

# Analytical Modelling of Average Bond Stress within the Anchorage of Tensile Reinforcing Bars in Reinforced Concrete Members

Maruful H. Mazumder, Raymond I. Gilbert, and Zhen- T. Chang

**Abstract**—A reliable estimate of the average bond stress within the anchorage of steel reinforcing bars in tension is critically important for the design of reinforced concrete member. This paper describes part of a recently completed experimental research program in the Centre for Infrastructure Engineering and Safety (CIES) at the University of New South Wales, Sydney, Australia aimed at assessing the effects of different factors on the anchorage requirements of modern high strength steel reinforcing bars. The study found that an increase in the anchorage length and bar diameter generally leads to a reduction of the average ultimate bond stress. By the extension of a well established analytical model of bond and anchorage, it is shown here that the differences in the average ultimate bond stress for different anchorage lengths is associated with the variable degree of plastic deformation in the tensile zone of the concrete surrounding the bar.

**Keywords**—Anchorage, Bond stress, Development length, Reinforced concrete.

## I. INTRODUCTION

ONE of the most important factors governing the robust performance of reinforced concrete (RC) structures is the bond between the steel reinforcement and the concrete, since it provides the means of transferring the tensile forces between the reinforcement and the concrete. The fundamental requirements of both strength and ductility (robustness) cannot be met unless the tensile reinforcement at each critical section of a RC member is sufficiently anchored (developed) on either side of the critical section. Codes of Practices (such as AS3600 [1], ACI318 [2] and Eurocode 2 [3]) specify a minimum development length,  $l_{sy,t}$ , for the reinforcement to be embedded on both sides of the critical section in order to ensure that the tensile reinforcement at the critical section will not only develop the yield stress ( $f_{sy}$ ) and but also will sustain it with increasing deformation.

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A good estimate of the average bond stress within the anchorage length of a RC member is important for the design of the anchorage to meet the requirements of strength and ductility. Uniform bond stress along the anchorage length is commonly assumed in order to calculate the anchorage lengths of bars, regardless of the variations in anchorage lengths or bar diameter. Although a uniform bond stress is often a reasonable assumption for a relatively short length of reinforcing bar embedded in concrete, it is not true when the embedment length is longer. The conventional pull-out tests [[4]-[7]] involves measuring the force required to pull a short length of reinforcing bar out of the concrete in a relatively small concrete specimen. The average ultimate bond stress determined from such a test overestimates the bond strength of longer anchorages in full-scale RC members.

An experimental study to assess the impact of static, cyclic and sustained loading on the anchorage requirements of modern high strength steel reinforcing bars in tension, including the cases of end development and lapped splices has recently been completed. The first of the four stages of the experimental research program was aimed at assessing the impact of static and cyclic loading on the development length of normal ductility Australian deformed bars. This paper presents the experimental results of the variations of average bond stress in full-scale RC specimens which were tested for, among other factors, the effects of varying the development length of the tensile reinforcing bars in the specimens. It describes an improved analytical model that gives a better estimation of the average bond stress and accounts for variations of anchorage length and other important structural parameters.

## II. ANALYTICAL DESIGN PROVISIONS OF BOND AND ANCHORAGE LENGTH

### A. General Overview

When the tensile force in a deformed reinforcing bar is increased and the adhesive bond between the steel and the concrete is broken, some frictional slip takes place before the full bearing capacity at the bar deformations (the rib) is mobilized. Within the development length of a deformed bar, the deformations progressively bear onto the surrounding concrete and the bearing forces  $F$  are inclined at an angle  $\beta$  to the bar axis as shown in Fig. 1 (a) [8]. The horizontal

component of this bearing force mobilizes the bond stress,  $f_b$  at the interface between concrete and reinforcement. The perpendicular component of the bearing forces exerts a radial force on the surrounding concrete. Tepfers [[9]-[11]] described the concrete in the vicinity around the bar as acting like a thick walled pipe as shown in Fig. 1 (b) and the radial forces exerted by the bar cause tensile stresses,  $\sigma_r$  that may lead to splitting cracks radiating from the bar if the tensile strength of the concrete is exceeded. Bond failure is typically initiated by the splitting cracks within the development length of an anchored bar (Fig. 1 (c) and 1 (d)) or within the lap-length at a lapped tension splices (Fig. 1 (e)). The bar pull-out due to increasing slip between the concrete and reinforcement is also concurrently associated with the splitting phenomena. Transverse reinforcement across the splitting planes ( $A_{tr}$  in Fig. 1 (c), 1 (d), and 1 (e)) delays the propagation of cracks and improves bond strength. Compressive pressure transverse to the plane of splitting delays the onset of cracking thereby improves bond strength.

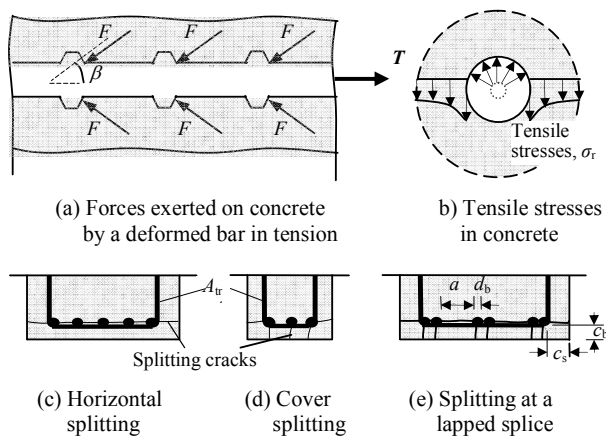


Fig. 1 Splitting failures around developing bars [9]

Codes of practice [[1]-[3]] specify a minimum development length,  $l_{sy,t}$ . The fundamental requirement is that for a reinforcing bar of diameter  $d_b$ , the ultimate bond force over the development length ( $\pi d_b l_{sy,t} f_{ub}$ ) must not be less than the maximum bar force ( $f_{sy} A_{st} = f_{sy} \pi d_b^2 / 4$ ). Therefore,

$$l_{sy,t} \geq 0.25 d_b f_{sy} / f_{ub} \quad (1)$$

Expressions for the development length in most codes of practice are similar to (1) with the average ultimate bond stress,  $f_{ub}$  directly related to the tensile strength of concrete,  $f'_t$  and modified by coefficients of varying form and complexity to account for the factors affecting bond strength. However, the specified values for  $l_{sy,t}$  in the major international Standards differ widely [12] and have been developed independently as empirical fits to experimental data obtained from tests involving monotonically increasing static loads often on small scale specimens.

### B. Empirical Analytical Models of Bond and Anchorage

In many of the analytical models, the angle  $\beta$  is used as the key parameter for describing the relationship between  $\sigma_r$  and  $f_{ub}$ . These empirical models also have in common the dimensionless parameter  $c/d_b$  as the correlation parameter to the normalized bond strength parameter,  $f_{ub}/(f'_c)^{0.5}$ , where  $c$  is the effective thickness of the concrete cover around a reinforcing bar which is taken as the minimum of the side or bottom thickness of concrete cover and half of the clear spacing between bars.

Ferguson and Briceno [13] developed equations for side split and face-and-side split bond failures assuming that the radial and longitudinal components of force between the bar and concrete are equal ( $\beta=45^\circ$ ). Ferguson and Krishnaswamy [14] used a slightly different approach and assumed that the splitting force is related to bond force but may not be equal to it (i.e.,  $\beta$  may be more or less than  $45^\circ$ ). They took the concrete tensile strength  $f'_t = 0.531(f'_c)^{0.5}$  (in MPa), a value based on split cylinder tests. One of the main limitations of their approach is that the splitting was assumed to occur instantaneously along the anchorage length. However, splitting is actually progressive, starting at the critical end of the anchorage. The crude assumption of the value of  $\beta$  also disregards the observed uneven distribution of bond stress along the anchorage length.

By applying the thick cylinder theory, Tepfers [[9]-[11]] derived a relationship between radial splitting forces at failure and  $f'_t$ . The radial stress,  $\sigma_r$  was regarded as the pressure acting against the thick walled cylinder while the inner diameter of the cylinder was considered as the reinforcing bar diameter. Tepfers [9] assumed the thick wall of concrete around the deformed bar in tension is cracked and the maximum depth of internal crack,  $e$ , was shown to be  $0.486(c+d_b/2)$ . The outer zone of concrete around the cylinder was assumed to be uncracked and fully elastic with the bond force carried to it through the cracked concrete. While comparing the bond action at the elastic, plastic and partly cracked elastic stages, Tepfers [9] assumed that the angle  $\beta$  is the same at all the above stages. Hence the criteria for the occurrence of ultimate load in the concrete ring around the reinforcing bar were set as follows:

- Elastic stage: Maximum tensile stress peak reaches the ultimate tensile stress for the concrete.
- Plastic stage: The tensile stress evenly distributed over the concrete cover reaches the ultimate tensile stress for the concrete.

Tepfers showed the maximum tensile stress due to the bond action was  $(\sigma_r)_{\max} = (1.664 f_{ub} d_b \tan \beta) / (c + d_b/2)$ . Bond failure is assumed to occur as soon as the maximum tensile stress in the concrete is equal to  $f'_t$ . That is:

$$f'_t = (1.664 f_{ub} d_b \tan \beta) / (c + d_b/2) \quad (2)$$

Since  $f'_t$  can be written as  $k_1(f'_c)^{0.5}$ , then

$$c/d_b + 1/2 = 1.664 f_{ub} \tan \beta / k_1(f'_c)^{0.5} \quad (3)$$

Tepfers' [9] model was verified [15] by using experimental results of Ferguson and Krishnaswamy [14] where the angle  $\beta$  was taken as equal to  $45^\circ$  and also assuming fully elastic stage in the uncracked concrete. The coefficient  $k_1$  was then determined from a linear regression analysis of the experimental results which provided the best estimate of the tensile strength ( $f_t$ ) over the range concrete strengths considered in the experiments. The main criticism of the Tepfers' [9] thick cylinder theory is that concrete does not behave wholly elastically in tension at failure. Hence, the application of the thick cylinder theory may not be entirely valid. Assuming a full plastic behavior in the uncracked section of the concrete, Orangun, Jirsa, and Breen [15] showed that the maximum tensile stress in the uncracked section is  $(0.972f_{ub}d_b \tan \beta)/(c+d_b/2)$ . For the range of concrete strengths considered by Ferguson and Krishnaswamy [14],  $f_t = 0.531(f_c)^{0.5}$  which results in a value of  $\tan \beta = 0.77$  for the full elastic behavior, whereas for the plastic behavior  $\tan \beta = 1.32$ .

In the cracking model developed by Canbay and Frosch [16], the tensile stresses are assumed to be constant across the failure plane and failure is assumed to occur when the entire splice region,  $l_s$  reaches its tensile capacity. For side-splitting failures (Fig. 1 (e)), the force required to cause splitting  $F_{\text{splitting}}$  was calculated using the following equation assuming that the tensile stress just before cracking  $f_t$  was uniform over the lapped splice length  $l_s$  and the splitting force on the horizontal side splitting crack was taken to be

$$F_{\text{splitting}} = l_s [2C_s + 2a] f_t \quad (4)$$

and the splitting force required to produce one of the vertical face splitting cracks in Fig. 1 (e) is

$$F_{\text{splitting}} = l_s [2C_b] f_t \quad (5)$$

Cracking associated with each of these splitting forces was assumed to occur when  $f_t = 0.5(f_c)^{0.5}$ . The radial splitting forces exerted on the concrete in the bar anchorage are generated by the resultant longitudinal bar force  $F_{\text{long}}$  which is calculated as the product of the area of the bars associated with the splitting crack under consideration,  $nA_b$  and the stress that is developed in the reinforcing bar(s),  $\sigma_{st}$ . That is

$$F_{\text{long}} = nA_b \sigma_{st} \quad (6)$$

where  $A_b$  is the cross-sectional area of a single bar, and for the horizontal side splitting crack considered in (4),  $n=6$ . For each vertical face splitting crack considered in (5),  $n=1$ .

With  $F_{\text{splitting}}$  and  $F_{\text{long}}$  representing the transverse and longitudinal components of the resultant bearing forces  $F$

$$\tan \beta = F_{\text{splitting}} / F_{\text{long}} \quad (7)$$

and, for any beam, the angle  $\beta$  can be readily determined. Canbay and Frosch [16] determined  $\beta$  for 203 unconfined test specimens (i.e., specimens without transverse reinforcement,

$A_t = 0$ ) subjected to monotonically increasing static loading and found the average value to be  $\beta = 36^\circ$ . With this value, they calculated the steel stress at splitting failure from

$$\sigma_{st} = F_{\text{splitting}} / (nA_b \tan \beta) \quad (8)$$

where  $F_{\text{splitting}}$  is the smaller of the values for side splitting and face splitting.

This analytical modelling [16] approach is attractive because of its simplicity and because, for the first time, it provides a physical model to determine the strength of an anchorage. However, it is based on a number of rather crude assumptions, the most important of which is the assumption that the splitting stresses are uniformly distributed over the length of the anchorage. While this is known to be reasonably good approximation for relatively short anchorages (i.e., unacceptably short anchorages from a design perspective), it becomes increasingly less accurate as the anchorage length increases. For practical anchorages, the full implications of the assumption are not known. The estimation of the radial pressure and the radial splitting forces may also be very complex for many practical anchorages. Only a few tests [[16], [17]] have been conducted to estimate the radial pressure and the radial strain in the concrete around an anchored bar using a simple uniaxial bar pull-out tests where a smoothed sliding surface (parallel to the axis of the bar) and development of a uniform bond stress was recreated through a carefully designed testing mechanism. However, in the case of RC members in bending, the force transfer between concrete and reinforcement is typically by the rib bearing and the sliding surface is not parallel to the axis of the bar. As loading continues, the progressive crushing of the concrete in front of the lugs of the reinforcement may variably change the sliding surface along the anchored bar, and the bond stresses and splitting stresses along the anchorage zone also change in a complex manner.

In reality, the angle  $\beta$  of the bearing force  $F$  changes in magnitude as bond failure develops due to progressive crushing of concrete in front of the bar deformation and this also changes the slip surface and mechanical wedging action between the bar and concrete [18]. As also mentioned by Tepfers [9], the concrete ring resistance to splitting falls at or near the ultimate state of bond failure and some slip takes place due to concrete crushing in front of the ribs. As a consequence, the angle  $\beta$  between the bond forces and the bar axis may change in this stage. For practical anchorages, the value of  $\beta$  determined at this stage may be indicative of the true deformation state of the concrete around the anchorage zone which typically falls between the upper and lower bond solution [9] of the elastic and fully plastic states in concrete.

In both the aforesaid analytical modeling approaches [[9], [16]], the resultant product is a unique value of  $\beta$  averaged from many experimental results. This may be highly inconsistent for estimating the true level of bond stresses within a particular anchorage length of a specimen that may suffer a different degree of plastic deformation in the uncracked concrete. Therefore, a true representation of the

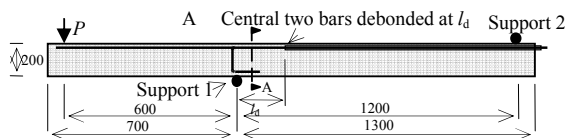
plastic deformation state in the uncracked concrete can be given by an estimate of  $\beta$  derived from Tepfers' [9] analytical model assuming both the elastic and fully plastic stage in the concrete. The value of  $\beta$  determined in this way, however, is different for different practical anchorages.

### III. THE EXPERIMENTAL PROGRAM

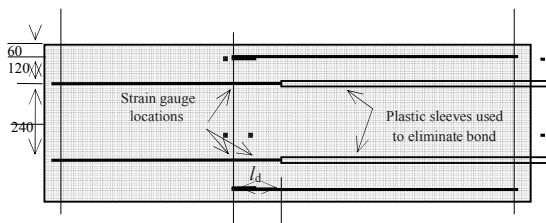
#### A. Test Specimens and Loading Regime

The development length specimens were 2000mm long, 600mm wide and 200mm deep. With supports 1200mm apart, these statically determinate members were cantilevered at one end by an amount of 700mm, as shown in Fig. 2 (a). The line load  $P$  was 600mm past Support 1. Each specimen contains four reinforcement bars in the top of the specimen, the outer two bars being terminated with a 180° cog immediately past Support 1. For the two centrally placed bars that carry bending in the cantilever, the bond between the concrete and the bars was eliminated from the point of development length,  $l_d$  past Support 1 through to the far end of the specimen by encasing each bar in a plastic sleeve. The bar within the plastic sleeve continues along the specimen, protruding from the right hand end. For the convenience during testing, the specimen is inverted so the anchored bars are located in the bottom of the specimen (see Fig. 2 (c)). A total of eighteen development length specimens were tested in Test Series 1. Fourteen specimens were tested under short-term static loading and the remaining were tested under a cyclic loading regime.

The static loading involved monotonically increasing the applied load on the specimen by controlling the rate of deformation at a suitably slow rate until failure occurs in the specimen, either by bond failure or yielding of the reinforcement. The cyclic loading involved repeatedly loading and unloading the specimen from 10% to about 50% of its static strength. Each cyclic loading specimen was subjected to in excess of 50,000 cycles of loading at a rate of 1.0 hertz.



(a) Elevation of specimen



(b) Plan of specimen



(c) Inverted testing arrangement

Fig. 2 Dimensions and loading arrangements of development length specimens

The deflection of the specimen at the point of load application, together with the slip at the end of the debonded bars, was measured throughout the test using LVDTs. Also measured throughout the test were the location of the primary cracks and crack widths. The variables considered were the bar diameter  $d_b = 12$  or 16mm; the development length  $l_d = 5d_b, 10d_b, 15d_b$ , and  $20d_b$ ; and the bottom concrete cover  $c = 25$  or 40mm. Strain gauges were used to monitor steel strains in the developing bar at the critical cross-section and mid-way along the length  $l_d$ , as shown in Fig. 2 (b). When interpreting the results of the static load tests, cracked section analysis can be readily undertaken to determine the stresses that develop in the developing bars at the critical cross- (Support 1) at all levels of applied loading up to and including anchorage failure.

#### B. Test Results of Development Length Specimens

The maximum load  $P_{max}$  applied to each specimen during the test is given in Table I, together with the calculated (by cracked section analysis) maximum stress,  $\sigma_{st}$  in the monitored bars at the critical section and  $f_{ub}$  mobilized over the development length. The factor of safety associated with the code approach (taken as the ratio of the measured  $f_{ub}$  to that specified by the Standard) is also shown in Table I. Table II shows the material properties measured at the time of testing.

TABLE I  
TEST RESULTS FOR DEVELOPMENT LENGTH SPECIMENS

Specimen no. and load type	$c/d_b$	$l_d$ (mm)	$P_{max}$ (kN)	At section at Support 1 under $P_{max}$		AS3600-2009	
				$\sigma_{st}$ (MPa)	$f_{ub}$ (MPa)	$f_{ub}$ (MPa)	Factor of safety
DL-1 (S)	1.56	160	30.5	308	7.69	3.93	1.96
DL-2 (S)	1.56	240	40.5	403	6.72	3.93	1.71
DL-3 (S)	1.56	320	48.3	478	5.97	3.93	1.52
DL-4 (C)	1.56	240	44.9	445	7.41	3.93	1.89
DL-5 (S)	1.56	240	42.8	425	7.09	3.93	1.80
DL-6 (S)	2.08	120	27.2	477	11.92	4.45	2.68
DL-7 (S)	2.08	180	32.5	565	9.41	4.45	2.12
DL-8 (S)	2.08	240	30.8	537	6.72	4.45	1.51
DL-9 (C)	2.08	180	20.7	369	6.16	4.45	1.38
DL-10 (S)	2.50	160	25.9	292	7.31	4.55	1.61
DL-11 (S)	2.50	240	34.8	387	6.44	4.55	1.42
DL-12 (S)	2.50	320	43.8	482	6.02	4.55	1.32
DL-13 (C)	2.50	240	31.9	356	5.93	4.55	1.30
DL-14 (S)	3.33	60	15.2	-	-	-	-
DL-15 (S)	3.33	120	21.4	422	10.54	5.21	2.03
DL-16 (S)	3.33	180	26.3	510	8.50	5.21	1.63
DL-17 (C)	3.33	180	21.8	429	7.15	5.21	1.37
DL-18 (S)	1.25	180	23.3	390	6.50	3.79	1.72

\* Static load test - (S), cyclic load test - (C)

TABLE II  
MATERIAL PROPERTIES OF SPECIMENS

Specimen no	$f_c$ (MPa)	$f_{ct,f}$ (MPa)	$E_c$ (MPa)	$f_{sy}$ (MPa)	$f_{su}$ (MPa)
DL-1 to 5	38.5	3.75	34700	546	731
DL-6 to 9	38.5	3.75	34700	561	721
DL-10 to 13	36.9	3.60	29300	546	731
DL-14 to 18	36.9	3.60	29300	561	721

When considering the minimum development lengths specified in codes of practice, it is generally agreed that a factor of safety in the range of 1.5 to 2.5 is reasonable. Selected data from Table I as illustrated in Fig. 3 shows that the increase of development length generally results a reduction of the factor of safety. This indicates that an increase in development length results in a non-uniform distribution of bond stress within part of the anchored bar and hence a reduction of the average bond stress.

The followings summarize the main experimental observations:

- The average bond stress at failure  $f_{ub}$  decreases as the development length  $l_d$  increases.
- For the static load tests, the factor of safety obtained from the Australian Standard, AS3600-2009 are generally in the acceptable range (except for DL-11 and DL-12 marginally under and DL-6 somewhat higher). With the exception of DL-4, the cyclic load tests are below the acceptable range.
- The values of  $f_{ub}$  for 16 mm bars are significantly smaller than that for 12mm bars, but the difference decreases for higher values of  $l_d$ .
- The effect of the dimensionless parameter,  $c/d_b$  on the average ultimate bond stress  $f_{ub}$  is significant for the smaller diameter bars (12mm) with the shorter anchorage lengths while for the larger diameter bars (16mm) the effect is insignificant.

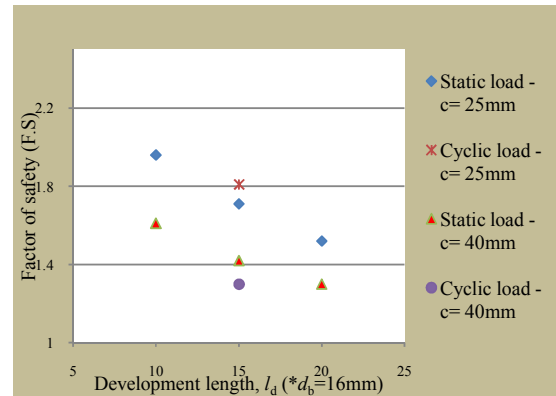


Fig. 3 Factor of safety vs.  $l_d$  for different concrete cover to reinforcement ( $d_b = 16\text{mm}$ )

### C. An Extension of the Analytical Modelling of Bond

The angle of inclination ( $\beta$ ) of the bearing forces determined at the ultimate state of bond failure is indicative of the extent of plastic deformation that occurs at that state. Fig. 4 is a plot of the normalized bond strength ( $f_{ub}/(f_c)^{0.5}$ ) against the dimensionless parameter  $c/d_b$  of the test specimens together with points obtained from Tepfers' [9] analytical solution using (2). The wide deviations of the experimental results from Tepfers' [9] solution, particularly at higher values of bond stress, indicates that the assumption of fully elastic behaviour of the concrete zone outside the assumed thick cylinder zone is not entirely valid. This also indicates that the angle  $\beta$  changes for different values of anchorage length and bar diameter resulting in variable degrees of plastic deformation in the tensile zone of the concrete.

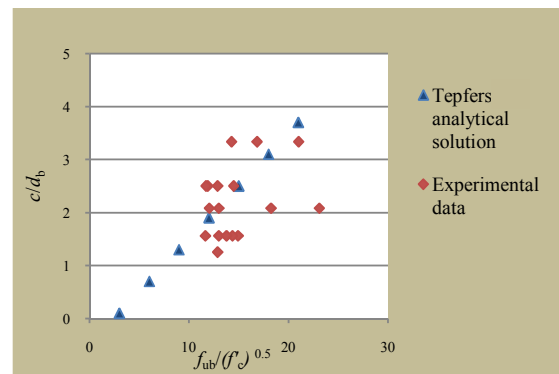


Fig. 4 Comparison between experimental results and Tepfers' [9] analytical solution

In order to minimize the difference between the experimental results of  $f_{ub(\text{exp})}$  and the calculated values  $f_{ub(\text{cal})}$  according to (2) and (3) (as proposed in [9]), an optimization technique has been implemented for each of the test results using Microsoft Excel optimization solver. The objective of the optimization was to find out the value of  $\beta$  that would give the best representation of the plasticity stage in the tensile zone of the concrete. The target for the optimization is shown in (9).



$$\text{Objective function: } f_{ub(\text{exp})} - (f_{ub,el} + f_{ub,pl})/2 = \text{minimum} \quad (9)$$

where  $f_{ub,el}$  and  $f_{ub,pl}$  is the Tepfers' [9] solutions of  $f_{ub}$  for partly cracked elastic stage and fully plastic stage respectively. The optimization allows the determination of the value of  $\beta$  that best represents the test results from a specimen with a particular anchorage length of bar. The optimized  $\tan\beta$  vs.  $c/d_b$  for the different development length specimens is shown in Fig. 5.

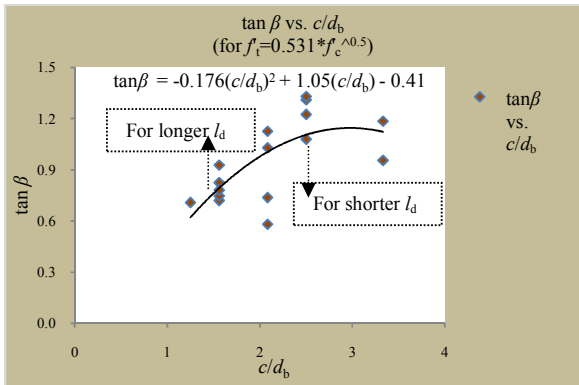


Fig. 5  $\tan\beta$  vs.  $c/d_b$  for selected development length specimens

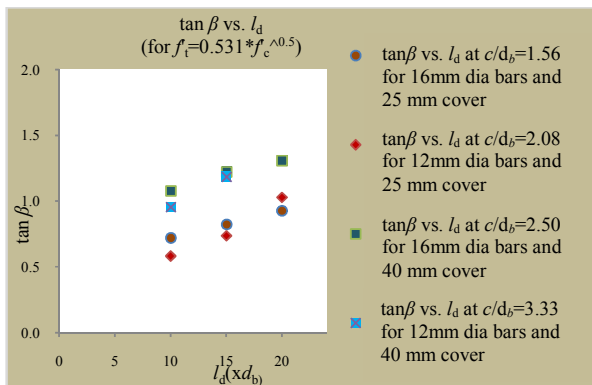


Fig. 6 Optimized  $\tan\beta$  vs.  $l_d$  for selected development length specimens

Fig. 5 shows that for the different values of  $c/d_b$  there is a wide range of values of the angle  $\beta$  (or  $\tan\beta$ ) which also varies with the variations of anchorage lengths. For the individual  $c/d_b$  ratios and for a given bar diameter ( $d_b$ ), Fig. 6 shows the variation of  $\tan\beta$  with variable development lengths ( $l_b$ ). The angle  $\beta$  (or  $\tan\beta$ ) optimized in this way represents the best approximation of the plasticity stage in the uncracked concrete surrounding the anchored bar. The relationships developed from the experimental results could be effectively used for realistic prediction of the angle  $\beta$  (for a particular design case depending upon different structural factors such as  $c/d_b$  and  $d_b$  and hence assist in developing more accurate formulations to estimate  $\beta$  and to provide a more reliable value of  $f_{ub}$ . However, further research is required to verify these relationships between the angle  $\beta$  ( $\tan\beta$ ) and the dimensionless parameters  $c/d_b$ , and the anchorage length.

#### IV. CONCLUSIONS

A common assumption reflected in modern design codes is that the average ultimate bond stress develops uniformly within the anchorage length and its value is independent of the actual anchorage length. It is evident from the experimental results presented in this paper that the average ultimate bond stress is in fact dependent on the anchorage length. In addition, other factors, including the dimensionless parameter ( $c/d_b$ ), have been shown to significantly affect the bearing angle  $\beta$  and hence the extent of plastic behavior of the concrete in the tensile zone also varies between the different specimens with different anchorage lengths of bars. Previous investigations using different analytical modelling procedures found that the angle  $\beta$  progressively changes with loading and this results in a varying degree of plastic deformation in the concrete depending on the anchorage length, bar diameter and other structural and material parameters.

For practical anchorages, the variable degree of plastic deformation in the concrete influences the average ultimate bond stress that can be mobilized. An extension of the analytical modelling procedure presented in this paper can be effectively used to represent the degree of plastic deformation in the tensile zone of the concrete. As these relationships are developed from the test results from full-scale RC specimens, they will provide a more reliable estimate of the bond stresses for a particular design condition and hence more reliable estimates of the safe anchorage length of reinforcing bars in concrete.

#### ACKNOWLEDGMENT

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