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Analytical Model to Predict the Shear Capacity of Reinforced Concrete Beams Externally Strengthened with CFRP Composites Conditions

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***Abstract*—**This paper presents a proposed analytical model for predicting the shear strength of reinforced concrete beams strengthened with CFRP composites as external reinforcement. The proposed analytical model can predict the shear contribution of CFRP composites of RC beams with an acceptable coefficient of correlation with the tested results. Based on the comparison of the proposed model with the published well-known models (ACI model, Triantafillou model, and Colotti model), the ACI model had a wider range of 0.16 to 10.08 for the ratio between tested and predicted ultimate shears at failure. Also, an acceptable range of 0.27 to 2.78 for the ratio between tested and predicted ultimate shears by the Triantafillou model. Finally, the best prediction (the ratio between the tested and predicted ones) of the ultimate shear capacity is observed by using Colotti model with a range of 0.20 to 1.78. Thus, the contribution of the CFRP composites as external reinforcement can be predicted with high accuracy by using the proposed analytical model.

***Keywords*—**Predicting, shear capacity, reinforced concrete, beams, strengthened, externally, CFRP composites.

I. INTRODUCTION

THE ACI318-08 [1] code method for design and analysis of shear and diagonal tension in beams is essentially imperial. It lacks the suitable physical model for the response of beams subjected to bending combined with shear. The shear strength provided by a particular section in beam is a combination of both Vc and Vs. Vc is the concrete combination, generally considered to be some combination of force transferred by dowel action of the main reinforcement, aggregate interlock a long a diagonal crack, and shear in the uncracked concrete beyond the end of the crack. The values of each combination are not identified. Fig. 1 shows these combinations. Because of that, attention was increased in developing a rational model to describe the shear behavior of beams subjected to combined effect of shear and bending. Truss model [1] is the best model to describe this behavior since it provides a clear insight into the flow of forces within the beam.

Truss model was first developed by Ritter [2], which was later developed and extended to be applicable in design of beams for shear, beams subjected to axial tension or compression, deep beams, brackets and corbels, and design for torsion. The truss of Fig. 2 can present the structural action of this model. In this truss, the concrete upper flange acts as the upper compression chord, the steel stirrups act as the vertical tension web members, the main reinforcement acts as the lower members, and the concrete acts as the diagonals between the inclined cracks. This configuration can be seen in Fig. 1. For the beams with no shear reinforcement, the theoretical shear capacity was determined using the ACI Code 318-08 [1] equation for concrete shear strength:

$$V_c = \left(1.9\sqrt{f_c'} + 2500\rho\frac{V_u d}{M_u}\right)b_w d \le 2\sqrt{f_c'}\,b_w d \tag{1}$$

where V_c is the shear strength of concrete (kips), f_c' is the concrete compressive strength (ksi), ρ is the longitudinal reinforcement ratio, V_u and M_u and are the ultimate shear and ultimate bending moment, respectively, taken at the point of crack initiation (i.e. at d/2) from the face of the support, d is the effective beam depth (in), and b_w is the width of the beam (in). The composite CFRP is an elastic material with a stress strain behavior. Using the mechanics of composite materials, the nominal shear strength of CFRP composite can be predicted from the following approach.

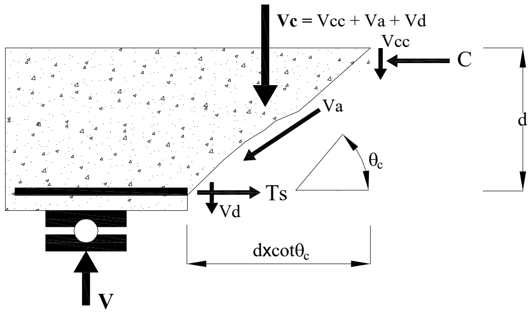

Fig. 1 Forces at a diagonal crack in a beam without web reinforcement

II. DEVELOPMENT OF SHEAR STRENGTH EQUATION PROVIDED BY THE CFRP COMPOSITE

The truss model will be used to study the shear capacity provided by CFRP composite. CFRP composite will be assumed to act as a vertical external stirrup with two legs. Fig. 1 shows representation for the model; as before, the main reinforcement will act as the tension lower chord, the uncracked compression zone of concrete will act as the upper

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compression chord, the concrete struts formed between the diagonal cracks will act as the diagonal compressive member in the truss. These diagonals will push a part the lower and upper chord of the truss, which represents the bottom and the top of the beam. The CFRP composite will be assumed to act as the vertical members in the truss that pulls the upper and lower chord of the truss together, which induced tensile stresses in the CFRP composite.

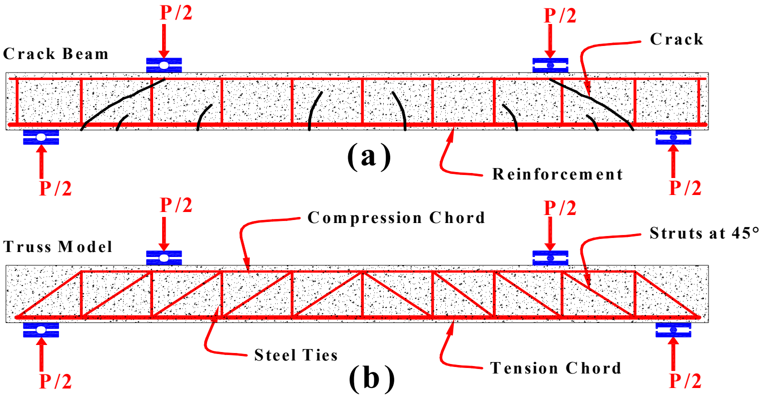

Fig. 2 Truss model of beam with web reinforcement: (a) beam with diagonal crack, and (b) simple truss model

Chajes et al. [3], Swamy et al. [4], Khalifaand and Nanni [5], and Matthys [6] had pointed out that shear failure of reinforced concrete beams strengthened externally with bonded composites generally has diagonal shear failure at an inclined shear crack similar to those of the unstrengthened beams. Therefore, the contribution of the external epoxy bonded composites can be calculated in a similar way as for the unstrengthened beam reinforced with internal shear reinforcement. Therefore, the contribution of the external shear reinforcement can be calculated in a similar way as for the internal shear reinforcement bars, the nominal shear strength of RC beams strengthened with composites of the critical section Vn can be seen as the sum of two terms.

$$V_n = V_c + V_{CFRP} \tag{2}$$

where V_n is the shear strength of concrete (kip), as defined by (1), V_{CFRP} is the additional shear capacity supplied by CFRP composite (kip). The second term VCFRP can be calculated based on the mode of failure of the CFRP composites as external reinforcement, that is tensile fracture or bond slip of composite materials. In reality, it is essential to calculate the ultimate contribution of the CFRP composites because it may be the result of tensile fracture or bond failure of bonded composite. Therefore, these two failure mechanisms are considered in model development for RC beams externally strengthened CFRP composites in the form of continuously bonded sheet or strips.

A. Bond Failure Due to Crack Bridging

A relative displacement of the crack faces occurs both horizontally and vertically during the shear crack development. As a result, the external epoxy bonded composites that bridged the crack may result in a debonding failure of CFRP composites due to peeling action. In such a condition, the contribution of the external epoxy bonded composites is governed by the ultimate load capacity which transmitted between the sides of the shear crack. This contribution depends on three parameters: (1) shear crack opening, (b) bond conditions, and (3) anchorage type. Based on these three parameters, the contribution of the external epoxy bonded composites in shear is very difficult and complicated to perform. Therefore, a simplified approach has to be adopted as well as an extensive validation will be carried out.

In the analysis of the bond conditions, the interest is paying attention on the global plane stress flow in concrete at the external composites/concrete interface rather than the local plane stress state close to a single strip. To determine the shear capacity provided by CFRP composite, the angle of inclination of the cracks and the vertical projection of diagonal cracks were assumed to be equal to θ_c and d, respectively, and then, the horizontal projection of the diagonal cracks will be equal to $d \times \cot\theta_c$ as shown in Fig. 3.

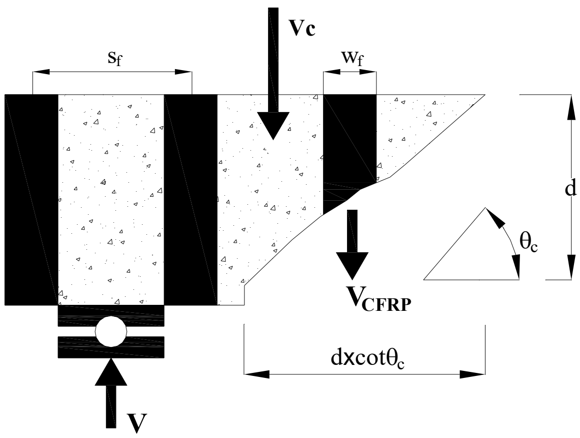

Fig. 3 Plot of evaluating bond action of CFRP composites

$$A_f = 2n_f w_f h_f = 2\frac{d\cot\theta}{s_f} w_f h_f \tag{3}$$

where A_f is the total bonded CFRP composites area, $n_f = d\cot\theta / s_f$ is the number of CFRP composites within the portion $d\cot\theta$ of the beam; h_f, w_f, and s_f are the CFRP composites height, width, and spacing, respectively. The total shear capacity provided by CFRP composite (V_{CFRP}) was defined as

$$V_{CFRP} = \frac{1}{2} A_f \tau_{bond} = 2\frac{d\cot\theta}{2s_f} w_f h_f \tau_{bond} = \frac{d\cot\theta}{s_f} w_f h_f \tau_{bond} \tag{4}$$

where τ_{bond} represents the bond strength between the CFRP composites and concrete as shown in Fig. 3. For this equation, the strength of the composites material and of composite adhesive interface is assumed to be stronger than that of the adhesive-concrete interface and concrete material. Thus, the bond strength τ_{bond} was defined as

$$\tau_{bond} = \beta\tau_{\max} \tag{5}$$

where $\tau_{\max}$ represents the maximum bond strength as shown in (6) which is proposed by Al-Rousan [7], and β is a reduction factor that accounts for the variations in the experimental work conditions and the length of the shear span as well as strengthening system, and it is equal to 0.75 for fully wrapped elements and 0.65 for beams with two or three sides bonded,

$$\tau_{\max} = 0.65\left(\frac{L}{b_f}\right)^{-0.3} \beta_w f_t \tag{6}$$

where L and b_f are the length and the width of the bonded CFRP sheet with concrete, f_t is the tensile strength of concrete which was related to the compressive strength of concrete ($f_c^{/}$) according to ACI 318-08 [1], and β_w and α are the CFRP to concrete width ratio factor and the parameter that controls the shape of the descending branch, and it is given by Lu et al. [8] as following

$$\beta_w = \sqrt{\frac{2.25 - b_f/b_c}{1.25 + b_f/b_c}} \tag{7}$$

where b_c is the width of the concrete specimen, and the angle of inclination of the cracks (θ_c) was assumed by Al-Rousan [7] as a function of shear span to depth ratio (a/d) as shown in Fig. 4, which can be expressed as

$$\theta_c = 60(a/d)^{-0.5} \tag{8}$$

Thus, the shear capacity provided by the CFRP composites due to debonding can be defined by the following equation

$$V_{CFRP} = 0.65\beta \frac{d\cot(60(a/d)^{-0.5})}{s_f} w_f h_f \left(\frac{h_f}{w_f}\right)^{-0.3} \beta_w f_t \tag{9}$$

B. Fracture of External Epoxy Bonded CFRP Composites

To avoid debonding failures, the external epoxy bonded CFRP composites could be properly anchored. In this case, the resistance mechanism of external epoxy bonded CFRP composites is in a similar manner to that of internal steel stirrups. Therefore, the shear failure is determined by fracture of the CFRP composites [5], [6]. As a result, the contribution of the external epoxy bonded CFRP composites (V_{CFRP}) in shear can be defined as

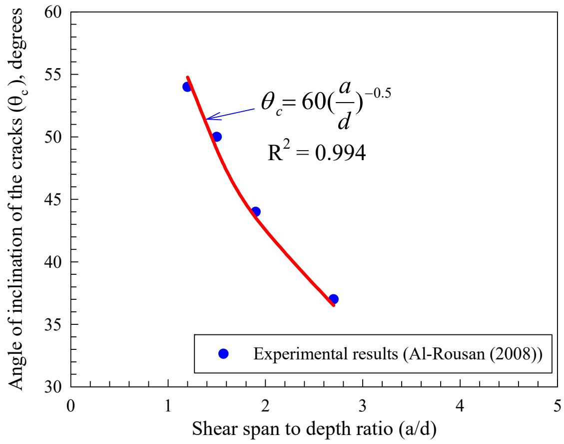

Fig. 4 Angle of inclination of the crack (□) vs. shear span to depth ratio (a/d)

$$V_{CFRP} = \frac{A_f f_{fe}}{s_f} \tag{10}$$

where $A_f = 2t_f w_f$ is the area of external epoxy bonded CFRP composites (for strips of thickness t_f on both sides of the beam), and $f_{fe} = \varepsilon_{fe} E_f$ is the effective strength, E_f being the CFRP composites modulus of elasticity and ε_{fe} an effective CFRP strain. The effective CFRP strain (ε_{fe}) is assumed to be smaller than the ultimate tensile strain of CFRP composites (ε_{fu}). Based on ACI committee 440 [9], the effective CFRP strain (ε_{fe}) can be computed as follows

$$\varepsilon_{fe} = 0.75\varepsilon_{fu} \tag{11}$$

Thus, the shear capacity provided by the CFRP composites due to the fracture of the CFRP composites can be defined by the following equation

$$V_{CFRP} = \frac{1.5 t_f w_f \varepsilon_{fu} E_f}{s_f} \tag{12}$$

Considering different CFRP composites modes of failure, the contribution of external epoxy bonded CFRP composites in shear is determined by the minimum value obtained from by the following equation

$$V_{CFRP} = \min(0.65\beta \frac{d\cot(60(a/d)^{-0.5})}{s_f} w_f h_f (\frac{h_f}{w_f})^{-0.3} \beta_w f_t ; \frac{1.5 t_f w_f \varepsilon_{fu} E_f}{s_f})$$

C. Some Recent Models

In recent years, Malek and Saadatmanesh [10], Triantafillou [11], Matthys [6], Khalifa and Nanni [5], and Colotti et al. [12] have carried out studies for predicting the ultimate shear capacity of beams strengthened with epoxy bonded steel reinforcement or FRP composites. These design guidelines are fundamentally based on the truss analogy. Consequently, ACI 318-08 [1] and EC2 Eurocode [13] are calculated the shear capacity of strengthened beams (V_u) as following

$$V_u = V_c + V_S + V_f \tag{13}$$

where V_c, V_s, and V_f are the shear capacity of the concrete, the internal steel stirrups contribution, and the contribution of the external shear reinforcement, respectively. The calculation of the concrete and stirrup contributions in models proposed in literature are based on the expressions provided by ACI 318-08 and EC2 Eurocode [13] for ordinary RC members. In this section, ACI Committee 440 [9], Triantafillou [11], and Colotti et al. [12] are selected in order to compare and critically examine the results obtained by the proposed model with those of the selected models in the light of the observed experimental results

III. Validation of Experimental Results and Recent Models

The first step in each theoretical modeling should be validity of the proposed model against a large number of experimental of the beams with a wide range of geometrical and mechanical characteristics. The proposed model has been applied to experimental results as well as the literature test results. For comparison purposes, the proposed model predicted values are compared with those of the ACI model, Triantafillou model, and Colotti model. It is cleared that in the ACI, Triantafillou, and Colotti models, the general design guideline is derived from the experimental data and they are only applicable to external FRP reinforcement. The results of the experimental validation are discussed below.

Table I shows a comparison of the results predicted by the four models $V_{f,exp}/V_{f,mod}$ (proposed), $V_{f,exp}/V_{f,ACI}$ (ACI), $V_{f,exp}/V_{f,Tri}$ (Triantafillou), and $V_{f,exp}/V_{f,Col.}$ (Colotti et al.). Note that the ACI and Triantafillou models, which were calibrated for CFRP, should be used with caution for the other types of composites as shown in Table I. The overall predictions by ACI and models also do not appear to be equally satisfactory with a mean $V_{f,exp}/V_{f,ACI}$ value of 2.12 and a COV of 96% for the ACI model and a mean $V_{f,exp}/V_{f,Col.}$ value of 0.58 and a COV of 50% for the Colotti model, while Triantafillou model had an acceptable range (0.27 to 2.78) and a mean of $V_{f,exp}/V_{f,Tri.}$ value of 1.22 for the ratio between the tested and predicted ultimate shear. The coefficient of variation (COV) of the beam test results is 42%. It is cleared that the proposed model shows the best agreement with the test results with an acceptable COV value. The Colotti model also shows a discrete agreement with the test data, but the model underestimated the experimental data. The ACI model, on the other hand, provides very conservative results.

Table I also shows that the proposed model offers the best output in terms of the mean and COV values. Based on the structural design point of view, Table I reflected that the proposed model has a higher COV of 38% (62% of the specimens agreed with the proposed model), but the value of 0.97 of the mean experimental/theoretical failure load ratio is reflected the excellent validation of the model. This higher value of COV is due to the wide range of investigated variables, test setup, types of composite materials, instrumentation used in collecting data, and type of tested beams.

TABLE I
COMPARISON OF RESULTS OBTAINED WITH DIFFERENT MODELS

	$\frac{V_{f,\exp}}{V_{f,\mathrm{mod}}}$	$\frac{V_{f,\exp}}{V_{f,ACI}}$	$\frac{V_{f,\exp}}{V_{f,Tri.}}$	$\frac{V_{f,\exp}}{V_{f,Col.}}$
Average	0.97	2.12	1.22	0.58
COV, %	38	96	42	50

IV. Conclusions

Based on the results of this investigation, the following observations can be drawn:

1. The ACI model had a wider range of 0.16 to 10.08 for the ratio between the tested and predicted ultimate shear at failure.
2. The Triantafillou model had an acceptable range of 0.27 to 2.78 for the ratio between the tested and predicted ultimate shear at failure.
3. The Colotti model had the best prediction for the ultimate shear capacity with a range of 0.20 to 1.78 for the ratio between the tested and predicted ones.
4. The proposed analytical model in this study can predict the contribution of the CFRP composites as external reinforcement with high accuracy.

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