

# An Optimized Design of Non-uniform Filterbank

Ram Kumar Soni, Alok Jain, and Rajiv Saxena

**Abstract**—The tree structured approach of non-uniform filterbank (NUFB) is normally used in perfect reconstruction (PR). The PR is not always feasible due to certain limitations, *i.e.*, constraints in selecting design parameters, design complexity and some times output is severely affected by aliasing error if necessary and sufficient conditions of PR is not satisfied perfectly. Therefore, there has been generalized interest of researchers to go for near perfect reconstruction (NPR). In this proposed work, an optimized tree structure technique is used for the design of NPR non-uniform filterbank. Window functions of Blackman family are used to design the prototype FIR filter. A single variable linear optimization is used to minimize the amplitude distortion. The main feature of the proposed design is its simplicity with linear phase property.

**Keywords**—Tree structure, NUFB, QMF, NPR.

## I. INTRODUCTION

NON-UNIFORM filterbank (NUFB) find wide applications in many areas of signal processing such as image coding, sub-band coding, audio analysis and broadband array signal processing [1-3]. In these applications NUFB provides adjustable time-frequency resolution as per the requirement. Over the years, a number of design methods have been proposed by different authors [3-7]. However, among these methods only few provide linear phase (LP) property. The linear phase property is crucial in applications like image coding to avoid artifacts in the reconstructed image [1].

The tree-structure method is one of the technique to design integer decimated linear phase non-uniform filter bank. In this approach non-uniform filterbank is realized by using two channel maximally decimated quadrature mirror filterbank (QMF) as a parent structure as shown in Fig. 1(a) [8,9]. In first level of decomposition either lower or upper band of QMF filter is decomposed into another QMF bank. This decomposition results half the time and double the frequency resolution. In this way at every level, decomposition changes the time-frequency resolution and required decimation factors can be achieved at the final output. The decomposition process of three band non-uniform filterbank for (4,4,2)

decimation factors is shown in Fig. 1. The z-transform of analysis and synthesis filters are given by Eq. (1-3) [10,11].

$$H_0(z) = H_L(z)H_L(z^2), \quad G_0(z) = G_L(z)G_L(z^2) \quad (1)$$

$$H_1(z) = H_L(z)H_H(z^2), \quad G_1(z) = G_L(z)G_H(z^2) \quad (2)$$

$$H_2(z) = H_H(z), \quad G_2(z) = G_H(z) \quad (3)$$

Depending on the filterbank used in the design, it can be either perfect reconstruction (PR) or near perfect reconstruction (NPR) [8]. The non-uniform filterbank in PR condition is very complex. Even, if the parent structure and cascading filterbanks are PR, one cannot always build perfect reconstruction filterbank (PRFB) until unless the filter used in the design is to be ideal which is unrealizable. Also, in PR condition high stopband attenuation is difficult to achieve and some times, it is necessary to introduce scale factors and delay at proper places so that the complete system is perfect reconstructed [8,11,12]. Therefore, NPR is a practical and realizable approach, in which the PR condition is relaxed, at the cost of allowing aliasing error and amplitude distortion. In this work, NPR non-uniform filter bank is designed using an optimized tree structure technique. A linear optimization is used to minimize the amplitude distortion. Window functions of Blackman family are used in the design of the prototype FIR filter [13,14].

The impulse response of the prototype filter  $p(n)$  of length  $(N+1)$  is given by:

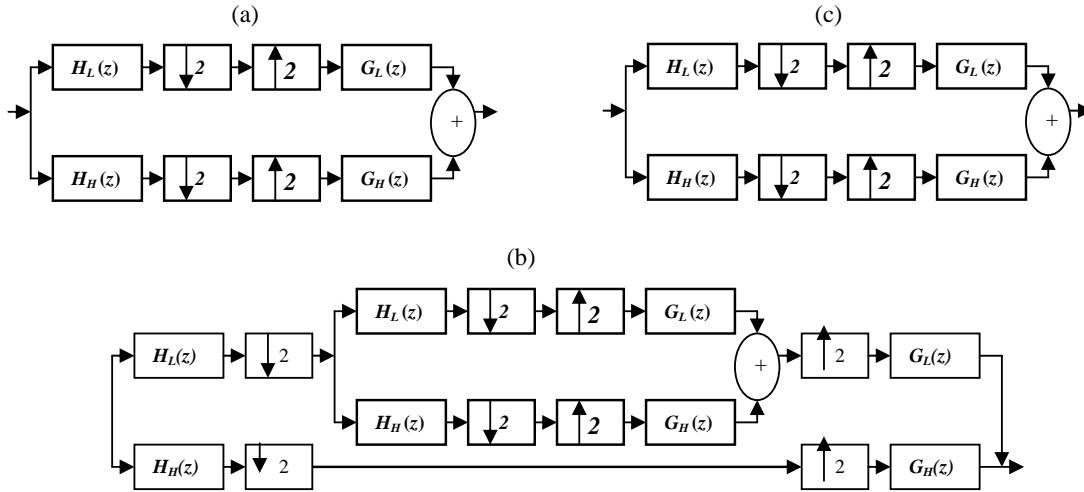
$$p(n) = h_i(n)w(n)$$

$$\text{where } h_i(n) = \frac{\sin(\omega_c(n - 0.5N))}{\pi(n - 0.5N)} \quad (4)$$

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**Fig. 1.** (a) A two-channel QMF bank. (b) A three channel QMF bank derived from the two-channel QMF.  
 (c) Three channel NUFB with decimation factor (4, 4, 2)

## II. OPTIMIZATION TECHNIQUE

The NPR QMF bank suffers from three types of distortions at the reconstructed output, *viz.*, aliasing, phase and amplitude. In NPR aliasing and phase errors can be eliminated by careful design of the analysis and synthesis FIR filter using a single prototype linear phase FIR filter. Amplitude distortion is not possible to eliminate completely, but can be minimized using optimization technique [10].

In this work, a linear iterative optimization of single parameter is presented. Filter length, passband frequency and stopband frequency are fixed before calling optimization. The reconstruction error given by (5) is selected as an objective function. As per the optimization routine the cutoff frequency of the filter is varied to achieve minimum value of amplitude distortion.

$$\phi = \max \left[ \left| \left( \sum_{n=0}^{M-1} |H_n(e^{j\omega})|^2 \right) - 1 \right| \right] \text{ for } 0 < \omega < \pi/M \quad (5)$$

where,  $M$  is number of bands in the non-uniform filter bank.

The summarized optimization routine is given below:

### START

- Choose the initial value of sampling frequency ( $F$ ), number of band ( $M$ ).
- Initialize the passband ( $\omega_p$ ) frequency, stopband ( $\omega_s$ ) frequency.
- Calculate cutoff frequency ( $\omega_c$ ) and Filter length ( $N+1$ ).
- Choose step size (step), search direction (dir), flag, minimum possible value of error (terror) and initial value of error (perror).

**Set the While loop with flag = 0**

- (i) Design Prototype low pass filter  $\{H_0(z)\}$  and complementary high pass filter  $\{H_1(z)\}$  using current value of cutoff frequency

- (ii) Determine the frequency response of the other filters ( $H_0(e^{j\omega}), H_1(e^{j\omega}), H_2(e^{j\omega})$ —) by using Eq. (1-3).

- (iii) Calculate the reconstruction error and objective function.

- Compare the current value of objective function with initial or previous value (perror) and minimum possible value (terror).

- If the current value greater than previous value or initial value, step size become half and changed the search direction (dir= -dir). Go to step (3).
- If the current value smaller or equals to the minimum possible absolute value (terror) than flag becomes set to '1'. Control comes out from the loop and go to step (v).
- If the current value equals to previous value, than flag set to '1' and control come out from the loop and go to step (v).

- Modified the value of cutoff frequency as ( $\omega_c = \omega_c + \text{dir} \times \text{step}$ ) and assign the current value of objective function as previous value and go to step (1).

### END OF THE LOOP

- Display the value of objective function as optimized value of reconstruction error.

### END

The above mentioned optimization routine is implemented on MATLAB 7.0 version on Pentium IV processor. The performance of the routine is given by the design examples.

### III. DESIGN EXAMPLES

In this section, two examples are illustrated and a comparison has been made with the earlier reported work. The same parameters are selected for the comparison as reported in [6,8,12].

**Example-1:** A 3-channel NUFB with decimation factors (4,4,2) has been designed using window functions of Blackman window family. The prototype filter has the specifications:  $N = 63$ ,  $\omega_p = 0.5\pi$ ,  $\omega_s = 0.563\pi$ . The obtained values of the different parameters are summarized in Table I. The magnitude responses of the prototype filter and analysis filterbank are shown in Fig. 2.

**Example 2:** A 4-channel NUFB is designed with (8,8,4,2) decimation factors. The prototype filter has the specifications:  $N = 57$ ,  $\omega_p = 0.5\pi$ ,  $\omega_s = 0.563\pi$ . The obtained values of the different parameters are summarized in Table II. The magnitude responses of the prototype filter and analysis filterbank are shown in Fig. 3. The magnitude responses of reconstruction error for different window functions are shown in Fig. 4 (a). Fig. 4 (b) shows the variation of reconstruction error vs. filter length.

### IV. DISCUSSIONS

It is clear from Table I that for the same decimation factor (4,4,2) the proposed technique provides smaller value of reconstruction error than the earlier publications [8,12]. Similarly, from Table II the results obtained for the 4-band NUFB indicates that the values of reconstruction error for all three window functions are smaller than the earlier reported work of Elias *et al* [6]. Moreover, the Blackman window based 3-channel NUFB provides the lowest value of reconstruction error in all window functions. At filter length of 63, the obtained value of reconstruction error is  $8.6 \times 10^{-4}$ . The variation of reconstruction error with respect to the filter length is of linear nature as shown in Fig. 4 (b).

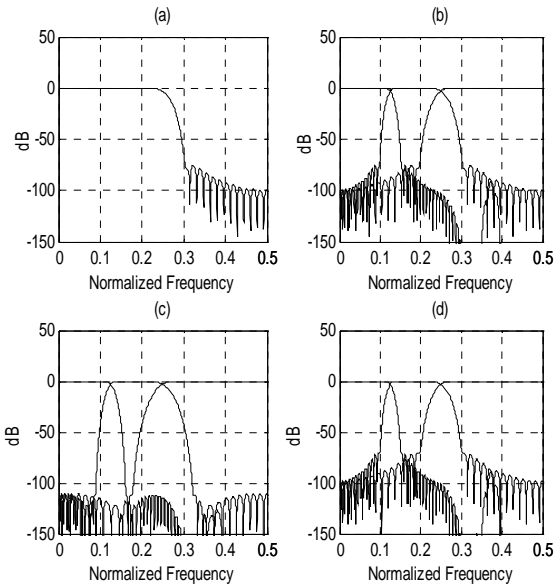


Fig. 2 Non-uniform filter bank for decimation factor (4, 4, 2). (a) Prototype filter (b) Blackman-window (c) Blackman Nuttall window (d) Blackman Harris 3 term

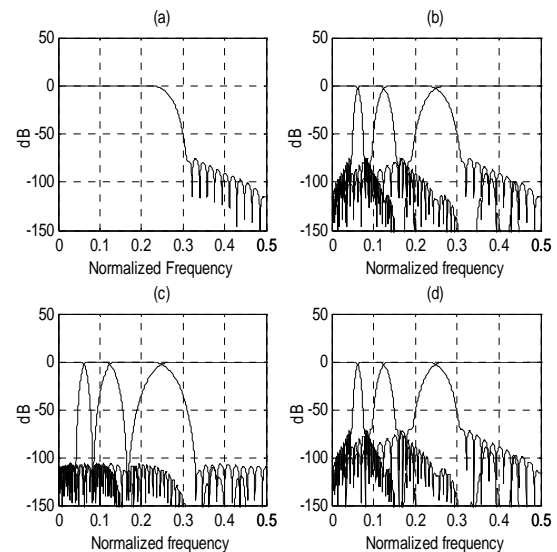


Fig. 3 Non-uniform filter bank for decimation factor (8 8 4 2). (a) Prototype filter (b) Blackman window (c) Blackman Nuttall window (d) Blackman Harris 3-term

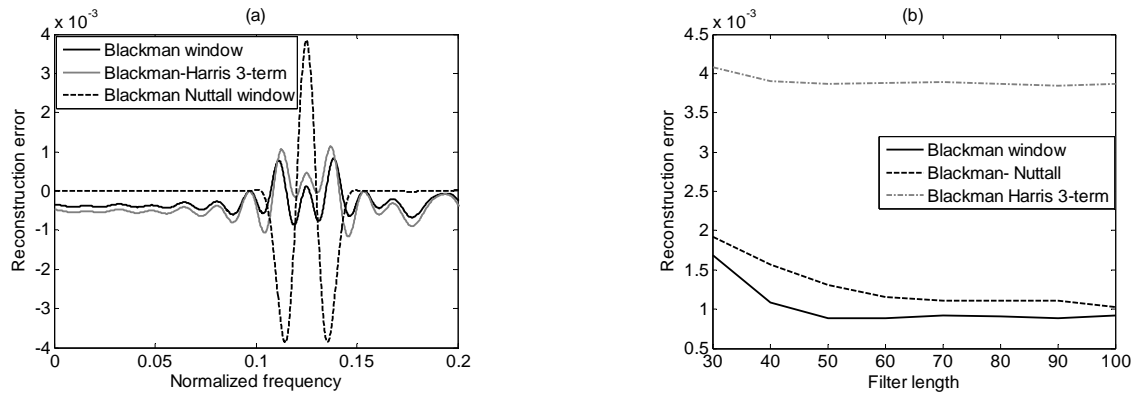


Fig. 4 (a) Magnitude response of reconstruction error (b) Plot of filter length vs. reconstruction error for used window functions

TABLE I  
PERFORMANCE COMPARISON WITH EARLIER REPORTED WORKS FOR THREE BAND NON-UNIFORM FILTERBANK (4,4,2)

	Band in Non-uniform filter bank	Technique used in design	Stopband Attenuation (dB)	Filter length	Reconstruction error
Reported work					
Xie <i>et al</i> [5]	Three band {4, 4, 2}	PM method	-110	63	$7.803 \times 10^{-3}$
Li <i>et al</i> [11]	Three band {4, 4, 2}	PM method	-60	64	$7.803 \times 10^{-3}$
<b>Proposed work</b>	Three band {4, 4, 2}	Optimized tree-structure			
		Blackman window	-75	63	$8.6 \times 10^{-4}$
		Blackman-Nuttall	-110	63	$3.85 \times 10^{-3}$
		Blackman Harris 3-terms	-70	63	$1.1 \times 10^{-3}$

TABLE II  
PERFORMANCE COMPARISON WITH EARLIER REPORTED WORKS FOR FOUR BAND NON-UNIFORM FILTERBANK (8,8,4,2)

Reported work	Four band	Tree structure method	-40	57	$4.6 \times 10^{-2}$
Elias <i>et al</i> [9]					
<b>Proposed work</b>	Four bands {8, 8, 4, 2}	Optimized tree-structure			
		Blackman window	-75	57	$9.1 \times 10^{-4}$
		Blackman-Nuttall	-110	57	$3.8 \times 10^{-3}$
		Blackman Harris 3-term	-70	57	$1.2 \times 10^{-3}$

## V. CONCLUSION

Efficient design for optimizing the prototype filter of NUFBs has been presented. Compared to the earlier reported work, the proposed method provides very small value of the reconstruction error with sufficiently high stopband attenuation. The proposed method can be used in applications like image coding and broadband signal processing where high stopband attenuation with negligible reconstruction error is desirable. The main advantage of this approach is its simplicity as only one parameter is to be optimized.

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