K⁻¹
0	0	80	4.32
1	41.1	90	3.42
2	81.8	100	3.02
3	121	123.2	2.62
4	157	150	2.48
5	188	173.2	2.41
6	213	200	2.37
7	229	223.2	2.35
8	237	250	2.35
9	239	273.2	2.36
10	235	298.2	2.37
11	226	300	2.37
12	214	323.2	2.39
13	201	350	2.40
14	189	373.2	2.40
15	176	400	2.40
16	163	473.2	2.37
18	138	500	2.36
20	117	573.2	2.33
25	75.2	600	2.31
30	49.5	673.2	2.26
35	33.8	700	2.25
40	24.0	773.2	2.19
45	17.7	800	2.18
50	13.5	873.2	2.12
60	8.50	900	2.10
70	5.85	933.52	2.08

For samples with other ρ_0, the thermal conductivity, k, at temperature below $1.5T_m$ (where T_m is the temperature of the maximum in K) is given by:

$$k = \left[\alpha' T^n + \beta T^{-1} \right]^{-1} \qquad (2)$$

where:

$$\alpha' = \alpha'' \left[\beta / n \alpha'' \right]^{(m-n)/(m+1)} \qquad (3)$$

where:

$$\begin{cases} \alpha'' = 4.79 \times 10^{-6}, & m = 2.62 \\ n = 2, & \beta = \rho_0 / L_0 \end{cases} \qquad (4)$$

The boundary conditions for the present fin geometry are defined as in (5):

$$T = T_b \qquad at \qquad x = 0$$

$$\frac{dT}{dx} = 0 \qquad at \qquad x = w \qquad\qquad (5)$$

III. NUMERICAL HEAT TRANSFER ANALYSIS

The present problem is aimed at the prediction of temperature distribution with new method of calculation for variable thermal conductivity in an extended surface while combined conduction-radiation heat transfer is taking place. One-dimensional conduction across the fin and radiation heat loss from side surfaces are taken into account. The governing for the present computation is considered as follows [10]:

$$\frac{d}{dx}\left(k \frac{dT}{dx} \right) + S = 0 \qquad\qquad (6)$$

where, S is radiation heat loss that has a negative sign. The discretization procedure, based on the finite volume technique [10], is carried out on the above equation to solve the problem numerically.

The treatment of radiation heat loss term is considered in (6). As shown in (1), this term is a function of the dependent variable T itself and it is then desirable to acknowledge this dependence in constructing the discretization equation.

The discretization equation is solved by the technique for linear algebraic equations and only a linear dependence can be accountable.

The comparison between (1) and (6) shows that S is the function of T^4 and thus it should be linearized. Therefore, the heat loss term is linearized by replacing T^4 by $\left[T^* \right]^4 + 4 \left[T^* \right]^3 \left[T - T^* \right]$ wherever it appears. Here, the superscript, *, refers to a previous iteration value. The linearization of S should be a good representation of the $S \sim T$ relationship. This technique of linearization is on the basis of the Taylor series [10].

Fin Efficincy: The fin efficiency is an important parameter which is provided to measure the fin thermal performance. The maximum rate at which a fin could dissipate energy is the rate that would exist if the entire fin surface where at the base temperature. A logical definition of fin efficiency is therefore [13]:

$$\eta = \frac{Q_f}{Q_{f,ideal}} = \frac{\int_0^w 2W\varepsilon\sigma T^4 dx}{2Ww\varepsilon\sigma T_b^4} = \int_0^1 \theta^4 d\xi \qquad\qquad (7)$$

In the equation (7) the Q_f is heat transfer from fin surfaces which is defined by Stephan-Boltzman law, and $Q_{f,ideal}$ is ideal heat flux from fin surface. To determine the value of fin efficiency the extrapolation on dimensionless temperature distribution diagrams is used to determine the function of dimensionless temperature regarding to dimensionless length.

IV. RESULTS AND DISCUSSION

In the present work, the new method of evaluation of temperature dependent thermal conductivity is considered. The temperature distribution on a heat pipe on the basis of this new method is evaluated. The present results are compared with previous method's results on the basis of linear function of temperature-dependent thermal conductivity. Also, the present results are compared with those of Naumann [6] and Arslanturk [7].

For more precise and also simpler analysis, two dimensionless parameters are defined as follows:

$$\theta = \frac{T}{T_b} \ , \ \xi = \frac{x}{w} \qquad\qquad (8)$$

where, θ is dimensionless temperature, ξ is dimensionless length. The fin material is considered to be Aluminum (Al).

The present predictions on the dimensionless aluminum fin temperature distribution in the axial direction (i.e., fin length, x) are depicted by Fig. 2, for new method of variable thermal conductivity. The fin length and emissivity of fin material are assumed as: $\varepsilon = 0.85$, $w = 0.04952m$. It can be seen that the temperature decreases as the fin length increases. Also, the present numerical results are directly compared with those available Bazdidi [9] for constant thermal conductivity.

Fig. 3 represents the dimensionless temperature distribution at a fixed length, $w = 0.054m$. These evaluations are attained on the basis of this new method of thermal conductivity of fin material and previous linear function of variable thermal conductivity [9]. In the Fig.3 β is a dimensionless coefficient for thermal conductivity function which is introduced and utilized by Arslanturk [7].

In the previous work [9] the variation of β for constant length is demonstrated. It is observed that as β is increased from -0.6 toward 0.6 the temperature difference between the fin base and its tip is decreased and consequently the fin can radiate to outer space with an overally higher temperature. This may lead to an increase in the fin overall efficiency [9]. Also new method of evaluation of temperature distribution on space radiating fin is considered. It is shown that the evaluated temperature distribution by this new method is 2% less than constant those of for constant thermal conductivity.

The values of conductivity of Al at $T = 100K$ and $T = 1000K$ can be taken from the literature [13] as $k = 300 W/m.K$ and $k = 200 W/m.K$, respectively. The average conductivity at the same temperature range is $k = 250 W/m.K$.
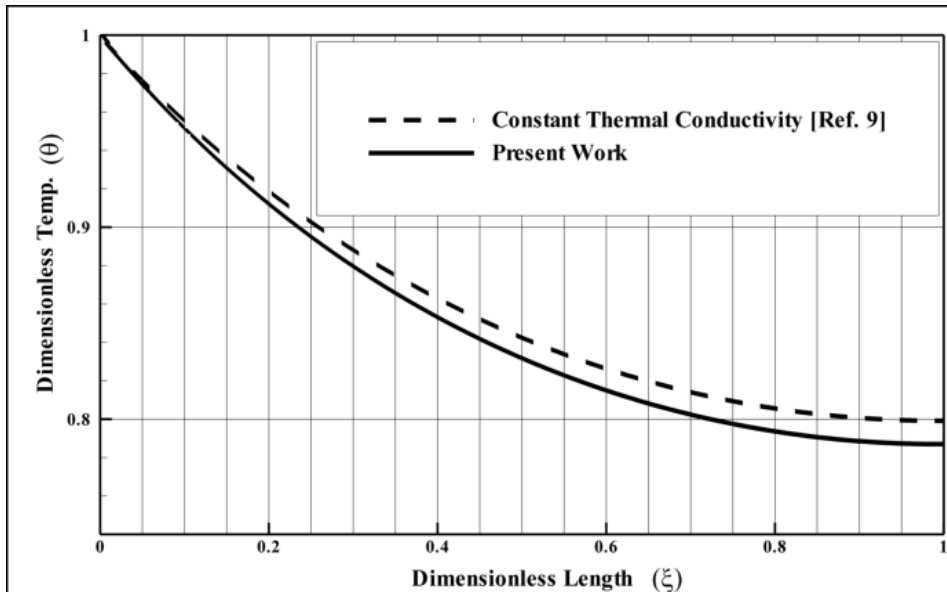
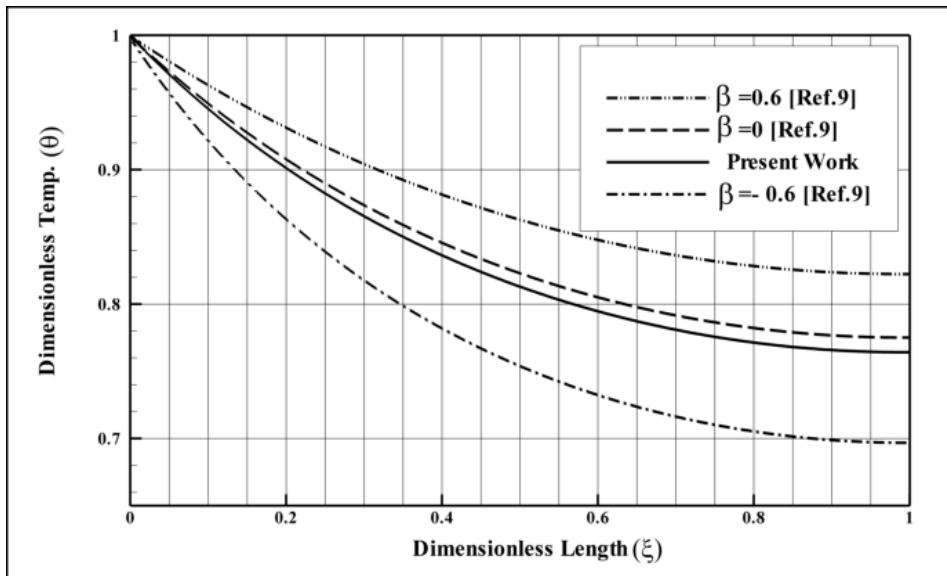Fig.2 Dimensionless aluminum fin temperature distribution at *w=0.04952m*

Fig.3 Dimensionless temperature distribution of aluminum fin at fixed length, $w = 0.054m$

TABLE II
DIMENSIONLESS TIP TEMPERATURE FOR ALUMINUM FIN FOR DIFFERENT LENGTHS

Method	$w = 0.045m$	$w = 0.04796m$	$w = 0.04952m$	$w = 0.054m$	$w = 0.058m$
Present work	0.8241	0.8084	0.7870	0.7641	0.7459
Constant Thermal Conductivity [Ref.9]	0.8369	0.8154	0.7991	0.7751	0.7593

The present dimensionless aluminum fin tip temperature predictions (i.e., at x=w) for constant thermal conductivity on the basis of Ref.[9] ($\beta = 0$ at Reference [9]) and variable thermal conductivity, with new evaluation of temperature-dependent thermal conductivity, for different fin length, are depicted in Table 2.

The length of fin, w is an important parameter and its effect on the fin axial temperature distribution, is displayed in Fig.4 for aluminum fin material. It is observed that as w is increased, the fin tip temperature and hence overall temperature along the fin length are decreased significantly.

Also, in this figure the dimensionless temperature distribution with constant temperature distribution is compared with variable temperature distribution along fin. It is demonstrated that the overall temperature along the fin length with constant thermal conductivity is more that variable one's.

The fin efficiency of aluminum fin is illustrated by Fig.5 in different length and for both variable and constant thermal conductivity. It can be seen that the fin efficiency is decreased while fin length is increased. Also, the calculated fin efficiency for variable thermal conductivity is less than constant thermal conductivity one's.
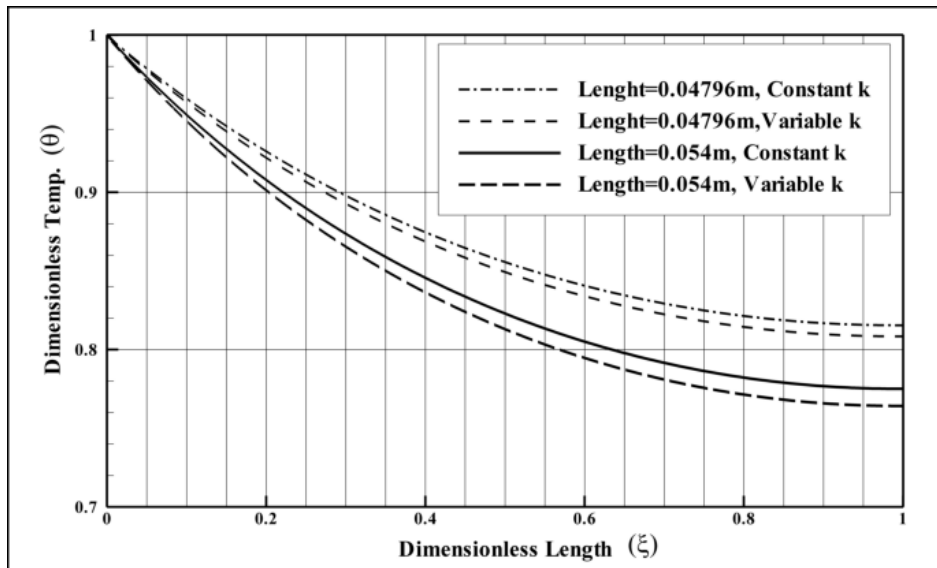

Fig.4 Dimensionless temperature distribution of aluminum fin for different lengths
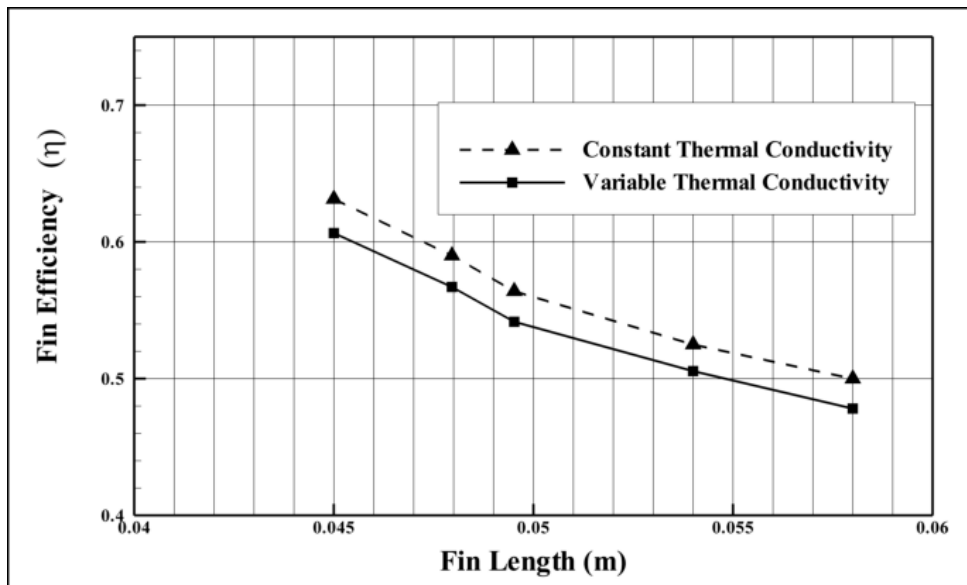

Fig.5 Space radiating fin efficiency in different length for constant and variable thermal conductivity

V. CONCLUSIONS

In the present work, a new function for variable thermal conductivity is introduced and its influence on a space radiating fin is considered and compared with previous linear function of thermal conductivity's results. The finite volume method has been accomplished to investigate the nonlinear governing equation numerically. Also, the linearization technique is used for the radiation heat loss term. The temperature distribution along a radiating fin and fin efficiency are predicted for different geometric and thermal parameters. The main conclusions may be drawn as follows:

- In new method of evaluation thermal conductivity of fin material, trend of dimensionless temperature distribution along fin identical to previous method of linear function of temperature-dependent thermal conductivity.
- Under the new method of variable thermal conductivity consideration, at the same length of fin, the fin tip temperature is less than fin tip temperature under the previous linear function.
- Fin tip temperature decreases if length of fin increases.
- Dimensionless tip temperature predictions are compared with previous method and constant thermal conductivity for different length and good agreement is shown.
- Fin efficiency is concluded by extrapolating a polynomial function on dimensionless temperature distribution along fin. It is observed that the fin efficiency decreased while fin length increased.

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