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An Output Oriented Super-Efficiency Model for Considering Time Lag Effect

Yanshuang Zhang, Byungho Jeong

***Abstract*—**There exists some time lag between the consumption of inputs and the production of outputs. This time lag effect should be considered in calculating efficiency of decision making units (DMU). Recently, a couple of DEA models were developed for considering time lag effect in efficiency evaluation of research activities. However, these models can't discriminate efficient DMUs because of the nature of basic DEA model in which efficiency scores are limited to '1'. This problem can be resolved a super-efficiency model. However, a super efficiency model sometimes causes infeasibility problem. This paper suggests an output oriented super-efficiency model for efficiency evaluation under the consideration of time lag effect. A case example using a long term research project is given to compare the suggested model with the MpO model.

***Keywords*—**DEA, Super-efficiency, Time Lag.

I. Introduction

DEA evaluates the relative performances of comparable units of an organization. DEA models use multiple input and output factors of DMUs in efficiency evaluation. Since the CCR model was developed by Charnes, Cooper and Rhodes [1], there are various extensions of the original CCR model [2]-[6]. In special, Banker, Charnes, and Cooper extended the CCR model to accommodate technologies that exhibit variable returns to scale [2]. In order to improve discrimination power among efficient DMUs based upon the CCR model, a super-efficiency DEA model in which a DMU under evaluation is excluded from the reference set was first suggested by Andersen and Petersen [7]. This super-efficiency model was developed under the assumption of constant returns to scale (CRS). The super-efficiency model under CRS does not suffer from the problem of infeasibility even though the super-efficiency model under variable returns to scale (VRS) can be infeasible.

A Generalized DEA model (GDEA) was developed by Yu, Wei and Brockett [8]. Post and Spronk proposed an interactive DEA procedure (IDEA) and introduced a combined DEA model that solves a single linear program to compute the efficiency values of all DMUs [9]. Özpeynirci and Kökslan [10] suggested a multi-period input (MpI) model to capture the time lag between the inputs and outputs in DEA. This model described a time lag model under the assumption that the inputs of multiple periods are consumed to produce the outputs of a period. Zhang and Jeong [11] also proposed a multi period output (MpO) model to consider the time lag effect in another viewpoint. This model was developed under assumption that the inputs of a period partially contribute the outputs of one or more subsequent periods from the input period. That is, these models are extended models to release a basic assumption of DEA models that inputs are converted to outputs in the same period. It is because this assumption may not be appropriate to some situations such as the performance evaluation of R&D, educational and marketing activity.

The results of MpO model classify DMUs into two sets of those that are efficient and constitute a Pareto frontier and of those that are inefficient. This model doesn't give any idea to compare performances of the efficient DMUs. Anderson and Petersen [7] suggested a super-efficiency model based on CCR model to obtain the discrimination among efficient DMUs. The approach enables an extreme efficient unit k to achieve an efficiency score greater than one by removing the k^{th} constraint limiting the score under 0-1 in the primal formulation.

Extension of MpO model is also available to get super-efficiency considering time lag effect. The purpose of this paper is to modify the MpO model in order that efficient DMUs can be discriminated. This paper suggests an input oriented super-efficiency model and output oriented super-efficiency model for considering time lag effect. That is, the suggested models allow efficient DMUs to have larger efficiency value than '1' and the models are based on the MpO model. Section II describes the suggested super-efficiency MpO models. A brief case example and discussions are given in Section III.

II. Super-Efficiency Multi-Period Output Model

The MpO model suggested by Zhang and Jeong [10] is to get efficiency values for considering time lag effect, based on the assumption that the inputs of a period can contribute to the outputs not only of the same period but also one or more subsequent periods of the period. That is, the MPO model assumes that the inputs of a period contribute partially to the outputs from the input period to PM periods after the input period.

The second constraint set of the MpO model restricts efficiency scores of all DMU not greater than '1' such as the CCR model. Thus, the MpO model can't discriminate efficient DMUs because all efficient DMUs have same value of '1'. That is, we can't compare the performance of efficient DMUs using the results of MpO model. To resolve this problem, this paper suggests a modified MpO model to get super-efficiency based on the super-efficiency model suggested by Andersen &

Y. S. Zhang is with the Chonbuk National University, Jeonju, Korea.
B. H. Jeong is with the Chonbuk National University, Jeonju, Korea (corresponding author; phone: +82-63-270-2329; fax: +82-63-270-2333; e-mail: jeong@ jbnu.ac.kr).

Peterson [7].

Let h_{jt} be the super-efficiency of the input used in the t^{th} period by DMU j. Let x_{ijt} denote the amount of the i^{th} input consumed by DMU j in the period t. Let v_{ijt} be the weight of the i^{th} input consumed by DMU j in the period t. Let $Y_{rj(t+p)}$ represent the amount of the r^{th} output produced by DMU j in the period t + p. Let u^{p}_{rjt} denote the weight of the r^{th} output produced by DMU j p periods after from the period t. Since there is a time lag of PM periods, the outputs corresponding to the input of the 1^{st} period are from the first period to the $(PM+1)^{th}$ period. The following model is an input oriented super-efficiency model for considering time lag effect.

[Model I]

$$Max \quad \sum_{t=1}^{T-PM}\sum_{j=1}^{n} h_{jt} = \sum_{t=1}^{T-PM}\sum_{j=1}^{n}\sum_{p=0}^{PM}\sum_{r=1}^{s} u^{p}_{rjt} Y_{rj(t+P)}$$

$$S.T.$$

$$\sum_{i=1}^{m} v_{ijt} X_{ijt} = 1, \; j = 1,...,n; t = 1,...,T-PM;$$

$$-\sum_{i=1}^{m} v_{ijt} X_{ikt} + \sum_{p=0}^{PM}\sum_{r=1}^{s} u^{p}_{rjt} Y_{rk(t+P)} \le 0, \; j = 1,...,n; k = 1,...,n \,\&\, k \ne j;$$

$$t = 1,...,T-PM;$$

$$u^{p}_{rjt}, v_{ijt} \ge 0, \forall p, r, i, j, t;$$

Model I is an integrated model to calculate super- efficiency scores of all DMUs for entire periods at once. The super-efficiency is obtained by maximizing the weighted outputs for periods from t to t+PM to reflect the time lag effect of the inputs of period t for t= 1, 2, .., T-PM. The constraint for each DMU under evaluation is removed from the second constraint set. Thus, some efficient DMUs can have efficiency value larger that '1'. That is, the Model I is a super-efficiency model for calculating super-efficiency for the input of DMU under consideration of time lag effect

However, the input oriented super-efficiency model can have an unbounded solution when data set has zero value in one of outputs. In the case that any DMU of the remaining DMUs has zero value in any output, the corresponding output weight can be increased without violating the second constraint set for the DMU with zero value. That is, super-efficiency of the DMU under evaluation is unbounded. The dual model of the super-efficiency model can be infeasible in this case. Thus, we can't get super-efficiency value in the case that zero values are included in output data.

This problem should be resolved to get super-efficiency values regardless of existence of zero values in output data. To do this, we suggest an output oriented super-efficiency model for considering time lag effect. We can obtain super-efficiency values using the suggested output oriented super-efficiency model. The following model II is the output oriented model for obtaining super-efficiency values of all DMUs under consideration of time lag effect.

The model II minimizes the weighted sum of inputs which contributes partially to the outputs produced at a specific period t by DMU j. That is, q^{*}_{jt} is a reciprocal value of the super-efficiency value. We can get super-efficiency value by calculating the reciprocal value of optimal value, q^{*}_{jt}, for the model II. That is, we can obtain super-efficiency values of all DMUs by using one of the input oriented or output oriented super-efficiency models.

[Model II]

$$Min \quad \sum_{t=1}^{T\text{-}PM}\sum_{j=1}^{n} q_{jt} = \sum_{t=1}^{T\text{-}PM}\sum_{j=1}^{n}\sum_{i=1}^{m} v_{ijt} X_{ijt}$$

$$S.T.$$

$$\sum_{t=1}^{T\text{-}PM}\sum_{j=1}^{n}\sum_{p=0}^{PM}\sum_{r=1}^{s} u^{p}_{rjt} Y_{rj(t+P)} = 1, \quad j = 1,...,n; t = 1,...,T\text{-}PM$$

$$-\sum_{i=1}^{m} v_{ijt} X_{ikt} + \sum_{p=0}^{PM}\sum_{r=1}^{s} u^{p}_{rjt} Y_{rk(t+P)} \le 0, \quad j = 1,...,n; k = 1,...,n; k \ne j;$$

$$t = 1,...,T\text{-}PM$$

$$u^{p}_{rjt}, v_{ijt} \ge 0, \forall p, r, i, j, t;$$

TABLE I
DATA OF A PROJECT IN RESEARCH CENTER SG09

Yr	Fund	Researcher			Papers		Patent	
	(m ₩)	Ph.D	MS	BS	SCI	Non SCI	Appl	Reg.
1	300	0	10	13	0	0	2	0
2	300	0	9	12	0	1	3	2
3	300	4	12	6	3	3	4	0
4	150	5	5	2	3	0	2	0
5	180	5	5	2	2	2	5	3
6	180	5	5	2	8	7	8	0
7	180	3	0	6	1	0	6	4
8	180	3	0	3	5	0	3	6
9	180	0	3	5	2	0	9	2
10	180	0	2	4	9	0	5	2

III. CASE EXAMPLE

The 21st century frontier R&D program is a long-term national R&D program of Korea, for selective and intensive development of strategic technologies to enhance national scientific competitiveness to the level of advanced countries. Currently, 24 research centers are being supported by this program since two project centers were launched in 1999. The number of subprojects supported by these 24 centers is totally 2,670 subprojects. However, the long term projects are required to consider the time lag effect in evaluating efficiency. The input and output data set of a center (SG09) among these 24 centers is used to show the implementation of the suggested model. The center SG09 has 18 subprojects which are supported for 10 years. For instance, Table I shows a sample data set for a subproject supported by the research center SG09. There are four input variables, research expenditure (million ₩), the number of researchers (Ph.D, MS, BS). The number of published papers (SCI journal and non SCI journal) and the number of applied and registered patents are considered as output factors of research activity. The data set, shown in Table I, of the center SG09 are used to get super-efficiency values using the suggested output oriented super-efficiency model. We used LINGO to run the model.

Tables II and III show the efficiency values and the super-efficiency values respectively for the inputs consumed in 8 years by each DMU. The super-efficiency values are obtained using the output oriented super-efficiency model. That is, the reciprocal of q^*_{jt} is a super-efficiency value for DMU j at period t. The inefficient DMUs get same efficiency scores with MpO model. Some super-efficiency values are large extremely, i.e., DMU A, F, K at period 1. The input oriented model gives unbounded solution in these cases. We can get ranks of all DMUs using super-efficiency. Table IV shows the ranking of DMUs by super-efficiency for each period.

TABLE II
EFFICIENCY SCORES OF 17 PROJECTS IN CENTER SG09 FOR EACH PERIOD

DMU	1	2	3	4	5	6	7	8
A	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
B	0.0000	0.0000	0.3205	1.0000	1.0000	1.0000	1.0000	1.0000
C	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6723
D	1.0000	1.0000	1.0000	0.8887	1.0000	1.0000	1.0000	1.0000
E	1.0000	1.0000	1.0000	0.5515	0.6380	1.0000	1.0000	0.8062
F	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
G	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
H	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7997
I	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
J	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
K	0.5125	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
L	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9213	0.8181
M	1.0000	1.0000	1.0000	0.7368	0.7368	0.8363	0.9458	0.9458
N	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7545	1.0000
O	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
P	1.0000	1.0000	0.9441	0.7221	0.6007	1.0000	0.5280	0.2877
Q	0.8338	1.0000	1.0000	1.0000	1.0000	0.5576	0.7336	0.7029

TABLE III
SUPER-EFFICIENCY SCORES OF 17 PROJECTS IN CENTER SG09 FOR EACH PERIOD

DMU	1	2	3	4	5	6	7	8
A	1547.988	1557.632	2.0358	3.0451	3.0145	2.3332	2645.503	17.976
B	0	0	0.3205	2.3332	2.3332	6.3987	1.9999	1.1538
C	2.1656	6.1146	5.9787	2.2044	2.1438	5.0296	3.1238	0.6723
D	1.9994	2.9986	3.1419	0.8887	2.8435	2.8201	3.4161	3.2288
E	1.4541	1.5217	1.5572	0.5515	0.638	2.0896	1.0344	0.8062
F	4166.667	2.1858	2.453	1.2221	1.3846	1.01	1.3734	1.7623
G	7.0782	7.0636	1.3997	1.7997	2.0414	2.8411	1.3984	1.5414
H	3.7965	4.0218	4.74	3.3107	3.3374	2.1486	1.4613	0.7997
I	8.4256	18.9682	15.9889	23.9737	5.9233	5.0468	13157.89	13157.89
J	0.5125	5.094	4.4439	4.9999	1.9893	2.2582	1.3332	4807.692
K	3225.807	3225.807	3225.807	1.2171	1.1103	1.0085	0.9213	0.8181
L	3.808	3.8579	1.5468	0.7368	0.7368	0.8363	0.9458	0.9458
M	1.0054	1.1536	1.3838	1.6658	1.9627	2.191	0.7545	1.1998
N	1.551	7.2787	10.4938	15.6199	3.9995	1.739	10416.67	1.3125
O	2.2939	1.6622	0.9441	0.7319	0.6008	1.3468	0.5283	0.2879
P	0.8338	1.2725	2.8007	1.0481	1.1533	0.5576	0.7336	0.7029
Q	3.6535	4.5879	3.7714	735.2941	917.4312	917.4312	1.2157	1.2869

TABLE IV
RANKING OF 17 PROJECTS IN CENTER SG09 BASED ON SUPER-EFFICIENCY SCORES FOR EACH PERIOD

DMU	1	2	3	4	5	6	7	8
A	3	2	11	6	5	7	3	3
B	17	17	17	7	7	2	6	10
C	10	6	4	8	8	4	5	16
D	11	11	8	14	6	6	4	4
E	13	14	12	17	16	11	12	13
F	1	12	10	11	12	14	9	5
G	5	5	14	9	9	5	8	6
H	7	9	5	5	4	10	7	14
I	4	3	2	2	2	3	1	1
J	16	7	6	4	10	8	10	2
K	2	1	1	12	14	15	14	12
L	6	10	13	15	15	16	13	11
M	14	16	15	10	11	9	15	9
N	12	4	3	3	3	12	2	7
O	9	13	16	16	17	13	17	17
P	15	15	9	13	13	17	16	15
Q	8	8	7	1	1	1	11	8

IV. CONCLUSION

This paper modified the MpO model to improve the discrimination power of efficient DMUs under consideration of time lag effect. The suggested model is an output oriented super-efficiency model. The model can get super-efficiency values without infeasible or unbounded problem. The suggested model was compared with the MpO model using the data set of long term projects.

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Yanshuang Zhang is a doctoral student of the department of industrial engineering, Jeonbuk National University. He earned his MS degree in the department (2011). He is interested in data envelopment analysis and multiattribute decision making.

Byungho Jeong is a professor of the department of industrial engineering, Jeonbuk National University**.** He earned his B.S. degree(1983) in dept. of industrial engineering, Hanyang university, Seoul, Korea, and M.S(1985) and Ph.D.(1988) degrees in dept. of industrial engineering, Korea Advanced Institute of Science and Technology, Seoul, Korea. His research area includes data envelopment analysis, multi-attribute decision making, analytic hierarchy process.