

# Analysis of Vibration Signal of DC Motor Based on Hilbert-Huang Transform

Chun-Yao Lee and Hung-Chi Lin

**Abstract**—This paper presents a signal analysis process for improving energy completeness based on the Hilbert-Huang Transform (HHT). Firstly, the vibration signal of a DC Motor obtained by employing an accelerometer is the model used to analyze the signal. Secondly, the intrinsic mode functions (IMFs) and Hilbert spectrum of the decomposed signal are obtained by applying HHT. The results of the IMFs constituent and the original signal are compared and the process of energy loss is discussed. Finally, the differences between Wavelet Transform (WT) and HHT in analyzing the signal are compared. The simulated results reveal the analysis process based on HHT is advantageous for the enhancement of energy completeness.

**Keywords**—Hilbert-Huang transform, Hilbert spectrum, Wavelet transform, Wavelet spectrum, DC Motor.

## I. INTRODUCTION

MOTORS are common electronic machinery and their stability is crucial for proper factory operations. If a generated signal occurs in the rotary process of the machine, the unobserved fault can be prevented. Many studies have analyzed signals derived from vibrations. The Fast Fourier Transform (FFT), for example, can be applied to analyze vibration signals [1]-[3]. The main weaknesses of this approach are that the fixed basis leads to decreased precision and that the approach itself incorrectly identifies the exact time of fault. In recent years, both WT and HHT, which do not have the aforementioned FFT disadvantages have been applied to analyze vibration signals [4], [5], and have proved to be more effective. However, few studies have analyzed the energy completeness of these approaches. Therefore, this study has compared the energy completeness of both WT and HHT in the signal analyzing process based on vibration signal measurement.

## II. WAVELET TRANSFORM

WT mitigates the disadvantages of FFT, since the basis of FFT is sine and cosine, while the basis of WT can be adjusted to the type corresponding to the signal. The basis of WT, termed the mother wavelet meets the requirements of integral zero and energy limit, as shown in (1) and (2). The mother wavelet changes the scale parameter and the translation parameter, and then generates the basis function of WT, as shown in (3). The

changes in the translation parameter, and the changes in the scale parameter are two times scale and zero point five times scale. The basis function of WT and the arbitrary function as shown in (4) are defined as  $X(t) \in L^2(R)$ , [6].

$$\int_{-\infty}^{\infty} \psi(t) dt = 0 \quad (1)$$

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty \quad (2)$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right), \quad a, b \in R \quad (3)$$

$$X(\omega)(b, a) = \frac{1}{|a|} \int_{-\infty}^{\infty} X(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (4)$$

where

$\psi(t)$  : mother wavelet transform,

$a$  : scale parameter,

$b$  : translation parameter.

## III. HILBERT-HUANG TRANSFORM

HT refers to the Time-Frequency Analysis proposed by Huang. Unlike WT and FFT, HHT can be applied to analyze both non-stationary and nonlinear problems. Both the bases of WT and FFT are presupposed, and the basis functions are calculated using a dot product. It is difficult to present the signal characteristics entirely as this leads to great differences in amplitude. On the other hand, HHT without a presupposed basis can improve the weaknesses of both WT and FFT. HHT is composed of empirical mode decomposition (EMD) and the Hilbert Spectrum [5], [7], [8].

### A. Empirical Mode Decomposition

The paper has defined the decomposition signal as  $X(t)$  by virtue of EMD. The decomposition for extracting the IMFs from a signal  $X(t)$  involves the following steps:

Step 1) Search the maxima and minima of the signal  $X(t)$

Step 2) The upper and lower envelopes using the curve fitting are generated by maxima and minima.

Step 3) The mean values of the upper and lower envelopes of the signal  $m(t)$  are calculated as (5).

Step 4) Subtract  $h_{1k}$  from  $X(t)$  and judge if  $h_{1k}$  is the first IMF. If  $h_{1k}$  is the first IMF, the first IMF is outputted. Otherwise the calculations continue. This process is the so-called Sifting Process.

Step 5) Subtract IMF from  $X(t)$  and get  $r_j$ , the residue of the signal decomposition, as shown in (7).

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Step 6) Observe if the residue satisfies the conditions of monotonic function and constant. If the residue satisfies the conditions, the empirical mode decomposition is completed. Otherwise the calculations continue.

The signal,  $X(t)$ , is decomposed into the  $j$  IMF layer and a residue layer. Theoretically, the add-up IMFs and residue should be equal to signal  $X(t)$ , as shown in (8).

$$m(t) = (\maxs(x) + \mins(x)) / 2 \quad (5)$$

$$c_1 = h_{1k} \quad (6)$$

$$X(t) - h_{jk} = r_j(t) \quad (7)$$

$$X(t) = \sum_{j=1}^n c_j(t) + r_n(t) \quad (8)$$

where

$X(t)$  : original signal,

$m(t)$  : mean value,

$h_{1k}$  : IMF of first layer,

$c_j(t)$  : IMFs,

$r_n(t)$  : residue.

#### B. Hilbert-Huang Transform

The  $Z_j(t)$  is obtained by Hilbert Transform of  $c_j(t)$ , as shown in (9). Theoretically, analytic  $Z_j(t)$  is the sum of its real part,  $c_j(t)$ , and the imaginary part can be rewritten in a polar coordinate system, as shown in (10), in which  $a_j(t)$  and  $\theta_j(t)$  represent the instantaneous amplitude and phase respectively, as shown in (11) and (12). The instantaneous frequency,  $\omega_j(t)$ , derived from the instantaneous phase is shown in (13).

$$Z_j(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{c_j(\tau)}{t - \tau} d\tau \quad j = 1, 2, 3, \dots, n \quad (9)$$

$$S_j(t) = c_j(t) + iZ_j(t) = a_j(t)e^{i\theta_j(t)} \quad j = 1, 2, 3, \dots, n \quad (10)$$

$$a_j(t) = [X_j^2(t) + Z_j^2(t)]^{1/2} \quad (11)$$

$$\theta_j(t) = \tan^{-1} \frac{Z_j(t)}{X_j(t)} \quad (12)$$

$$\omega_j(t) = \frac{d\theta_j(t)}{dt} \quad j = 1, 2, 3, \dots, n \quad (13)$$

where

$Z_j(t)$  : imaginary part of  $c_j(t)$ ,

$a_j(t)$  : instantaneous amplitude,

$\theta_j(t)$  : instantaneous phase,

$\omega_j(t)$  : instantaneous frequency.

#### IV. MEASUREMENT AND ENERGY DISCUSSION OF DC MOTORS

The detection procedure for the vibration signal measurement does require downtime and hence did not affect the regular rotation of the motor. The hardware structure of the

DC motor in which the required equipment were utility, DC power supply, accelerometer sensor, data acquisition system, personal computer and tested object, and DC motor. In order to measure the vibration signal of the 24V DC motor, the accelerometer sensor was attached to the DC motor, and the A/D conversion was performed by employing the data acquisition system. Finally, the digital data was stored in the personal computer so as to execute HHT and WT analyses.

HHT was applied to analyze the vibration signal of the DC motor. First, EMD was employed to decompose the vibration signal of the DC motor, and then the IMFs were obtained. Second, the IMFs were transformed into the Hilbert spectrums through the Hilbert Transform, as shown in Fig. 1. Finally, the Hilbert spectrum of the IMFs was constituted to acquire the Hilbert spectrum of the vibration signal of the DC motor. The dB10 of WT was applied to transform the vibration signal of the DC motor into Wavelet spectrum. By comparing the applications of HHT and WT, signals showed a clearer frequency characteristic based on HHT acquiring not only a clearer spectrum distribution but mitigating the disadvantage of unclear frequency distribution of traditional WT.

The multiresolution reconstruction of WT increased the amplitude compared with the DC motor vibration signal. WT analyzed the signal by means of the mother wavelet function, nevertheless, the dissimilarity of the mother wavelet function resulted in an amplitude difference. The square difference of the analyzed result of the commonly-used mother wavelet function and the vibration signal of the DC motor were calculated to present the amplitude difference of the analyzing processes, as shown in Table 1. The amplitude difference of HHT was  $8.66 \times 10^{-6}$ , while the smallest amplitude difference of WT was dB6 ( $3.86 \times 10^{-3}$ ). In other words, no matter how the mother wavelet functions were alternated for analyzing vibration signal of the DC motor, the result revealed that the amplitude completeness of WT in the analyzing process was inferior to that of HHT. Therefore, the amplitude completeness of HHT in the analyzing signal was not only superior but also more applicable than that of WT.

#### V. CONCLUSION

This study adopted vibration signal measurements to execute on-line detection of DC motor vibration signal in order to compare the signal energy difference between WT and HHT. The results revealed that the WT analyzing process led to a blur phenomenon in the Wavelet spectrum and hence misled the judgment of the detectors. However, the clearness of the Hilbert spectrum based on HHT was superior to that of WT and the error of the decomposed signal energy constituents based on HHT was lower than that of WT. Therefore, concerning the time-frequency analysis of HHT, energy remains invariant in the process of analyzing vibration signals.

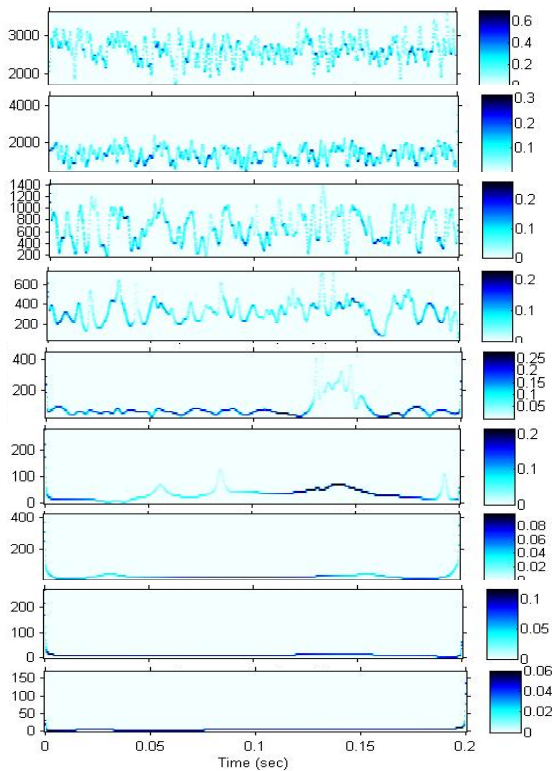


Fig. 1. Hilbert spectrum of each layer of intrinsic mode function.

#### REFERENCES

- [1] G. S. Maruthi, and K. P. Vittal "Electrical fault detection in three phase squirrel cage induction motor by vibration analysis using MEMS accelerometer," in *Proc. 2006 IEEE Power Electronics and Drives Systems conf.*, pp. 838-843.
- [2] H. Ocak, and K. A. Loparo "Estimation of the running speed and bearing defect frequencies of an induction motor from vibration data," in *Proc. 2004 Mechanical Systems and Signal*, pp. 515-533.
- [3] H. Zhang, K. Bradley, and P. Zanchetta "A None-intrusive Load and Efficiency Evaluation Method for In-Service Motors Using Vibration Tests with an Accelerometer," in *Proc. 2008 IEEE Industry Applications Society Annual Meeting conf.*, pp 1-6.
- [4] M. Kawada, and K. Yamada "Visualization of Vibration Phenomena on Model Turbine Rotor Using Cross-Correlation Method Based on In-Place Fast Haar Wavelet Transform," in *Proc. 2008 IEEE Power and Energy Society General Meeting conf.*, pp 1-6.
- [5] R. Yan, and R. X. Gao, Senior Member, " Hilbert-Huang Transform-Based Vibration Signal Analysis for Machine Health Monitoring," *IEEE Trans. Instrumentation and measurement*, vol. 55, pp. 2320-2329, Dec. 2006.
- [6] A. Roy, C.-H. Wen, J. F. Doherty, and J. D. Mathews, " Signal Feature Extraction From Microbarograph Observations Using the Hilbert-Huang Transform," *IEEE Trans. Geoscience and Remote Sensing*, vol. 46, no. 5, May 2008.
- [7] N. E. Huang and S. R. Long, "The ages of large amplitude coastal seiches on the Caribbean coast of Puerto Rico," *Journal Phys. Ocean.*, vol. 30, pp.2001-2012, Aug. 2000.
- [8] N. E. Huang, Z. Shen, and S. R. Long, "A new review of nonlinear water waves: The Hilbert spectrum," *Annu. Rev. Fluid Mech.*, vol. 31, pp. 417-457, Jan. 1999.

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